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## **A Case Study of the Professional Development of a Main Scale Teacher of Mathematics.**

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### **Abstract**

*I have attempted to provide an overview of the professional development of a former student through his involvement in a research project. This involved the introduction of thinking skills approaches into his teaching and the monitoring of his thoughts as to how this impacted on his teaching through keeping a teacher diary.*

### **Introduction**

Since joining the Education Department of Newcastle University five years ago, having been a 'proper teacher' up until that time, I am continually reminded of what a privilege it is to be in this post. Initially, I was delighted just to have my own office (a first) and easy access to a telephone and despite the down sides of the job (I will not list them!) the positive side continues to grow. So what are these privileges and how might they compare with your list? In approximate chronological order they are:

- Introducing keen committed graduates to the teaching profession;
- Visiting different schools to meet, and observe, students and mentors;
- Going to conferences to discuss issues with others in similar posts;
- Providing support to newly qualified teachers through INSET courses;
- Seeing past students act as excellent mentors and role models;
- Making the space to research, and read, about mathematics education;
- Being a part of past students' continuing professional development.

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It is this final point which I intend to develop here. The majority of our students gain posts within a 30 mile radius of Newcastle. This means that it has been relatively straightforward to keep in touch with ex-students. However my involvement in a three year teacher training agency (TT A) research project (one of four nationally funded school based research consortia) means that I have been able to work with some former students more closely than usual.

### **The research rationale**

One of the main aims of all four consortia, in keeping with many of the recommendations of the Hillage report (DfEE, 1998), is to explore how research processes and findings can enhance practice and help to raise standards. The North East school based research consortium (NESBRC) comprises six schools across three local education authorities, the focus for our study being the effective implementation of thinking skills. A wide range of curriculum areas have been involved, at different levels across the consortium, including art, English, geography, history, modern languages, RE, science and of course mathematics. The main vehicle for introducing thinking skills in mathematics has been the King's College designed Cognitive Acceleration through Mathematics Education (CAME) material (Adhami, Johnson & Shayer, 1998) - also known as 'Thinking Maths'. The aim of each of the thirty lessons is to develop pupils' reasoning and thinking skills. To achieve this the lessons are designed to encourage the discussion and justification of ideas both in small and large groups. The emphasis is not on the mathematical content of the lesson but the processes that the pupils go through in order to accommodate their findings. The programme demands that the role of the teacher change significantly from that of a more didactic lesson. (For a fuller overview of the CAME project the reader is directed to the very readable introduction to the teaching materials). However a number of generic thinking skills strategies (Taverner, in press) have also been adapted to facilitate an infusion approach in mathematics teaching. The mathematics teachers involved in the research consortium have been introduced to the CAME material through a combination of University and school-based sessions, some taking advantage of the opportunity to link it into Newcastle University's Master's programme. There have also been whole consortium '24 hour' (Friday evening to Saturday afternoon) INSET sessions at a local hotel. We have found this to be a very beneficial way of focusing on research and content issues away from the pressured school environment.

One of the outcomes of these 24 hour sessions has been the introduction of various collection and data analysis research techniques. The teachers expressed an interest in researching their practice and pupil outcomes in a number of ways including:

- Pupils complete a learning log after thinking skills lessons;
- Teachers keep a diary focusing on their perceptions of thinking skills lessons;
- Some lessons videoed in order to look more closely at different features and phases of the lesson.

### Teacher's professional development

The effectiveness of INSET provision be it a day or a series of sessions has recently been debated more widely. The DfEE consultation paper (Feb 2000) 'Professional development' poses about twenty five questions. They range from the relatively uninspiring "How can INSET days be used effectively and imaginatively in school?" - which in itself makes a number of assumptions, to the more exciting "Should experienced teachers be given a 'sabbatical' period away from the classroom for developmental activity and research?"

Colleagues here in Newcastle (Higgins and Leat, 1997) have been interested in what constitutes an effective model for teacher development for some time. It is clear that no one model will suit all teachers. Much will depend on 'who they are', 'where they are', and 'what they want to change'. While this may seem obvious, it is worth re-stating. Much teacher development input is not always suitably differentiated, built on prior knowledge and experience and presented to appeal to a range of teaching styles. Rather a 'one size fits all' approach could be said to exemplify many one-off INSET day inputs.

### Case Study

The main purpose of this paper then is to look at a particular mechanism for teacher development within the context of thinking skills in mathematics lessons. This model has also been used successfully in other disciplines. The focus is to discuss how the research tools mentioned above (pupil logs, teacher diaries and video), and the teacher diary in particular, have contributed, if at all, to the professional development of one of my former students whom I shall call John Cranston.

Although it is not the purpose of this paper to focus on the pupil logs they have provided a fascinating insight into pupils' perceptions of lessons and so for interest and completeness the questions in the logs are given below.

- What did you learn this lesson?
- What did your teacher do to help you?
- Could you transfer what you learned here to another situation?
- Any other comments?

I hope to report on the pupil logs later in the year.

John Cranston started to keep his diary as soon as he became involved in the consortium which was July 1998.

### Process v Content

Many entries reveal the tension between 'getting through the content' and focussing on the processes that go on in the lesson as is revealed by this early entry.

*"We did not get through as much work as 1 would have liked" (July 98)*

Several months on, the pressures of the national curriculum and its assessment remain dominant.

*"I am aware that the mathematical content is not as important as the understanding of thought processes. However it is hard to move away from the teaching of 'maths' to understanding when exams etc are looming on the horizon" (Jan 99)*

This concern about the rate in which material was covered was linked to other issues such as dissemination within his department. John doubted his ability to persuade colleagues of the worth of thinking skills (TS). The need for hard quantitative data dominated his thoughts in order to balance his belief that colleagues would be cynical of its potential impact.

*"1 don't think that 1 will be able to convince the doubters of the merits of TS with the evidence based solely upon the way my opinions have changed over time. " (Oct 98)*

This is, perhaps, a familiar scenario - the pressure of league tables placing an undue emphasis on short term results - and is beyond the scope of this paper.

### Role of Support

However in his diaries John Cranston has also usefully identified incidents which have moved him forward but has yet to consider how these may be operationalised in order to persuade other members of the department to trial lessons. An additional obstacle is that, even in this democratic department, he is a relatively junior member of staff.

A critical developmental episode is identified as when the school coordinator organised a 24 hour 'away day' to allow colleagues from different departments to discuss and compare progress. This proved to be particularly beneficial as John was becoming a little isolated within his department.

*"the weekend away was very useful ... I am more keen to teach and discuss TS" (Oct 98)*

There had also been opportunities to meet on a regular structured basis with other mathematicians. This is also recorded as being of help.

"Have attended a number of MEd sessions at the university now, I feel

that they are paying real dividends. "

but before I can get overly reassured about the quality of the input I provide he continues,

*"It is reassuring to discuss the CAME lesson with other interested maths teachers. " (Feb 99)*

### Reflection on practice

The diary also became a useful mechanism to record views on his own practice beyond the TS lessons.

*"The CAME lessons have really made me think about my practice - the questions I and the pupils ask, as well as the quality and importance of discussion in my lessons. " (Feb 99)*

It has not however been plain sailing, as with many attempts to change an a roach to teachinf and learnini' there are urs and downs aloni the

way. However the diary provides an opportunity to re-visit the journey and note how far one has travelled albeit sometimes by what feels a rather circuitous route.

*"The key to the lesson appeared to be the TRANSFER discussion at the end which did yield some pleasing results ... some very interesting comments from a weaker member of the group. I must improve this discussion next time." (Jan 99)*

However six months later there is no evidence of complacency but a continued awareness of the processes within the lesson.

*"I was not happy with the way that I managed discussions, the pupils were certainly not engaged by the lesson." (June 99)*

This is interesting in that he is taking all the responsibility upon himself for any short comings in the lesson rather than default to blaming the materials, the pupils, the weather etc. However reviewing the other data sources for the lesson to which the above extract refers is enlightening. The video, and pupil logs, reveal that John had reorganised the seating arrangements - showing how such changes can upset the ecology of the classroom. His diary entry goes on to record specific shortcomings rushing his contributions, cutting pupils off etc and an on-going tension between content and process outcomes. However, the day this lesson took place was also a non-uniform day - an event that can put extra pressure on anyone's class management skills.

### Other sources

It is not only self-initiated data that has helped in the professional development of John Cranston. Other useful research opportunities have been exploited through our links with the Educational Psychology (EP) course based at Newcastle

University. Intending EPs have to complete a collaborative, small scale action research project as part of their course and many express an interest in looking at classroom processes. Consequently, one pair looked at teacher questions and pupil response times in a CAME and a 'normal' mathematics lesson taught by John Cranston. Their report proved to be one of the 'critical incidents' that pushed John forward. He, rightly in my view, perceived himself to be an effective classroom teacher however an independent analysis of one lesson revealed the limited number of open questions posed. As he later records in his diary, when considering the type of questioning that went on his lesson,

*"I would like to think that I do encourage these things however I am sufficiently broad minded to accept statistical results to the contrary .. I was very proud of the single open question that one pupil asked!" (Oct 98)*

### Model for Professional Development?

It is clear, even from these brief extracts, that the journey to adding a different dimension to one's teaching repertoire is not a steady one. It is rather like riding a bike to work. Although at first you may feel a little wobbly, the novelty pushes you on to overcome initial fears. However, parts of the journey are up-hill and you feel not just like getting off and pushing but hitching a lift as well. On rainy days you may opt out altogether (this should be seen as common sense not as a failing). As your fitness levels improve, and you feel better in yourself, and perhaps receive passing compliments from colleagues then it seems more worthwhile. The hill which used to prove such a challenge becomes manageable. There will always be days when the weather is dreadful, you have too much to carry or you have a puncture but the confidence and fitness you have built up allows you to get right back on again the next day.

### Other benefits

Undoubtedly, involvement in such a research project can have benefits beyond the potential to become a more effective classroom teacher. John has recently gained a promoted post in another of our consortium schools, and coincidentally he has accumulated significant evidence to support any application to cross the threshold. This needs evidence in five categories which are summarised below:

- Knowledge and understanding - thorough knowledge of the teaching of mathematics including wider curriculum developments;
- Teaching and assessment - use a range of strategies, monitor progress against prior attainment;
- Pupil progress - equal or higher achievement to that of similar pupils nationally;

Wider professional effectiveness - take responsibility for own development and make active contribution to school policies and aspirations;

- Professional characteristics - challenge and support all pupils, take positive action to improve the quality of pupils' learning.

It is clear that involvement in this project admirably fulfils the opportunity for teachers to gather evidence to support their application to cross the threshold. However, any teacher interested in a more reflective approach to their teaching could be encouraged to keep the type of records which have been discussed here. Perhaps it is something, as teacher educators, that we could usefully encourage all beginning teachers to do - it could become a useful part of the induction process, acting as a more reflective lesson evaluation log. Maybe we, as teacher educators, could also benefit from such a practice ourselves.

### Benefits for the researcher

As a PGCE tutor, I feel that this research involvement has been of equal, if not more, professional development for me as it has for the other teachers involved. It has made me look carefully at offers to deliver oneoff INSET sessions in schools. Undoubtedly there is more impact in the type of professional development described above than having the whole staff addressed for a morning before they have to dash back to the reality of 'cutting and sticking' appropriate worksheets for that week's lessons.

### Final thoughts?

As for John Cranston, he soon takes up his new post as second in department at another NESBRC school, where thinking skills in mathematics is more firmly established. Many of the department are actively involved in researching their own practice. They use a range of research opportunities including teacher diaries and videoing lessons for later reflection. John recognises the role that keeping a teacher diary has played in recognising the progress he has made in his, already excellent, teaching and how it has helped him to identify the key incidents which moved him forward. He plans to continue the process ...

References

Adhami M, Johnson D & Shayer M (1998) *Thinking Maths The programme for accelerated learning in mathematics*, Heinemann

Department for Education and Employment (1998) *Excellence in Research on Schools*, London HMSO

Department for Education and Employment (2000) *Professional development support for teaching and learning* (refDfEE 0008/2000)

Higgins, Sand Leat D (1997) Horses for courses or courses for horses: what is effective teacher development? *British Journal of In-service Education*, 23, No.3

Taverner, S (in press) Challenging 11-14 year olds *Mathematics in School*

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## So you think counting is easy!

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### Introduction

Learning to count is not as easy as you might think. It is complex and takes place over a period of time and it is not a quickly acquired skill. Many writers have documented the problems and issues surrounding counting - see for example the work of Young-Loveridge (1999), Thompson (1997), Resnick (1983), Ginsberg (1983) or Fuson et al (1982). There are several sub-skills involved and total mastery of counting is generally not achieved quickly in a short period of time although many teachers of young children will know of particular children who have made rapid changes in their perceived achievement levels in counting. Deciding whether or not a child can be said to be able to count is also not a simple task and almost bound to be the subject of judgements that may not attract universal agreement from the people most closely involved with the particular child. Some may think that the child can be said to be able to count when others think they still need to acquire other sub-skills.

For example, consider the sub-skill of being able to count backwards. If we can only go up the numbers and not back down them again, can we say that we can count? If not, we might ask why it is such a common feature of many mathematics courses. In these, there is nearly always an exercise which asks the children to 'launch the space rocket' by counting backwards from 10 down to zero.

For trainee teachers, learning to teach children to count is also a complex and time-consuming process. Put alongside the need to learn about children's acquisition of number concepts whilst at the same time trying to put the theory into practice is demanding, especially in the first years of training.

"The research on pre-schoolers' mathematical competence shows that new-entrant teachers need to be aware of the rich informal knowledge of mathematics which children bring with them to school. Teachers can then organise the early mathematics curriculum so as to capitalise on that knowledge."

Young-Loveridge, 1987, p163

This then poses a problem for teacher trainers who are charged with trying to prepare new-entrant teachers to deal with all these issues.

### The Context

As a teacher educator from the UK, I recently arranged an exchange with a colleague doing a similar job in New Zealand. As part of my new role, I was teaching several groups of first year, undergraduate pre-service teachers prior to their going into NZ schools. The focus of their first school mathematics lesson was to be a one-to-one experience with a child of around five years of age who had only recently started school. Their challenge was to help the child in their learning about counting. My job was to prepare them to make the most of their time with the child. (For UK readers, it is important to be aware of this context and not to make assumptions about the levels of counting experiences of five year olds in the UK and in NZ as being totally equivalent.)

I was well acquainted with Freudenthal's dictum that "If you want to teach anyone anything in mathematics, first find out what they know". The pre-service teachers were being guided towards preparing for an exploratory interview which would enable them to do this and we needed to recognise the issues surrounding a young child's acquisition of the skill of counting.

## The Lesson

We went through Gelman and Gallistel's five principles of counting:

- the one-one principle;
- the stable order principle;
- the cardinal principle;
- the abstraction principle; and • the order irrelevance principle;

Gelman & Gallistel (1978) pp77-82

Illustrating each as we went. All seemed to be going well until we tried to explore what these meant from an adult point of view.

The students had been given a suggested interview schedule to follow

Which included such questions such as "What number comes after 51" and "What comes just before 7?". It was here that the first alarm bells began ringing for I recognised that most of the very young students, those who were not parents and therefore had no experience of their own children, seemed to think that these questions would be so easy that it was almost not worth asking them. However, they did think that perhaps the one that said, "Suppose you have four beans and then I give I give you 3 more beans. How many beans do you now have?" would be worth asking.

It struck me that they did not really appreciate just how difficult it is for young children to come to terms with all that counting involved and so I decided to put them into as near a similar position as I could.

### New number names in a stable order

Learning new number names in a stable order is neither a quick nor easily mastered skill and I really wanted to try and by-pass this particular stage. If I was to do this, I needed them to use some new names in a stable order that they already knew and could use fluently but one that wasn't the usual "one, two, three ... ". I decided to use the tonic sol-fah words from music so the new names in order were:

Doh re me fah sol lah tee

and we would use these for counting. Immediately, there were protests about there not being ten number names. By passing the detail that in fact we only have nine names and place keeper, I explained how ten was an arbitrary number that happened to match the number of our fingers. It is highly unlikely that had we evolved from octopuses or spiders that counting would have been in tens - it would almost certainly have been in eights.

Everyone was quickly able to repeat the words in the correct order and to demonstrate their one-one correspondance skills with objects up to tee. We could also agree on cardinality by saying that the word we ended with was the size of the set. For example, each of our hands has sol fingers. But what happens after tee?

Moving beyond single number words.

To make sense of this, we began by considering our traditional decimal numbers looking first at the numerals. After 9 is 10 which uses the first number symbol plus a place holder which we call zero. After this we have the first symbol (1) plus each of the original number symbols in order: 11, 12, 13, etc. Everyone could see the logic. But what about the words because this after all was what we were most interested in.

Ten is followed by some fairly peculiar words for the numbers which come between this and twenty. We decided to look a little further on than ten. What happens to numbers of higher values? As we come to a decade, we seem to add the suffix 'ty' as in six!y, seven~ eigh~ and nine!! . Looking a little closer, we can see that we do it also for 'fourty' although we spell it slightly differently as forty and again for 'fivety' which we spell as fifty. Why doesn't this work for the early numbers? Logically shouldn't we have one-ty, two-ty and three-ty? The answer of course is yes but the English language is not totally logical and some rather



idiosyncratic number names have evolved.

We agreed to ignore these peculiar words and decided to construct a totally logical number system by simply adopting the familiar suffix of 'ty' to indicate a new set of numbers. We could not call them decades since they are not sets of ten but at least we could now count beyond single word numbers.

Doh re me fah sol lah tee dohty

But now what? Looking back at our decimal numbers which take the new 'ty' suffix, we could see that each is followed by repeating the new 'ty' word plus each of the original single words in the same order:

Sixty (on its own), then sixty-one sixty-two sixty-three sixty-four etc. until we get to sixty-nine, then add 'ty' to the next single word and repeat the process. Seventy      seventy-one      seventy-two      etc.

Applying this to our new number names and stable order, counting becomes reasonably straight forward:

Doh / re / me / fah / sol / lah / tee / dohty

dohty-doh / dohty-re / dohty-me / dohty-fah / dohty-sol / dohty-lah / dohty-tee / etc

And with just a little more effort, we were able to go on further and construct the equivalent of our well known hundred square. However, and very importantly, we knew that the children would be working purely orally so we did not allow anyone to use a, written version.

The group was now becoming quite proficient at counting up in this peculiar system but there were more protests about it being unnecessarily difficult. I pointed out that we are fortunate to be working in English for even though it may have some peculiar words for numbers between 11 and 20. We should spare a thought for the French whose system is even more peculiar than ours! They too share our penchant for idiosyncratic words for the numbers in the teens but then appear to abandon logic totally when it comes to words for their decades. How does vingt relate to deux? Perhaps we can see something between trente and trois but saints preserve us from having to use four twenties (quatre-vingt) instead of eighty and why on earth do the nineties have to be four twenties plus the teen numbers again as in quatre vingt treize for 93? Any reader wanting a fuller discussion of this particular issue is recommended to read Muira et al (1988 and 1994) quoted in Thompson (2000, in press).

One of the students was originally Austrian and so grew up learning German as her mother tongue. She commented subsequently,

"I found that it was a bit like learning to speak another language, it was new and I was unfamiliar with it. And even though I could relate it to my past experience of learning to count in English (my mother tongue is German) I was still struggling to make sense of it and say it in the right order. "

Female, late teens

### **Moving on**

By now, we could say the words of doh-re-me up to some really big numbers. So what? Could we say that we can count? We returned to the interview schedule and tried a few of the suggested questions for 5 year olds but substituted the new words and sounds. "What comes immediately after fah?" A few, mostly those with musical backgrounds, could bring sol quickly to mind. For the rest, it was back to the new number names and all around the classroom we could see and hear people repeating the order, "doh, re, me, fah, sol ... " as they studiously tapped or uncurled one finger at a time. Next, "What comes just before tee?" Again the same process of tagging individual fingers.

### **What about the Cardinality principle?**

We counted sets of fingers on one hand (sol); and on both hands (dohtyre). We started asking questions around the room such as, "How many people at this table?" or "How many panes of glass in this window?" until finally, "How many people in this classroom?" (26). Almost everyone had to stand and point to each person one at a time whilst chanting the numbers in sequence - a very explicit example of the one-one principle. However, there was little agreement on a single answer with several different answers being offered. Why? We tried to unpick the problem by counting altogether, slowly and out loud. It transpired that many had not remembered that the 'decade' numbers such as sixty, seventy, etc. or in doh-re-me, dohty, rety, etc. needed to be said on their own before starting to add the single words again. In base ten we have to say twenty before we can start again using twenty one, twenty two, ... etc. Do children do the same? Eventually we agreed on mety-re and, since we had already agreed that the last number we said was the size of the set, we had confirmed our cardinality principle again.

### What about arithmetic?

Trying questions of arithmetic was quite difficult as we found when we tried the question about beans. 'Suppose you have fah beans and then I give you me more beans. How many beans do you now have?' Oh dear - chaos reigned! Almost every single student reverted to using fingers and, it seemed, had to do the counting out loud! First on one hand, "Doh, re, me, fah". This was retained on one hand with the appropriate number of fingers (4) standing up. Then the other hand, "Doh, re, me" and they had three fingers standing. They knew instantly that it was 7 in traditional decimal counting but the answer needed to be in doh-re-me numbers so what did they do? They reverted to the count-all strategy and even though it was not necessary, almost every single person re-set their fingers to show 7 as  $5 + 2$  and then returned to the stable order, one-one and cardinality principles. "Doh, re, me, fah, sol, lah, tee" Choruses of "Tee" rang out followed almost immediately by peels of laughter as they watched each other trying to do it. We repeated this with similar questions such as, "What is sol more than rety-fah?" and after a little while, they were all convinced that they could now count in doh-re-me.

At this point, I threw in a googly (a cricketing term for a ball that spins the opposite way to normal). "What is dohty more than sol?" The vast majority began the same process of finger counting, converting to decimal numbers, finding the answer then translating back into doh-re-me to eventually get the right answer. I tried another version of the same question, "What is dohty more than rety-sol?" This was seen as getting harder except that a couple of students were able to get the answer very quickly. "It's mety-sol" They were assailed by their peers who want to know how they did it so quickly. "It's easy - it's like adding ten isn't it! 10 more than 17 is 27 and 10 more than 34 is 44. Dohty is like our 10 and so you just change the 'ty' word to the next one!" Daylight dawned and we had several examples where they all wanted to confirm their newly acquired skill of being able to add dohty to any number.

By now, everyone was ready to move on. We could do that and we could see why children find it hard. But what about our interview questions? Didn't we also ask them to count backwards from 5 or 10 or even from 17? Could our doh-re-me counters do the same? Counting back from dohty was easy for the musicians again but not so easy for the rest. And what about from rety?

The next interview question was, 'Can you count in twos for me?' with the expected answer, "2, 4, 6, 8, ... " This was thought to be really easy until they tried it in doh-re-me. Everyone was back to fingers and saying the whole sequence very quietly to themselves like a mantra but saying every other number name out loud so that *it appeared* as if they could count in res.

### Adding two numbers

On the blackboard, I put some of the initial strategies that children use when adding two single digit numbers such as  $5 + 3$ :

- count- all;
- count on from the first; and
- count on from the larger.

In case these are not familiar to the reader, I have outlined them below.

Count-all: first count out each of the numbers, usually with concrete materials such blocks or toys or fingers. Next combine them into a single coherent group then count them all. For example, asked to add 3 and 4, the child counts out 3 fingers, then 4 fingers and finally counts all of them to get 7.

Count-on from the first: the child who can do this now knows about the cardinality principle and recognises that the first set ends at that count number and so simply continues the sequence from that point. For example, asked to add 3 and 5, the child starts with the 3 and, usually holding up a set of 5 fingers before continuing the count and says, "four, five, six, seven, eight" taking care to keep the one-one principle in mind as she counts on to eight.

And finally,

Count-on from the larger: This is a more sophisticated version of the previous strategy. Here the child is intuitively aware of the commutativity of addition ( $3+5=5+3$ ) and recognises the efficiency of starting with the larger number first regardless of whether it is given as the first or second number. So  $3 + 5$  becomes something along the lines of: starts with the 5 in her head then holds up three fingers and says, "six, seven, eight". For a fuller and more in-depth look at this issue, see Thompson 1999.

Knowing which is larger is not something we can take for granted and I needed my students to appreciate this. I threw very quickly at a student, "Which is larger fah or sol?" Most were unable to answer this without first saying the names in sequence until they reached one of the given numbers. They tried to test each other with similar questions and my point was made very graphically.

#### **After the school visit.**

Once we were all back in the university after their first interview lesson with their particular five year old, I asked them to write down how it had gone. Many, though I must admit not all, were ready to say how much more meaningful they found their child's answers. They said things such as:

"During an exercise in class where "Doh Reh Me ..." was used to replace 1,2,3 ... I began to understand how difficult it is for a young child to fully understand and use and name numbers. Before this experience I hadn't given much thought to the subject and had assumed that children picked up numbers and naming numbers easily through every day life."

(female, early twenties)

"After (learning) Doh Reh Me ... I quickly realised that learning how to count numbers is rather difficult."

(male, late teens)

"The exercises with counting in Doh Reh Me were helpful as an indication of how hard it is for the child to learn to count. I found it particularly hard to count in this way, as I had to constantly convert the Doh Reh Me's to numbers to follow their sequence,"

(female, early twenties)

And for me, the most revealing of these answers ....

"I have learnt that I must have patience and give adequate waiting time

for an answer, instead of jumping in and trying to help the child when they are still thinking."

(female, late teens)

#### **Data results and interpretations**

The students ( $n=83$ ) were asked to complete a questionnaire related to their experiences. A return rate of 60% meant an opportunity sample of 50. Their responses are analysed below.

Asked how useful they found learning in to count in doh-re-me, an overwhelming majority (88%) said they found it useful or very useful. Of the remainder, only one reported that they found it 'a real waste of time'. A follow up question revealed that 66% of them thought it helped them to relate to how they thought a child of five would feel. Another 44% mentioned that they found it hard and 14 % mentioned personal memories of learning to count themselves. It should be noted that the categories for this follow up question were not mutually exclusive and so the same person may have made multiple answers.

They were asked if they could recognise any of the counting principles in their interview with their 5 year old interviewee and perhaps the most interesting thing to emerge was not their naming of one of the Gelman-Gallistel principles but the fact that almost half (48%) either couldn't give one or described something different such as 'Couldn't say what was just before 5' or '... she was unable to bridge past 10'. This may indicate that these pre-service teachers themselves had not yet got a firm grasp of the principles and how to recognise them.

They were then asked the same questions as those in their schedule for interviewing a 5 year old except that the questions had been translated to Doh-re-me. Where they asked the 5 year old, '*What comes after 4?*', they were asked, 'What comes after fah?' 92% were able to answer this correctly. However, when they were asked, 'What comes before metyfah?' (*What comes before 28?*), the success rate fell to 66%. For those who got it wrong, the most common error was to give mety-soh as the answer. This is the number immediately AFTER mety-fah suggesting that it is probably a mis-reading of the question, a common error amongst children!

The next question was the one they originally thought might be worth while. '*You have 5 beans and I give you 7 more. How many beans do you now have?*' Translated, this became 'You have sol beans and I give you tee more. How many beans do you now have?' And only 44% could get this right with dohty-fah. Looking at why so many had got it wrong, the most frequent incorrect answer was dohty-sol which is one too many and may be explained by the previously noted error of forgetting to include the decade number on its own.

Very simple arithmetic caused some real problems. For the child we asked that they, '*Add together 19 and 10*' but for the students this became, 'Add together rety-me and dohty'. The keen eyed reader will have noted that this is not a direct translation since dohty is not 10 but by asking the students to add 8 in base 8, the coherence of the question was preserved. Only 30% could get this correct and there was very little coherence amongst the other answers except to note that 42% didn't offer any answer at all.

The hardest question we asked the 5 year olds was '*What is 31 and 36?*' This was only to be asked to any child who was clearly capable of succeeding and who had already shown that they knew what was involved by correctly answering previous questions. For the students, this was translated into 'What is mety-me + fahty-fah?' This proved to be step too far for most students as only an minority (26%) offered any sort of response and only 1 (2%) got the answer correct. Two digit addition before they were comfortably confident with one digit work follows much previous research findings about young children moved too quickly onto two digit calculations. The answer given here was dohtydohty-me and ought probably to have had a new name for the third place digit to match the decimal 'hundred'. Since this was never discussed or suggested by any of the students, the addition of another dohty was accepted as correct.

The last two questions were equally poorly answered and it is probably reasonable to assume that fatigue had set in by now. 12% could correctly answer, 'What is tee fewer than dohty-re?' (*What is 7 fewer than 12?*) with 88% not offering any sort of answer. Finally they were asked if they could count backwards from rety in their heads. 66% said they couldn't and only 4% said they could.

### **Making sense of the experience**

The findings of this small experiment are neither earth shattering nor particularly scientifically based. The sample was only an opportunity sample and there was very little rigor in the design of the experiment. The descriptions and analyses of the data are intended as an illustration of just how something as apparently simple as counting is, once you get under the skin of it, a complex and easily misunderstood skill. For these particular trainee teachers, the experience appears to have been educative and useful. Their responses were, I believe, genuinely honest and the exercise may well be worthy of consideration by others.

Teacher training is already under the spotlight in many countries and no doubt will increasingly be so in the future. This will almost certainly mean that whatever experiences are included in the training programmes, they will be monitored and scrutinised for superficial relevance and practicality. I believe this particular activity and all those similar to it such as the Alphabetland exercise in England's National Numeracy Strategy training

materials (DfEE 1999) will pass such scrutiny.

In my experience, our bright eyed and enthusiastic entrants to the profession are desperate to learn the essential skills so that they can get into classrooms and can start interacting with young children. We not only need to give them the tools with which to do this effectively but also the understanding of what goes on when the young are coming to grips with a whole new world of school in general and mathematics in particular. Learning to count is not as easy it first appears!

## References

DfEE (1999) The National Numeracy Strategy, Three-day course: Notes for tutors (pp58), London

Fuson K. C., Richards J and Briars D.J. (1982) *The acquisition and elaboration of the number word sequence*. In C.J.Brainerd (Ed.) *Children's Logical and Mathematical Cognition: Progress in Cognitive Development Research*. New York. Springer-Verlag

Gelman, R. and Gallistel C.R. (1978) *The Child's Understanding of Number*, Harvard Press, Cambridge, Ma. USA.

Ginsberg H.P., (1983) (Ed.), *The development of mathematical understanding* Academic Press, New York

Miura, L.T., Kim, C., Chang, C. & Okamoto, Y. (1988) Effects of language characteristics on children's cognitive representation of number: cross-national comparisons, *Child Development*, 59, pp. 1445-1450.

Miura, L.T., Okamoto, Y., Kim, C., Chang, C., Steere, M. & Fayol, M. (1994) Comparisons of children's cognitive representation of number:

China, France, Japan, Korea, Sweden and the United States, *International Journal of Behavioral Development*, 17, pp. 401-411.

Resnick L., (1983) A developmental theory of number understanding. In H.P.Ginsberg (Ed.), *The development of mathematical understanding* (pp 1 09-151) Academic Press, New York.

Thompson I. (1997) (Ed.) *Teaching and learning early number*. Open University Press, Milton Keynes, UK

Thompson, I. (1999) Mental Calculation Strategies for Addition and Subtraction: Part 1 *Mathematics in School*, November 1999 pp 2-4

Thompson, I. (2000) (in press) Implications of Research on Mental Calculation for the Teaching of Place Value *Curriculum*

Young-Loveridge J.M. (1987), Learning Mathematics, *British Journal of Developmental Psychology* 5, 155-167.

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# The Influence of Mathematics Teachers on Student Teachers of Secondary Mathematics

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## Abstract

*This study describes the nature of written guidance provided by mathematics teachers to a cohort of student teachers of mathematics and identifies the nature and influence of such guidance on the student teachers. The findings suggest that the mathematics teachers were generally advising student teachers most frequently about aspects of class management. Other aspects of mathematics teachers' craft knowledge were shared, but nested within a framework of class management. The aspects of guidance offered within this framework were generally narrowly focused on the traditional mathematics teaching craft skills of explanation, examples and exercises. There was some attempt to exhort student teachers to use a wide range of pupil activities, but little specific guidance was made available.*

## Introduction

This is part of an ongoing research programme (Smith, 1996a). The underlying value judgement is that an improvement in student teaching of secondary mathematics will be achieved by increasing the variety of appropriate learning experiences that pupils are to engage upon.

The intention of this study is to examine the nature and extent of the major influences on student teachers of secondary mathematics with regard to their selection of teaching approach, teaching materials and pupil activities. A range of sources such as the mathematics teacher, school-based mentor, Higher Education (H.E.) subject tutor, other student teachers, professional journals, and other literature generally provides such guidance. In some cases, the mathematics teacher may also be a school-based mentor; I have distinguished between these roles where appropriate.

The influence of the mathematics teacher is actively sought by Initial Teacher Education partnerships in the UK, sometimes being characterised as a conscious attempt to assist student teachers to gain access to the mathematics teachers' craft knowledge (e.g. McIntyre et al, 1994).

Some thinking and research on these issues appears to incorporate several assumptions.

- That all teachers' craft knowledge is of equal value
- That, taken as a whole, craft knowledge forms a harmonious body of knowledge- in-action (Schon, 1991, p.49)
- That craft knowledge is revealed in a standard way to the trainee.

Whilst being carefully diplomatic, the view that all teachers' craft knowledge is of equal value is clearly not the case, depending as it must on the individual teacher's experience, communication skills, personal qualities and their relationship with the student teacher. This is not to pass judgement on the quality of advice from teachers, or their intentions, merely to question the *equality* of value of that advice in terms of its usefulness to any particular student teacher.

The second view of coherence in craft knowledge is questionable, since while there may be some generally agreed principles in education, there are very many remaining controversies. An example being the ongoing debate between those who believe in setting and those who believe in mixed ability teaching of mathematics.

The third view, that craft knowledge is revealed in a standard way to the trainee is readily contradicted, for example a variety of approaches was found in the Oxford Intern Scheme (itself one scheme among many);

*" .... it was clear that subject teachers worked with interns in many different ways. The extent of classroom support and guidance varied and subject teachers described many different approaches .... "(McIntyre and*

*Hagger, 1996, p. 94.)*

There are considerable difficulties for student teachers in attempting to access teachers' professional knowledge. This is due to the intricate and contested nature of mathematics education, conflicting advice from different sources, the difficulty of learning from observation of skilled practice, the ambiguity of professional language such as "practical work", "exposition", "challenge" etc. and the well-documented difficulty that many teachers have in articulating their craft knowledge, (see Tomlinson, (1995, p.33) for a more detailed discussion). The professional craft knowledge itself is likely to be highly complex and provide few simple generalisations about how to do anything well in teaching, (Brown et al, 1993, p.113).

Apart from a few studies, (e.g. at Primary level, D. McNamara, 1994, pp. 107-122) investigations have paid little attention to the

nature of such advice or the extent to which the guidance provided by mathematics teachers actually impacts upon student teacher" beliefs and actions in the classroom.

This study has therefore attempted to

Identify the nature of guidance provided by mathematics teachers, exploring it for coherence, contrasts, ambiguities and tensions .

- Determine the relative influence of the mathematics teachers on individual student teachers.

with regard to the teaching approaches and pupil activities adopted by the student teacher for secondary mathematics.

It is believed that the study will contribute to the ongoing debate about the nature of the roles of H.E. and partnership schools in the initial education of teachers and perhaps to greater understanding of professional learning in general.

### The Context

The research was undertaken with an entire cohort of student teachers of secondary mathematics on a one year postgraduate certificate course based at Sheffield Hallam University for one third of the course and at its Partnership Schools for the other two thirds. The sample is therefore non - random, but may well relate to other cohorts in similar situations, i.e. other partnerships following the UK's nationally predominant "Higher Education led" model, rather than being collaborative or separatist, (see Whiting, 1996, pp. 15-17).

The university works in partnership with over 100 local schools to provide a 36-week academic year teacher education programme that is two thirds school based and one third university based. The pattern of school time and university time over the 36 weeks is approximately as indicated below:

Week Number	Monday	Tuesday	Wednesday	Thursday	Friday
1 and 2	University	University	University	University	University
3	School	School	School	School	School
4 to 7	University	University	University	School	School
8	University	University	University	University	University
9 to 18	School	School	School	School	School
19 to 20	University	University	University	University	University
21 to 24	University	University	University	School	School
25 to 34	School	School	School	School	School
35	University	University	University	University	University
36	School	School	University	University	University

This complex pattern has been devised by the partnership of university and schools to meet the various requirements of student teachers, the Teacher Training Agency, governmental circulars and the university, etc. and varies from year to year. The student teachers have a broad range of ages, backgrounds and experiences, some are direct from their first degree and others have worked in a range of industries.

### Evidential base

In order to develop opportunities for triangulation, evidence relating to mathematics teachers' influences was collected in three varying ways;

1. Documentary analysis of lesson observation feedback forms which are used to provide student teachers with written guidance on their teaching.
2. Questionnaire survey completed by the mathematics teachers and school-based mentors who were offering the student teachers advice.
3. Interviews in which the student teachers' lesson plans were used to stimulate recall of planning decisions regarding the selection of pupil activities.

### Written guidance

It is important to consider the possibility that the majority of advice regarding teaching approaches and pupil activities



is verbal and in advance of the session, rather than in writing after the event. However, it is the written record which student teachers take away and reflect upon in the medium and longer term. For this reason a documentary analysis was undertaken of written advice given to the cohort of 12 one year Post Graduate Certificate in Education secondary student teachers of mathematics. The available documents were carbon copies of mathematics teacher and school-based mentor comments on observed lessons (10 per student teacher) and copies of end-of-practice summaries (2 per student teacher). The focus of attention in the analysis of these documents was on advice relating to the choice of teaching approach and pupil activity.

Since 1997 teachers have used a proforma to structure their written feedback on lessons. The spaces on the proforma are headed:

- Knowledge and Understanding
- Planning Teaching and Class Management
- Assessment, recording, reporting and accountability.

There is a small amount of training in the use of this pro forma as part of the general induction to mentorship (i.e. 1 day). The focus of attention in the analysis of the proforma documents was on advice relating to the choice of teaching approach and pupil activity. This tended to come under the heading of "Planning Teaching and Class Management".

In these documents there was little evidence of mathematics teachers suggesting alternative pupil activities or teaching approaches. By far the most frequently occurring advice given was in regard to carrying out the chosen approach more efficiently. In particular, 79% of the documents comment on the student teachers' exposition, examples and selection of pupil practice materials (i.e. exercises). A good example of this type of comment being;

*Mathematics teacher: "Explains well, questioning the pupils to push them on. Explains well on the board getting the class to think. Content and level are suitable for class; enough to challenge them, but many good explanations were 'wasted' due to the class not fully listening. " with Year 9 set 2 out of 4.*

One third of the students' block practices passed with written comments confined solely to exposition, examples and exercises. There were some exceptions to this general finding, for example:

*School-based mentor: "I would like her to consider being more adventurous in her approach and to try to provide more variety. With our long (75 minutes) lessons there is often the need to change direction or introduce new ideas in order to lift the lesson. In her active tutorial lessons she has planned a variety of activities, but less so in Maths lessons. "*

As can be seen, this is a comment urging' the student teacher to use a variety of pupil activities but it does not suggest particular activities. In 50% of the documents there was some comment on pupil activities, but the focus was nearly always on the general quantity of the activities rather than their quality, specific nature or possible alternatives. This is the case even when activities have not worked well, for example:

*Mathematics teacher: "I observed a variety of activities during which the student teachers were actively engaged. One of the activities (Cuisenaire rods) didn't seem to work very well and I wasn't sure if it was seen as useful by the students." Y12 GCSE 'res it' Maths class*

Because the aims of this project involve trying to widen the range of teaching approaches used by student teachers, my judgement was that these documents were very disappointing. They did not provide student teachers with advice on alternative approaches or activities to adopt in the classroom. Indeed, the main focus was on improving a relatively narrow range of teaching skills; i.e. a didactic approach of explanations, examples and exercises. This concurred with many, but not all, of my own experiences of working in school with mentors and student teachers. The profusion of comments about improving explanations, examples and routine practice exercises might have been encouraging student teachers to perceive the craft knowledge of mathematics teaching to lie only in these particular aspects of the work. It was also suspected that some mathematics teachers did not offer alternatives because they perceive mathematics teaching to solely consist of "the Ex's", i.e. explanation, examples and exercises. Whilst the vast majority of lesson reports focused largely on the 3 Ex's I judged it to be of concern that one third of the written reports focused exclusively on these aspects.

However, as mentioned previously, it is speculated that comments on the selection of teaching approaches and pupil activities might more likely to be verbal rather than written. Providing tangential support for this view, in a study of professional conversations involving student teachers Hilary Constable and Jerry Norton noted that conversations with school staff were most frequently about lesson evaluation, preparation and planning, (Reid et al, 1994, p.128). The practicalities, timing and mechanisms in place would make such comments more likely to be made in advance of the

teaching, rather than after the event. If this were the case it would provide at least a partial explanation for the general lack of comments on the choice of pupil activity.

### Teacher Survey Questionnaire

Each of the student teachers in the cohort worked with a small number of classes in their placement schools during the first half of the year. The mathematics teachers who normally took these classes were asked in a questionnaire to identify the range of advice that they had provided to the student teachers. The total number of mathematics teachers invited to participate was 61. The total number of respondents was 26. Thirteen of these respondents had received training as school-based mentors. A provisional summary of the findings was circulated to respondents and further comment invited, so as to obtain some respondent validation of the findings.

The views of trained school-based mentors and other mathematics teachers did appear to diverge on a number of issues. In comparison with other the teachers, school-based mentors seemed to be

- More strongly inclined to identify student teachers' strengths.
- More likely to offer student teachers challenges.
- More directly involved in the assessment of student teachers.
- More likely to see themselves as 'guides to the school'.
- More likely to see themselves in a coaching role with the student teacher.
- More strongly identified with a reflective practitioner model of development.
- More likely to attempt to exemplify good practice.
- More involved in the planning of the student teachers' learning experiences.

These differences could be thought surprising because the very limited time involved in school-based mentor training cannot be expected to make a great impact in comparison to the long term experiences of a professional career. However, it is possible that school-based mentors are different from other mathematics teachers in the sense of generally having *opted into* the teacher-training role and in tending to be more senior members of staff and having had more experience of actually mentoring student teachers.

However, no firm conclusions are drawn from this small-scale observation of qualitative responses and little corroborative evidence could be established from the other perspectives within this study.

As might be expected, all of the mathematics teachers advised the student teachers on the title of the mathematical topics to be taught and advised the student teacher on the resources available to them in the department. The vast majority also gave time allocations for topics. All claimed to be involved in checking that lesson planning had been satisfactorily completed. Usually this involved the student teacher planning a rough outline of the lesson, checking with the mathematics teacher and then planning in more detail. There were variations on this approach, for example

*"I ask the student to show me his plans. I don't specify how detailed they should be. In practice if he is less confident about something, the plan appears first in outline. "*

Sometimes only certain lesson plans were examined by the mathematics teacher because the particular topic was believed to be difficult. Sometimes lesson plans were all checked until the mathematics teacher became confident in the student teacher's planning abilities. With weaker student teachers this meant a continuing check right through the practice. About half of the mathematics teachers said they exemplified their lesson planning processes for the benefit of the student teacher, others stated that it depended upon the success or otherwise of the student teachers' planning.

Mathematics teachers were asked to identify the most important advice that they gave to student teachers about planning for teaching. The most frequent response (28%) to this related to keeping pupils on task, e.g.

*"Ensure all pupils will be occupied with relevant work for the whole lesson"*

*"Have plenty in reserve"*

*"Pupils may work faster than expected. "*

This could be seen to have benefits both as a control mechanism and as a strategy to ensure efficient learning in the time available. Another category of frequently occurring advice (20%) was with regard to the pupils' ability and planning for differentiation. Student teachers were also advised to "know your topic" and to "look at a variety of resources". Only one comment of

*"Clear explanation with examples must be given to pupils. "* explicitly echoed the repeated concerns expressed by mathematics teachers in the writing of reports after observing a student's teaching.

A major effort is made in the university part of the course to help student teachers to develop a well-informed awareness of a range of teaching strategies and mathematical learning activities along with discussion of the relative merits of each. When mathematics teachers were asked if they advised the student teacher on the selection of suitable pupil activities, the response was somewhat mixed. Some mathematics teachers claimed to offer advice, others not and some claimed to outline a variety of options and left the selection to the student teacher. Others claimed to give advice in response to their perceptions of the student teachers' needs and one mathematics teacher to the stage of development:

*"When I was a student I wanted to find all kinds of activities myself and try using them. I didn't want to be told to use specific resources. My current student is the opposite and so I encourage him to look elsewhere, think about ways of approaching the topic. I think he lacks confidence in this area and so I am suggesting more ideas than I (had) expected to. As his practice goes on I would like to move from saying, 'How about trying. ...' to discussing a variety of possible approaches which he has suggested."*

Specific advice claimed included simple exhortations to include a variety of activities:

*"Mixture of activities bearing in mind the ability / behaviour / attitude of the classes."*

*"Try to offer a range of learning opportunities, reading, listening, practical, IT, etc."*

*"Use a variety of teaching styles, aural work, writing, whole class, individualised."*

*"Try to use a variety of activities in order to maintain pupils interest."*

*"Pupils need a range of activities."*

Some claimed advice about activities was more specific, such as:

*"Timing is all important. A lesson should not end with an activity simply petering out."*

*"With regard to pupil activities, try the activity yourself before giving to pupils."*

*"Look at the needs of certain pupils, e.g. disabilities (physical and reading for example)."*

*"Ensure activities are suitable for all or can be adapted."*

*"Use ... activities where pupils are working in groups or pairs and where they are 'discovering' maths rather than being told."*

Some proffered advice could be rather too brief to be helpful without further elaboration, such as;

*"A lesson should have a beginning, middle and an end";*

This brief statement of advice may be interpreted in a variety of ways, from being a simple statement of the obvious to a more complex statement about structuring lessons into three individual phases each having their own characteristics and consequential demands on the teacher. If nothing else, it is an illustration of the difficulty that some teachers have in communicating their general teaching craft knowledge. One mathematics teacher's responses seemed to indicate that a variety of approaches and activities are unnecessary and perhaps undesirable:

*"Demonstrate the method of solution of maths problems relevant to the ability of the group."*

The underlining was the mathematics teacher's own. This advice might suggest to student teachers that there is a single method of solution for each type of mathematical problem, that the mathematics teacher knows this method, the student teacher ought to know it too, and the only effective way to teach is the transmission model.

Mathematics teachers were also asked which documents they encouraged the student teacher to refer to when they are planning a lesson, with the following responses.

Documents	Percentage of mathematics teachers
Departmental Scheme of work	91%
Pupil text books	57%
National Curriculum	52%
A "wide variety" is recommended	39%

Teachers' guides	22%
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The departmental scheme of work was sometimes described as itself drawing on a wide range of materials, e.g.

*"Departmental scheme of work which includes all the others. " Mathematics teachers clearly felt that they had useful advice to offer student teachers about the approach to adopt with their particular classes. "Each group has a different character and your style should reflect this. "*

The suggestions were generally made with an eye to maintaining class management, but there was much direct support for experimentation and encouraging the student teacher to use a variety of approaches and activities. Sometimes this was with the implication that the student

needed to "see which works for him/her" and that perhaps, once this was found, further experimentation might not be needed. A mathematics

teacher comment that would fairly represent the whole was~

*"Advice could include (the) suitability of certain groups to undertake practical/group activities. General advice on (the) suitability of groups to certain situations. "*

About half of the respondents also suggested that the student teacher would need to be more formal with the classes than they themselves would be, an example of this being:

*"As I know the class and the class know what I expect, I can adopt a less formal approach on occasions. "*

Interestingly, about one third of respondents claimed that the approach that they advised student teachers to adopt was the same as the one that they would use themselves. This raises a number of questions

Is any allowance made for the student teachers' inexperience?

Does the mathematics teacher believe that his or her own particular classroom approach should be emulated?

Is the suggested approach the only one that the mathematics teacher can imagine?

In comparison, one mathematics teacher wrote:

*"Greatly - I can think on my feet, change activities to suit the class - I could not do this when I was on teaching practice. "*

It was considered that the receptivity of the student teacher to the mathematics teacher's proffered advice might well be influenced by the nature of their relationship with that person. Mathematics teachers and school-based mentors were asked to select descriptions of the role relationships that seemed to fit their way of working with student teachers. The role descriptions employed key metaphors frequently found in the literature (e.g. Calderhead and Shorrock, 1997, pp. 198-204), such as an apprenticeship model, listening friend, guide, fixer, coach, etc. Mathematics teachers were asked to respond to these descriptors using a five point Likert scale (strongly agree, agree, neutral, disagree and strongly disagree).

The most consistently favoured descriptor of the role relationship (11 strongly agree, 15 agree, 0 neutral, 0 disagree and 0 strongly disagree) was: *"Your role is to identify strengths and encourage the student. "*

The statement showing the most variability (4 strongly agree, 5 agree, 12 neutral, 4 disagree and 1 strongly disagree) was: *"Your role is to identify strengths and to challenge the student. "*

There was a slight balance overall in favour of the statement. Taken together, these statements may indicate hesitancy on the part of mathematics teachers to take up this notion of challenge. Yet there are suggestions in the research literature (e.g. Elliot and Calderhead, 1993, p. 184 in McIntyre D et al, 1994 and Daloz, 1986, pp. 212-215) that challenge, particularly when combined with support, is essential for professional growth. This hesitancy is further illustrated in the general consensus of agreement (9 strongly agree, 14 agree, 1 neutral, 2 disagree and 0 strongly disagree) with *"Your role is to diagnose weaknesses and support the student. "* and the general disagreement (1 strongly agree, 2 agree, 7 neutral, 10 disagree and 6 strongly disagree) with: *"Your role is to diagnose weaknesses and to challenge the student. "*

Clearly the notion of challenge is meeting with different responses from mathematics teachers and there is some modification to this response in particular circumstances. A focus of interest in this study is whether student teachers are being challenged to produce a wide range of teaching approaches and pupil activities. However, there is evidence here that some mathematics teachers are being hesitant about setting any challenges of any nature. This may reflect a lack of shared professional understanding and agreement about the nature of 'challenge', or the concept of challenge being perceived as somewhat threatening. There is an indication here that as a profession, teaching may need to be more clear on the twin notions of challenge and support.

## Student Teacher Interviews

The purpose of the research interview was to investigate how the student teachers actually arrive at a selection of teaching material to use in the classroom, i.e. in the cognitive processes that student teachers use in the 'pre-active stage of teaching' (Brown and Borko, 1992, p.212). It was also intended to begin to answer some further questions that had arisen during the study

Do student teachers really choose between alternative pupil activities, or do some feel that they have no choice?

No choice because they cannot think of any alternatives?

No choice because they feel constrained (possibly mistakenly) by the expectations of the staff?

No choice because they never think to look for an alternative?

Are their choices really made a long time ago, rather than in the stages of planning a lesson?

In other words to try to find out what influences are at work in the process of making the selection and how these influences finally result in some selection occurring. A concurrent task was to identify ways in which the student teachers themselves describe and classify mathematical learning activities that they set as tasks for pupils.

These were semi-structured interviews constructed and piloted following guidance suggested in Drever, 1995, Chapter 3. The student teachers were asked to use their teaching practice file to promote a "stimulated recall" of the planning process. So as not to distort the data to fit pre-determined categories, the classifications of pupil activity were allowed to emerge from the descriptions given by the student teachers at the data analysis stage. It was explained that there was no attempt to assess the plans in any way, but that the research interest lay in the decision - making process that led up to the plans being finalised.

The particular plans that were discussed related to two key points in the teaching practice. The first was right at the start of the practice where it might be expected that the student teachers would be most likely to have looked to others for guidance. The second point was immediately after the Christmas break, where the student teachers were most likely to have had to plan with minimal support from others because both school and university were closed for the vacation. In order to get a range of responses student teachers were asked about their planning of the first lesson for each of their classes at these two key times, but in practice there was surprisingly little to distinguish the two sets of results. With hindsight, this may have been due to an unforeseen similarity between the two times of the year, in that these were both times when the student teachers were particularly concerned to establish firm control in preparation for the weeks ahead.

### Findings from the student teacher interviews

It is appropriate to bear in mind the limitations of the interview process here; asking students to recall their lesson planning decisions is not likely to be 100% accurate, even with the document to remind them. After all, recalling the *process* is not the same as recognising *the product*. Students' attempts to recall the process of planning and decision making may well result in imperfect perceptions and impressions rather than actualities. Attempting to recall such decisions for the benefit of a tutor may well be different from recalling for the benefit of a more disinterested party.

However, on a straightforward count of identified influences, the major sources of pupil activities in order of importance were:

the student teacher's own ideas (57%)  
ideas developed from the pupil textbook (20%)  
mathematics departmental staff / school-based mentor (9%)  
university sessions (8%)

5. others, i.e. peers, reference texts, in-service courses (6%) I shall now elaborate each of these influences in turn.

During pilot interviews, student teachers had claimed that the vast majority of teaching ideas were their own. When conducting the main interviews I felt it important to press the interviewees on this to obtain more detail. Where the interviewee claimed that a teaching idea was their own I asked them if they could allocate any aspect of the idea to their own learning experiences, to their views on the nature of mathematics, to an adapted idea from a text book, or to any other source. However, even after doing this, the vast majority of the residual pupil activities still remained claimed as the student teacher's own, independent of external advice. Given the emphasis allocated in university sessions to the discussion of example activities, resources and sources of ideas, it is felt unlikely (although possible) that any of the student teachers would have believed that they were expected to devise the majority of their own pupil tasks.

McNamara discovered in a related study at primary level that

*"Much of the information and advice which student teachers make use of in their practical teaching probably becomes 'absorbed' by them and taken on board or accepted as part of their working knowledge which, at a later time, they no longer specifically identify as being provided by a nominated source." (McNamara, 1994, p.115).*

I found this to be the case at secondary level too. Sometimes a student teacher would lay claim to an activity as their own, or could not identify the source of the idea when it had definitely been presented to them in a college session (i.e. I presented it to the group myself and have checked the student's attendance record).

The second level of frequency of influence was that of pupil texts. In the interviews student teachers gave the impression of seeing pupil texts as being a 'safe option'. This appeared to be due to a variety of factors including

A pupil text provides student teachers with a template consisting of on track to the content, the content itself and practice exercises with which to build the lesson.

Pupils' attention is directed towards the text and therefore away from the student teacher, releasing some of the psychological pressures of facing a class.

Pupils are often familiar with a text - based approach.

the mathematics teachers often advised a text - based approach

Often the student teachers had learned much of their own mathematics through the use of texts, had been relatively successful in doing this and in consequence were likely to value a text - based approach.

Student teachers were sometimes unsure of the mathematics content and relied heavily upon the textbook as an authoritative source.

The departmental, mathematics teacher, school-based mentor and pupil influences have been grouped together and form the third most frequently mentioned set of influences. Perhaps surprisingly, the influence of pupils was not often mentioned directly but more by implication in the consideration of management aspects of the planning. The student teachers clearly placed a strong emphasis on trying to plan lessons for which they imagined pupil management to be most feasible. For example one student teacher mentioned that when planning 100 minute long lessons for a Year 10 Set 6 out of 8 class:

*"Pupils constrained what you could do, it had to be a control oriented approach. I needed to have alternative approaches available in case something did not work. "*

This emphasis on management was reinforced by messages received from the school staff, e.g. *"Get them in and get them sat down and working through textbooks"* was the mathematics teacher's advice quoted by a student teacher working with Y10, Set 4/5. Similarly *"Keep them busy and keep expositions short"* was the advice quoted by a student teacher for Year 8 Set 5 out of 5.

Most student teachers claimed to have been given a free choice of methods to use, provided that the content from the departmental scheme of work was covered. This was the case even where departments had very detailed and apparently directive schemes of work.

There were some exceptions, for example a mathematics teacher who was apparently concerned for his class to be kept up with a parallel set in the other halfyear and directed the student teacher's pace and methods to achieve this goal.

Several student teachers commented that they had consciously modelled their teaching approaches on role models that they had selected from the school staff. One student teacher commented that the mathematics teacher always patterned her lessons with a variety of activities; the student teacher perceived this to be a key factor in a successful approach and therefore modelled her teaching on this example. Another student teacher claimed to have abandoned his own predominantly worksheet based approach when he was impressed by a mathematics teacher using a highly interactive approach with a jointly shared class. Little direct use was made of particular pupil activities that had been suggested in college based sessions. This may in part be due to

There was not a close match between the teaching required of the student teachers and the activities that had been suggested in college.

It is intended that the pupil activities suggested in college might act as exemplars of the kinds of activities that are possible and which have good potential, rather than being intended to provide full curricular coverage, (e.g. Smith, 1996b).

The suggested pupil activities did not match the student teacher's vision of how they imagined working in the classroom and in consequence the activities were rejected without trial.

Some of the activities presented in college may have required greater experience; confidence with the mathematics and confident class management skills than the student teachers felt that they possessed at the time.

One student teacher carried out a college-suggested activity as his first ever lesson with his Year 10 Set 2 (out of 5) class. The activity was to introduce Loci through using the pupils themselves to act out the roles of points obeying rules; e.g. asking pupils to stand in the room obeying the rule *"Stand so that you are 1m away from the wall"*.

*In hindsight, this was a risky thing to do first lesson, but it was what I wanted to do and I wouldn't change it now. "*

Others tended to avoid taking chances at these key times of the year i.e. start of the practice and start of the term, e.g.

*"I looked for fairly safe lessons at these particular times, but felt more imaginative at other times. "*

## Conclusions

With regard to the influence of the mathematics teachers and schoolbased mentors, it is clearly not possible to generalise from

this sample of 26 mathematics teachers to all the mathematics teachers in our partnership schools, and certainly not to all mathematics teachers. It is quite possible that the respondents have self-selected themselves and are atypical, perhaps with a greater interest than others in initial teacher training, or perhaps being more able or willing to express their views. This appears to be a typical difficulty in researching teacher thinking

"... much *if* not all, of our work is based on the teacher thinking of articulate teachers. The silent voice of the ordinary teacher is not illuminated in the stories we tell." (Pope, in Day et al, 1993, p.28).

However, one can speculate that the more communicative teacher is likely to be more willing to give advice and more able to influence the student teachers' classroom approach and choice of pupil activity. The evidence accumulated from the three techniques used in this study suggests that mathematics teachers are motivated to advise student teachers most frequently about aspects of class management. Other aspects of mathematics teachers' craft knowledge are also shared, but are mostly nested in a framework of class management. The aspects of guidance offered within this structure are generally focused on explanation, examples and exercises. There is some attempt to exhort student teachers to use a range of activities, but little guidance is offered about specific activities.

The emphasis upon class management is no surprise, (see Haggerty, 1995, pp.-79 for a comparable finding) but the extent to which it appears to dominate and even exclude other advice was unexpected. With the benefit of hindsight it may have been the case that the university needed to offer explicit guidance to mentors and class teachers on the need to offer advice over a wider number of issues. (With the arrival of new Q.T.S. guidelines in 1998/99, this issue began to be addressed). Student teachers in this study claim to have relied predominantly on their own devices to produce pupil activities, suggesting that direct influences on their choice of activities are weak. It may be that external influences remain, but are influences on the *type of activity* undertaken rather than the selection of a specific activity. This might provide an opportunity for further research.

### Suggestions for Further Research

With a lack of direction from school on the issue and a limited quantity of suggestions from HE, student teachers generally claim to have chosen to devise their own pupil activities. This raises the related issues of what student teachers are trying to achieve with their activities and how they design the activities. Clearly there are obvious targets, such as the teaching of lesson objectives. *What kinds* of activities do they design to meet these targets? What principles, besides the maintenance of order, are being used to choose between competing alternatives? How do student teachers articulate these principles? Where do the principles originate? Given an ambitious suggested pupil activity, how do student teachers modify it to make it more "workable" for them in the classroom? What is the thinking behind such modifications?

### References

- Brown, Catherine A and Borko, Hilda: (1992) Becoming ~Mathematics Teacher, in Grouws Douglas A (editor) (1992) *"Handbook of Research on Mathematics Teaching and Learning"*, National Council of Teachers of Mathematics, Macmillan, New York, ISBN 0-02-922381-4.
- Brown, Sally & McIntyre, Donald: (1993), *Making Sense of Teaching*, Open University Press, Buckingham.
- Calderhead, James & Shorrock, Susan B (1997) *Understanding Teacher Education*, Falmer Press, London.
- Daloz, Laurent A (1986) *Realizing the Transformational Power of Adult Learning Experiences*, Jossey-Bass, San Francisco & London.
- Day, C., Calderhead, J., & Denicolo, P. (editors): (1993), *Research on Teacher Thinking: Understanding Professional Development*, The Falmer Press, London & Washington DC.
- Drever, Eric: (1995), *Using Semi-Structured Interviews in Small Scale Research*, S.C.R.E. Publications, Edinburgh.
- McIntyre D, Hagger H and Wilkin M (editors): (1994), *Mentoring: Perspectives on School Based Teacher Education*, Kogan Page.
- McIntyre D, Hagger H (editors): (1996), *Mentors in Schools: Developing the Profession of Teaching*, David Fulton Publishers.
- McNamara, D: (1994) , in Wilkin M, Sankey D (editors): (1994), *Collaboration and Transition in Initial Teacher Training*, Kogan Page Ltd.
- Reid, I., Constable, H., Griffiths, R., (editors): (1994), *Teacher Education Reform: Current Research*, Paul Chapman Publishing, London.

Schon, Donald A: (1991), *The Reflective Practitioner*, Harper Books. Smith, D. N.: (1996a), "College Ideals and School Practice", *Mathematics Education Review* 7, 25-30.

Smith, Jim: (1996b), *Getting Started*, Mathematical Association, Leicester. Tomlinson, Peter: (1995), *Understanding Mentoring: Reflective Strategies for Teacher Preparation*, Open University Press.

Whiting, C et al, (1996), *Modes of Teacher Education Project:*

*Partnership in Initial Teacher Education*, University of London.

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## Opening a Can of Worms: Investigating primary teacher's subject knowledge in mathematics

Maria Goulding and Jennifer Suggate University of Durham

### Abstract

*In this paper, we describe how one institution went about the auditing process of primary students' subject knowledge and what was learned about students' difficulties and errors. The scope is limited in that we have not made the connection between students' subject knowledge and classroom performance which was made in the study at the Institute of Education in London (Rowland, T., Martyn, Barber, Heal, 2000), mainly because of the difficulties of assessing classroom performance reliably. However, our analysis of primary students' subject knowledge within the same national framework offers interesting points of comparison between the two studies, both in method and findings.*

### Introduction

No one questions the idea that what a teacher knows is one of the most important influences .. on what students learn. However, there is no consensus on what critical knowledge is necessary to ensure that students learn mathematics' (Fennema and Franke, 1992)

One of the reasons for this is undoubtedly the difficulty of disentangling the different elements that make up a teacher's knowledge. It also seems very likely that teachers will utilise and display different facets of their knowledge in different contexts and that their knowledge will develop over time. Some theorists have, however, provided frameworks to describe teacher knowledge (Shulman, 1986, Peterson, 1988) and there have been some studies (Askew, Brown, Rhodes, Johnson, Wiliam, 1997) which have looked at teacher's mathematical knowledge in relation to measured progress of pupils' learning.

Shulman's categories of teacher knowledge, and in particular, his distinction between *subject matter knowledge* and *pedagogical content knowledge* are particularly useful for this paper, since they informed the thinking behind the intervention to be described. *Subject matter knowledge* is 'the amount and organization of the knowledge per se in the mind of the teacher' Shulman 1986 9 and *Peda o ical content*

*knowledge* consists of

" the most powerful analogies, illustrations, examples, explanations and demonstrations - in a word the ways of representing the subject which makes it comprehensible to others .... [it] also includes an understanding of what makes the learning of specific topics easy or difficult..." (Shulman, 1986, p9)

Although these components of knowledge are presented as separate categories, the relationship between them and the process of transforming the one into the other would seem to be crucial (Shulman, 1987). Some primary teacher trainees may have a very limited set of ways of thinking about and doing mathematics, and will need to broaden and develop alternative representations in order to teach. Others may already have a broader range of representations and their knowledge may be organised relationally (Skemp, 1976) and so their development of pedagogical subject knowledge may be more straightforward.

It seems obvious that how teachers know their mathematics is important. In the study at King's College (Askew, Brown, Rhodes, Johnson, and Wiliam, 1997), teachers who did not necessarily hold advanced qualifications in mathematics but whose knowledge was nonetheless connected and who believed in the capacity of pupils to become numerate proved to be the most effective. It seems very likely that for some teachers, connectedness may develop as they teach as a result of inservice training, personal research in their lesson planning and the process of interacting with pupils. Indeed the highly effective teachers in the King's College study were much more likely to have taken mathematics in-service courses than the other teachers in the study.

All this implies that an important task in teacher education is to start students down the road of developing and improving their understanding of underlying principles and connections within mathematics. What is less clear is what mathematics should be used as a focus for this work. For students training to be generalist primary teachers, how far should we go beyond the common content of the primary curriculum in order to develop students' conceptual grasp? Where should we place the emphasis? Which principles need to be very firm and which can we expect to be less secure?

Despite this uncertainty, policy makers in England and Wales, in their requirements for courses of initial teacher training, are sufficiently confident to prescribe the knowledge and understanding which will 'underpin effective teaching' at KS1 and 2 (DfEE, 1998). This amounts to a list of mathematical topics to be audited, together with related examples from the KS1 and 2 programmes of study: Where gaps in the subject knowledge and understanding are found, providers must make sure that these are filled by the end of the training course. This list of topics could be seen as an attempt to delineate the content of Shulman's *subject matter knowledge* and to justify this content by showing which elements of the primary curriculum it underpins.

This underpinning is not always convincing. For example, students are expected to know and understand 'the correct use of  $=$ ,  $\sim$ ,  $\therefore$ ' and 'to follow rigorous mathematical argument' but no obvious corresponding KS 1 or KS2 examples are given. For other statements the link is clearer e.g.

'familiarity with methods of proof, including simple deductive proof, proof by exhaustion, and disproof by counter - example'

is matched with

'proving that numbers divisible by 6 are also divisible by 3'

'proving, for example, that there are only 11 unique nets of cubes' 'disproving, for example, that any quadrilateral with sides of equal length is a square'.

In this paper, we will describe how one institution went about the auditing process of primary students' subject knowledge and what was learned about students' difficulties and errors. The scope is limited in that we have not made a connection between students' subject knowledge and classroom performance made in the study at the Institute of Education in London (Rowland, T., Martyn, Barber, Heal, 2000), mainly because of the difficulties of assessing classroom performance reliably. Our analysis of primary students' subject knowledge within the same national framework, however, offers interesting points of comparison between the two studies, both in method and findings.

### Context for the Study

In this institution, tutors had been offering primary student teachers opportunities to deepen their mathematical subject knowledge for several years before the government's initiatives were mooted. Students were invited to choose topics and starting points to investigate at their own level. They were expected to dig beneath the surface of what they already felt they knew, investigate avenues that were new to them and reflect on their learning process. We specifically asked them to reflect on the role of discussion in developing new understandings and the role of the more experienced other, either peer or tutor, who helped them in this process. At the end of the period, individuals or groups, depending on the ways they had worked, made presentations about the mathematics they had been working on as well as their reflections on this learning processes. It was clear from tutors' informal and students' written evaluations that some students seemed to relish this opportunity and others thought it was a waste of time.

With the introduction of the requirements for courses of Initial Teacher Training, we have adapted this subject knowledge initiative to meet the new demands. One of the big differences is that now gaps in subject knowledge have to be identified by means of an audit, and subsequently filled. Whereas before there had been choice, the emphasis now has to be on coverage and demonstrated competence. Apathy for some has been replaced by anxiety. Field notes taken from the sessions in the first year of implementation revealed particular anxieties with algebra (nth terms and the equations of graphs) and proof. We have found ourselves pushed into instrumental teaching on more than one occasion. With proof, we have used the Greek idea of *diknyme* proofs (showing something to be so, to make visible or evident) to get around the difficulties with symbolism. We found ourselves saying 'Don't worry, a visual proof accompanied by some explanation will do for the audit, if you can't understand a symbolic one.' Paradoxically, this provided some students with a good excuse to avoid algebra at all costs rather than using the context of proof to improve their facility with algebra.

This study will present data from the set of students (201) who took the audit in 1999. They were students on the one-year PGCE (postgraduate certificate in education) course, final year students on the three-year BA (Ed) degree, and students in the first year of the BA (Ed). The PGCE and final year BA (Ed) students had seven hours teaching specifically in subject knowledge as well as their methods course, and access to self study materials, before they took the audit. We encouraged peer tutoring and saw many examples of this happening both in and out of taught sessions. The first year students had followed a new one year course 'Mathematics for primary teaching' which concentrated largely on the mathematics in the primary curriculum and the research into children's understanding of topics within it. Since all the elements from the DfEE's subject knowledge list were touched upon in this course, but approached from the standpoint of the primary curriculum, we decided

to run the audit for these first years as well as the other assessment components of this module, and report it separately.

Developing procedures for the assessment of subject knowledge was a matter of making professional judgements that could be justified to outside bodies now that the audit was statutory. In the year of this study, the PGCE students and BA(Ed) students were given time after the audit to go over their problems, and to go through correct versions of similar questions in one to one interviews with tutors. We used tighter procedures for the BA(Ed) students in year 1. Those obtaining 90% or over were simply required to do their corrections. Those obtaining between 60% and 90% were offered some extra teaching time, after which they had to explain their errors in a one to one interview and then work through and explain similar questions. Students obtaining less than 60% or still revealing weaknesses after the interview sessions were offered the opportunity to sit the audit in subsequent years.

#### Aims Of the Study

This study was conducted to help the mathematics team evaluate its present provision and improve it in the future. We were interested in diagnosing the students' weaknesses and strengths, and identifying the most appropriate mathematical experiences for them while still fulfilling the external requirements to audit and remedy weaknesses in subject knowledge. We wanted to see how feasible it is to reconcile these aims.

We were also driven by curiosity; we wanted to know what makes certain topics so difficult, even for adults who have achieved at least the critical qualification of a grade C or its equivalent in the General Certificate of Secondary Education Mathematics examination. We suspect that there are some big ideas that are intrinsically very difficult to understand well and that students who appear to have made progress in school mathematics still have a very shaky understanding of them. Drawing attention to specific examples of such mis-understandings or incomplete understandings would be used as a teaching strategy with future cohorts of students.

Experience in the teaching sessions led us to anticipate errors in questions on fractions, algebra and proof. For the first years, we suspected that some students would have developed a good understanding of the primary curriculum and the connections within it but still have difficulty doing certain elements on the audit.

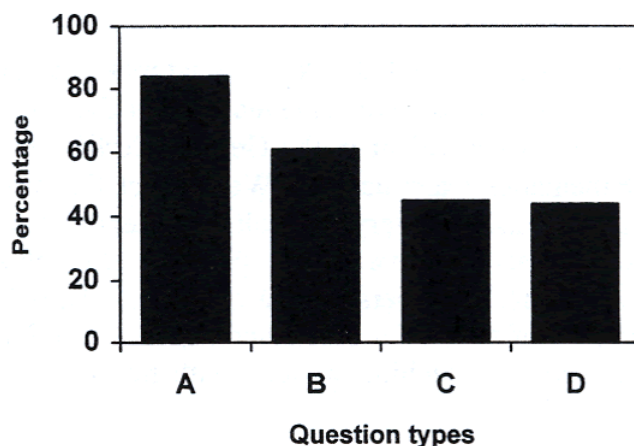
In conversation with colleagues offering similar courses, it would appear that many of our problems with the DfEE's subject knowledge component are shared, not least with the content specification. We all seem to have difficulties with some students' feelings and beliefs about mathematics, and we are all struggling to find sensitive ways to audit subject knowledge without glossing over serious weaknesses. Uncovering these weaknesses has however been an illuminating experience for many of us, and it has raised many questions about what students should know and how they should think if they are to be able to teach primary mathematics well.

The audit consisted of 16 questions to be done in up to 2 hours. In this paper, we will present an error analysis of the audit for all the groups taking it in the year of the study, with a discussion of what this can tell us about the amount and organisation of the students' subject matter knowledge.

#### Results of Analysis of Audit Errors

The audits for the 201 students were analysed to identify the mathematical topics that caused most problems and the most common sources of error.

**Figure 1 Questions causing difficulties to over 40% of the students**



#### Question types

- A Appropriate degree of accuracy in area calculation
- B Proof
- C Calculating volume and mass
- D Ordering small numbers

**Figure 2 Question causing difficulties to between 20% and 40% of students**

Finding mean and median	32%
Meaning of correlation	24%
Ratio of areas after enlargement by scale factor of 2	24%
Relations between fractions, decimals and percentages	23%
Providing a context for division by a fraction	23%
Providing a context for subtracting a negative number	23%
Simultaneous equations (graphical or algebraic)	21 %

#### Discussion of errors

It is clear that a significant numbers of students had difficulty with a range of mathematical topics drawn from the subject knowledge list. This could of course reflect on the quality of the questions, as much as on their knowledge, but since we also had teaching sessions and one to one interviews after the audit, we also have some idea of how deep some of the difficulties lie. Some interpretations of the nature of errors are offered below.

#### Question 10

##### Percentage making error

Some children were measuring their desk to the nearest centimetre. They found it was 53 cm by 62 cm. State the possible limits to the length of the sides. Work out the area to an appropriate degree of accuracy: Percentage making error 84%

##### Common errors:

Upper limits 53.4, 62.4: Percentage making error 58%

Inappropriate or no approximation given: Percentage making error 65%

The difficulties with the upper limits seem to arise from the strategies used for rounding. Since the students were used to rounding 53.5 upwards they found it hard to accept that it could be a limit for the measurement of 53 cm. Only when asked what '53.41, 53.45, 53.49, 53.4999...' would be rounded down to' did they appear to accept this idea. It may be that for some of them the idea of measurement being continuous is shaky and they may only have practical experience of measurements taken to one decimal place.

We wanted the students to work out the upper and lower limits for the area and then make a sensible estimate. Of the students who did work out the limits some left it at that whilst others went on to give the answer  $3286.5 \pm 57.5 \text{ cm}^2$ . We allowed this but were really looking for something like  $3280 \pm 50 \text{ cm}^2$  or better still,  $3300 \text{ cm}^2$  or  $3.3 \text{ m}^2$ . All this may reveal that students are wedded to the idea of giving very accurate answers even in a measurement situation where approximation is required. It could also be because they did not know what was

required from the question. In the post audit discussions, when reassured we were looking for a 'sensible' estimate, they did not feel the need to be as precise as they had been in the written paper.

### Question 13

Prove that if any two odd numbers are added together, the result will be an even number: Percentage making error 61%

#### Common errors

Use of only specific number or diagrams: Percentage making error 19%

Use of two equal odd numbers: Percentage making error 16%

Incomplete algebraic proof: Percentage making error 32%

A significant proportion of students making errors on this question did not seem to see the need for general reasoning. They would give specific numerical examples e.g.

$7 + 11 = 18$  or diagrams without any accompanying explanation:

□□□□□ + □□□□ = □□□□□□□□ ■

□□□□□ ■    □□□□ ■    □□□□□□□□□ ■

and seemed to think we were being fussy when we insisted on some dotted lines in between the pairs to show that the diagram represented any two odd numbers rather than the specific ones they had drawn. Their patient smiles seemed to say 'but you know what we mean'.

Some of the students who tried an algebraic proof wrote

$$2n + 1 + 2n + 1 = 4n + 2$$

and defended their use of only  $n$  by saying 'but  $n$  can be any number' as if the two  $n$ 's could represent different values. Many of them went no further with the algebra - the 2 in the final expression seemed to indicate that it represented an even number because it ended in 2.

This difficulty in completing the proof was also present in the answers of students who did distinguish between the two odd numbers. Two typical responses were

Student A  $2a + 2b + (2)$  this proves that the result will be even

Student B  $2n + 2m + 2$

$$2(n+m) + 2 \text{ even}$$

Our later discussions indicated that for some students the 2 at the end of the expression made the number even, for others it was simply the presence of 2s in each part of the expression. Those who argued that the sum is even because each part of the expression is even had a stronger case although they were assuming without any argument that the sum of two or more even numbers is even.

There was a great reluctance to factorise over three terms:  $2n + 1 + 2m + 1 = 2(n + m + 1)$

and to realise that this expression represented a multiple of 2 and was therefore even. In fact the appearance of the 1 inside the bracket changed some students' minds and they thought the number was now odd.

### Question 11

The diagram shows a lead ingot (cuboid of dimensions 40 mm, 60 mm, 25 cm). the density of lead is  $11.4 \text{ g/cm}^3$ . Find the mass of the ingot: Percentage making error 45%

#### Common errors

Changing  $\text{mm}^3$  to  $\text{cm}^3$  : Percentage making error 9%  
 Other problems with units : Percentage making error 21%  
 Use of mass = volume / density : Percentage making error 18%

Many of the problems with this question involved problems with units, notably changing  $\text{mm}^3$  to  $\text{cm}^3$  by dividing by 10. In the one to one interviews, this did not seem just to be a careless mistake and many had no strategy for working out how many  $\text{mm}^3$  there were in  $1 \text{ cm}^3$  rather they regarded it as a number to be remembered. They seemed to be helped by reference to rulers and centicubes centicubes.

In the calculation of the mass given its volume and density, again there seemed to be no strategy for deciding whether to multiply or divide. Even from the students who had this correct, there were few who could explain why they had chosen the operation of multiplication. It seemed to help many of them to go back to the definition of density and using a centicube, to explain that if it were made of lead that it would weigh 11.34 grams, and that every cubic cm in the ingot would also weigh 11.34 g. Multiplication then seemed a very reasonable strategy.

### Question 1

Arrange in order from largest to smallest

0.203;  $2.35 \times 10^{-2}$ ; two hundredths;  $2.19 \times 10^{-1}$ ; one fifth, 0.026,  $\frac{2}{9}$  : Percentage making error 44%

### Common errors

Ordering figures : Percentage making error 34%

Of the students who made mistakes in this question, the problem did not seem to be so much with converting the numbers into decimals but with putting them in the correct order once this was done. As in question 10, there was a lack of confidence with numbers with different numbers of digits in decimal places including those with more than one or two places of decimals. Many students, including those who had this question correct, admitted to being reliant on their calculators to convert the standard form numbers or to use a rule like 'moving the decimal point'. This particular item throws up a notable difference between our study and that in the Institute where the item on ordering decimals had the highest facility. There may be differences in the cohorts or the teaching experience that can explain this, although we cannot discount differences between the assessment items themselves.

### **Summary**

Of these four questions, there are obviously links between 1, 10 and 11 in that a confident understanding of decimal numbers and the continuous number line runs throughout measurement. Also of concern is an appreciation of the approximate nature of measurement and of compound measures such as density.

The problems with proof seem to be of a different nature, and we found it much more difficult to help students with this before and after the audit. From the taught sessions, it would seem that deductive proof is much more problematic than proof by exhaustion or disproof by contradiction. The proof question also threw up fear of and difficulties with algebra, which were not highlighted to such a degree with other algebra questions on the paper. The fact that reasoning and argument had one of the lowest facilities in the Institute study adds strength to our concern.

Of the questions that caused difficulties to over 20% of the sample, those in the area of number were a concern. The common wrong response of  $0.3 = \frac{1}{3}$  was carelessness in some cases but in others the relationship between tenths, hundredths etc. and decimals did not seem very secure. Being unable to provide word stories for division by a fraction and subtracting a negative number clearly showed up a limited understanding of the operations of division and subtraction. None of these errors can be taken lightly as these number concepts are all fundamental to the primary mathematics curriculum.

### **Distribution of marks for BA(Ed) year 1 students**

Concentrating on the errors made by the students taking the audit tends to conjure up a negative image, but in fact many students did very well on the audit and when explaining errors did so with confidence and understanding. The distribution of the first year students' marks (the only group to be given a percentage in accordance with the tighter procedures) reveals that well over half the students scored 80% or more:

Mark	50-59	60-69	70-79	80-89	90-100
No of students	25	31	13	8	4
%	5%	10%	16%	38%	31%

These figures may seem encouraging but in itself tell us nothing about the nature of the students' underrating.

### Conclusions

Although we may have raised anxiety levels amongst the students by giving the audit as a written test, there should have been very few surprises in it for those who had followed the support materials and sample questions. Nonetheless it uncovered a substantial number of errors which we were then able to follow up. In fact we feel that this follow-up process was extremely valuable and we doubt if the students would have put so much effort into it if they had not had to convince us of their understanding in order to complete the audit successfully.

In number, the problems highlighted an incomplete understanding of decimals per se and in the context of measurement and a limited understanding of number operations. More probing questions on the audit may well have revealed the problems with fractions which we had anticipated, and the links to decimals and percentages which seemed to cause difficulties in teaching sessions. We feel that for a substantial number of students these connections still need strengthening but we feel relatively optimistic that this can happen for the majority, particularly with the emphasis on numeracy now in primary schools. Much will depend on how this strategy is implemented i.e. if there is adequate emphasis on number concepts being developed and applied in measurement, and the sensible use of calculators as a teaching aid in the later years of KS3.

We are less confident that the majority of students will develop a secure understanding of proof and that they will feel comfortable with using algebra to generalise and to produce their own algebraic proofs. Although we were able to guide students through proofs which would give us the evidence required to pass the subject knowledge audit, we are not convinced that their understanding is deep and secure or that they see the need for rigour. This may be because we have not yet found the best way in here. With number we feel that dealing with subject knowledge at the student's own level can work reasonably well and complements the work done in the professional mathematics course. With proof and algebra we need to strengthen this link even more by using examples from the primary classroom as starting points. This year we have started to use activities drawn from several articles (Harding 1999, Orton, 1997, Stoncel, 1994) from professional journals to stimulate the students' thinking, and also to convince them that the seeds of proof and algebra can be sown in the primary school.

The problems with separating out the subject knowledge element will be much less of a problem for the BA (Ed) students taking the new compulsory first year course, which integrates subject matter knowledge and pedagogical subject knowledge whilst still offering the audit separately from the module assessment. At the moment we still intend to continue offering a separate subject knowledge element plus audit to the one year PGCE students.

We still have very mixed feelings about the whole process of auditing students' subject knowledge. We are not sure if we are emphasising what the students can do at the expense of how they are thinking, although we feel that the combination of written test plus follow up teaching and interviews is defensible. The advantage of the written test is that it puts students on the spot and gives us material to work on. This is not only with the students in their follow up interviews but also with successive cohorts where we can use errors as starting points for discussion.

However the most important part of the process may be that the students' self-awareness of their own cognitive processes has been awakened, particularly by having to explain their reasoning to tutors in the follow up sessions. The original framework proposed by Shulman was helpful to us in the initial stages of this study but Peterson's (1988) now seems more appropriate. She argues that 'effective teachers need three kinds of knowledge: how students think in specific content areas, how to facilitate growth in students' thinking and their own meta - cognition. Mathematics knowledge isolated from children's cognition and teacher's metacognition does not appear important to her.' (Fennema & LoefFranke, 1992).

Many would argue that all three kinds of knowledge are and should be integrated into methods courses. We feel that in practice it may be difficult to give each adequate attention and that the student teachers' metacognition may be neglected. We share the concern (Rowland, Martyn, Barber and Heal, 2000) that the audit process may simply reinforce an instrumental approach in weak students. There may, however, be some advantages. Some mathematical ideas, for example generality, reasoning and proof, may receive more attention than they might

otherwise. Moreover, given that we have to do an audit, assessing the mathematical content in ways that require students to reflect on and justify their methods, may give us our best opportunity to open up the student teacher's thinking to themselves and to us. This may be a far more important process than making sure that they can produce correct answers on the audit paper.

## References

Askew, M., Brown, M., Rhodes, V., Jollison, D., Wiliam, D., (1997) *Effective Teachers of Numeracy, Final Report*, King's College, London. Department for Education and Employment (1998), Circular 4/98, *Teaching: High Status, High Standards*.

Fennema, E and Loef Franke, M.,(1992), 'Teachers' Knowledge and its Impact', in Grouws, D.A. (ed.) *Handbook of Research in Mathematics Teaching and Learning*, p 147- 164 ..

Harding, B. ' Proof and refutation', *Mathematics Teaching*, 169, December, 1999

Orton, J., 'Matchsticks, Patterns and Generalisations: *Education 3-13*, March 1997.

Peterson, P.L. (1988), 'Teachers and students' cognitional knowledge for classroom teaching and learning,' *Educational Researcher*, 17(5), 5-14. Rowland, T., Martyn, S., Barber, P and Heal, C (2000), 'Primary Teacher Trainees' Mathematics Subject Knowledge and Classroom Performance', in Rowland, T and Morgan, C., (eds.) *Research in Mathematics Education Volume 2*, British Society for Research into Learning Mathematics.

Shulman, L.S.(1986) 'Those who understand, Knowledge growth in teaching,' *Educational Researcher*, 15(2), p 4-14.

Shulman, L.S. (1987) 'Knowledge and Teaching: Foundations of the new reform', *Harvard Educational Review*, 57(1), 1-22.

Skemp, R.R. (1976) 'Relational understanding and instrumental understanding', *Mathematics Teaching*, 77,20-26.

Stoncel, D., 'How far can you go?', *Mathematics Teaching*, 147, June 1994.

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## Implementing the National Numeracy Strategy in small rural schools

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### Abstract

*This study looked at how some small rural schools implemented the NNS, focusing on classes of three or more year groups. It was undertaken for two reasons. The first was the need for advice and guidance by undergraduate QTS students enrolled on the mathematics education modules I taught, to ensure that they were adequately prepared for teaching practices in such contexts. The second was the scarcity of such practical advice available, both within the NNS training materials themselves, and within wider reading. A number of teachers were interviewed and their strategies for organising and carrying out the daily mathematics lesson with a whole key stage class were catalogued and analysed. What emerged was an overall feeling that, although the implementation of the strategy within a mixed age class was problematic, it was not unmanageable. A number of useful strategies were put forward and can now be shared with my students and other colleagues.*

### Introduction

#### Historical background to the introduction of the National Numeracy Strategy

In recent years it has become evident from a number of international surveys of maths achievement, in particular the Third International Maths and Science Study (TIMSS) of 1996, that English children were performing badly in maths, in comparison to children in other countries. English year 5 pupils were shown by the TIMSS study to be " ... amongst the lowest performers in key areas of number out of nine countries with similar social and cultural backgrounds" (DfEE, 1998: 4). Reynolds and Muijs (1999) suggest that the causes of such poor performance are likely to have been due to educational rather than to socio-cultural factors. The reason given for this was that the same English children were performing very well in science in comparison to other countries. In support of their claim they cited a study by Mortimore et al (1988) which showed that educational factors do seem to affect maths more than other core subjects.

Whether or not this is the case, it was findings such as these and the "...accumulation of inspection .... and test evidence ... " (such as that outlined Standards in primary Mathematics Ofsted 1998) pointing to the " ... need to improve standards of numeracy" (Dille 1999: 2) that led to the setting up of the National Numeracy Project (NNP) in September 1996. The Numeracy Task Force (NTF) which was put in charge, was given the brief " ... to develop a Numeracy Strategy to raise standards of Numeracy in order to reach the National Numeracy target by 2002" (Dille 1998:4). This target, of 75% of eleven-year-olds achieving the standards expected for their age, had been set by the government at that time. The NNP involved a sample of schools throughout the country and is still ongoing. In September 1999 the National Numeracy Strategy (NNS) was finalised and all schools were asked to use the documentation provided by the NTF when planning for maths. This documentation consisted of a file (the National Numeracy Framework or NNF) containing sections of advice and key learning objectives, termly planning grids and supplements of examples of activities related to every learning objective for each year group. A system of cascade training was set up to ensure that all teachers were prepared for the new system.

There was some advice for small rural schools, within the NNF' for dealing with mixed age classes (see Dille 1999: 25 -26). The NTF highlighted the special nature of such schools in their final report on the implementation of the NNS when they said that

*"Training and other guidance sent to all schools should take into account the need to help teachers teach the daily lesson ... in very small schools- ... circumstances in which the range of achievement is likely to be larger than average. "* (Dille 1998: 4)

#### Teaching in mixed age classes

*Although much research has been carried out into small rural schools and mixed age classes, the focus of many of these studies has tended to be the performance of the children in such schools or classes as compared to those in large schools or single age classes. Cockcroft (1982: 106) listed a number of*

*problems that small schools face in the teaching of mathematics, one of which was the likelihood of a wide age range within the classes of these schools. This must have been seen as being particularly problematic by the Cockcroft committee as, earlier in the report, on the evidence produced from a survey carried out by HMI (1978) ("evidence ... that the performance of children in classes of mixed age can suffer. " ), Cockcroft felt justified in stating "we do not therefore consider that this form of grouping offers any advantages for the teaching of mathematics. "(1982:103)*

The findings of a survey carried out by Hargreaves et al (1996) support Cockcroft's view on the possible lack of expertise in maths in small schools. Only about 20% of the teachers included in their study felt that they were competent to act as a maths co-ordinator. However, they also found that "*Competence and confidence ratings were generally high...*" (p.89) across the curriculum, and that few felt the need for more training in maths, the majority rating themselves as competent to teach their own class.

Research carried out by Galton and Patrick (1990) led them to the tentative conclusion that children in small schools performed as well as those in larger schools. As these children are more likely to be placed in mixed age classes, this would seem to contradict the HMI findings of 1978. Veenman (1997) argues, on the basis of his two studies carried out in 1995 and 1996, that " *... there is no empirical evidence ... showing student learning to suffer in combination classes.* " (p.262)

(A combination class is the American equivalent of an English mixed age or vertically grouped class, usually containing two grades)

Mason and Burns (1997) findings were consistent with studies such as this, and others, regarding achievement but they concluded "*that combination classes have at least a small negative instructional effect ...*" (pA2) - an effect due to the difficult context that the teacher is working in. They listed the problems of the combination class teacher as including the need for additional planning time and more grouped instruction. The latter led to less time being available for instruction for all the children and less time for individual attention to be given. This, in turn, lowered teacher motivation.

Bennett et al (1983) carried out a survey of practice in mixed age classes in primary schools. They found that head teachers taking part saw '*... no great difficulty due to mixed age classes.*' (p.54) They also found that grouping by ability and individual work were characteristics of maths in the junior stage. There was general agreement between the head teachers taking part that mixed age class teaching: is more stressful; requires more preparation and increased organisational ability; creates 'task to child match' difficulties; and requires extra resources.

Mason and Good (1996) found that teachers of combination classes in their American study tended to keep the two year groups separate, each group being alternately instructed and then given independent 'seat work' to do. However the single grade teachers favoured the whole-class teaching approach, using small group work occasionally. If this was the case in small rural schools in England, a change of practice would be required by the introduction of the NNS.

Galton et al (1998) looked at the way classroom practices in small rural schools have changed over the ten years since the introduction of the National Curriculum. They found that

*"The patterns of teacher-pupil interaction ... are, with a few exceptions, very similar to those found in the earlier PRISMS study of small schools which took place at the beginning of the 1980s and which were, in turn, close to those recorded by other researchers in studies of larger urban and suburban settings during the same period."* (p. 58)

These patterns of interaction did not include high levels of whole-class teaching, as the Alexander report (1992) had had to encourage teachers to move towards such a system. However, Alexander's advice had clearly not been followed by the teachers in Galton's study, possibly, according to Galton, because these teachers "*maintained their prior values and practices*" (1998:58), favouring independent and small group work. The move towards whole class teaching advocated by the NNS really *was* going to require major changes in the way mixed age class teachers in small schools organised learning.

#### **Details of the study undertaken**

The type of data to be collected throughout the study was qualitative, consisting of individual class teachers' views on the problems involved in implementing the NNS with a whole key stage and their methods of dealing with these problems. It was decided that a structured interview would be the best method to employ for collecting data.

The schools to be approached to take part in the project had to meet a set of criteria and the final number participating was twelve. Only teachers of those classes that consisted of three or more year groups were interviewed, as it was felt that it was in this context that official advice from the NNS team was lacking. In all, seventeen teachers were interviewed.

### **The findings**

*(discussed under the headings covered by the interviews)*

#### Teachers' initial views on the NNS

There were both positive and negative comments from the teachers in both key stages. 56% of the comments made were negative and 64% of these were specifically to do with implementing the strategy in a mixed age class. The main cause for initial concern was the range of ability that exists in a mixed age class - particularly in the KS 1 classroom where reception children are not obliged to follow the NNS but do need to address the Early Learning Goals (DtEE 2000). Two KS1 teachers felt that the reception children would need to be static for too long in order that the Y1 and 2 children should receive their entitlement. One teacher voiced initial concern over children with SEN. She was worried that they would not be able to cope with the pace of the maths lessons. Good planning and organisation were obviously going to be paramount here yet four reported that little help with whole key stage planning was available initially.

A number of teachers foresaw problems with whole class teaching particularly during the oral and mental starter. One thought that this part of the lesson may bore the most able, oldest children if it was to be understood by the least able, youngest children. Another felt that some would 'switch off if they couldn't keep up with the pace. Three teachers raised the lack of appropriate training for teachers with whole key stage classes as an issue. There were few leading maths teachers in such contexts and the waiting list for such sessions was long. The opportunity to watch how whole class teaching *could* be done effectively was not readily available.

In spite of their worries about the problems involved with a mixed age class many of the teachers felt very positive towards aspects of the NNS, in particular the emphasis on mental maths. Certainly the introduction of the National Literacy Strategy the year before had prepared them for the level of planning needed and the proposed format of the daily maths lesson. However it was interesting to note that similar numbers of teachers had opposing views on the 'drip feeding' approach. They had all been used to teaching maths in half-termly blocks which allowed plenty of time for consolidation before moving on to a new subject. The framework, to some, seemed to offer little time for children to grasp a topic fully. However, to others it offered an opportunity to break out of the old system which, in some cases, led to 'doing a topic to death' and less popular subjects being avoided. The strategy would ensure that everything that should be covered, would be covered.

#### Additional adult support available

In all but three of the KS 1 classes visited the class teacher had additional adult support of some kind. All teachers with support said that they could be unable to cope effectively without this resource. All had strategies that they could resort to if additional support was unavailable for any reason, but quality teaching assistant (TA) support was seen as an essential factor in the effective delivery of the daily mathematics lesson as outlined in the NNF. Of the class teachers that had no additional help, two only had nine children to deal with and agreed that this lessened the burden of dealing with a whole key stage considerably. However, one teacher had 23 children and reported that she had found it very difficult to cope with the demands of the whole key stage. In KS2 classrooms less than half had adult support and this was often limited to support staff for particular children.

As in all classrooms where TA support is used for supervision of children's activities, the additional adult needed to be briefed before the lesson. In the case of a maths activity, class teachers needed to give detailed information about the learning outcomes of the task, the vocabulary and resources to be used and the activity to be done. In some cases this entailed the class teacher in a great deal of work in writing down full details of what had to be achieved. In others, the class teacher had to use playtime to pass on the information. Sometimes pages from the NNS supplement of examples were copied to ensure that the adult knew exactly what to do. Constant use of the TA in a supervisory role also meant that some time had to be found, after the lesson, for discussion between the class teacher and the TA about the children and activity supervised.

#### Organisation of the Daily Mathematics Lesson for a mixed age class

*Involvement of reception children*

The majority of KS 1 teachers used the T A, when available, to supervise the reception children for some, or all, of the time. This included carrying out the plenary session for these young children as it was felt that, particularly at the beginning of the school year, they were not able to participate in a plenary with older children. They needed a plenary that involved number rhymes and stories or some practical activity. They found it difficult to sit and listen to the older children explaining what they had learnt. It was reported by some that the older children often found listening to what the reception children had done, less than useful. In all KS 1 classes the reception children experienced a three part maths lesson by the summer term.

The amount of integration with the Y1 and 2 children varied somewhat. Where additional help was available the reception children were included in the oral and mental starter, after which they moved on to work on a maths activity under T A supervision. In three of these schools the children remained with the T A for the main activity and plenary (although towards the end of the first year one class teacher felt able to include the reception children in a whole class plenary). In four others the children stayed with the T A for the whole main activity section of the lesson, unless they were the class teacher's focus group, in which case the T A would move to work with another year group. Of these four schools two included the reception children in the whole class plenary and two asked the T A to carry out a separate plenary for the reception children. Where no additional help was available one school included the reception children throughout. However, the remaining two schools used a somewhat different system, described by one teacher as her 'Back to Back' system. This can best be shown in figures 1 and 2 below.

**Figure 1 – The 'Back to Back' system\*** (Reception not included in plenary / used all year)

Reception	Structured play activity (non-maths based)	Oral/mental starter and introduction to main activity	Group activity Plenary	
Y1 & Y2	Oral/mental starter and introduction to main activity	Group activities	Plenary	Independent non core subject work

\* This system was also used by the same teacher when she had taken the whole of KS2 for maths. Here the class was split into upper and lower KS2 and the teacher used the same staggered approach, spending 1 hour and 20 minutes over the maths lesson and additional independent activity combined. (The school had trialled a 'specialist approach' for the teaching of maths and English - one praised by Ofsted during their inspection - whereby the two teachers had spent all morning teaching either English or maths to the whole school.)

**Figure 2**

(System used for the first part of the year until the reception children had enough mathematical understanding to join in the whole lesson)

Reception	Structured play activity (non-maths based)	Oral! mental starter and introduction to main activity	Group activity
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Y1 & Y2	Oral! mental starter and introduction to main activity	Group activities	Plenary
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### *The oral and mental starter*

Having a whole key stage meant that questions within the oral and mental starter had to cover a wide range of levels. In five classes these were directed to a particular year or ability group all of the time but, more commonly, some of the questions were targeted whereas others were open to all the children. The teachers who did this felt that those not being targeted may well 'switch off' if they were not expected to answer. However, a KS 1 teacher preferred always to direct questions, as the reception children would have been worried if faced with a question too far above their level.

A number of schools relied on the T A to observe the children during the oral and mental starter to log which children kept answering incorrectly or selecting the easier questions when given a choice. In other words they were being used in an assessment capacity. This entailed discussion time being made available, after the lesson, for the T A to feed back this information. This is probably also the case in single age classes, however, a narrower range of questions would be needed in that context.

All class teachers tended to use the same individual resources with all year groups during this part of the lesson. A wide variety of resources were used. Three KS 1 teachers stressed the need for the oral and mental starter to catch the children's interest if the reception children were to remain fully involved.

### *The Main Activity*

The majority of the teachers organised the grouping for maths on an age basis, with each year group kept together. Some further split the year groups into upper and lower ability groups to allow for differentiation of tasks. The rest mixed the year groups and -based their maths groups on ability. The NNS guidance suggests that teachers should only differentiate at three levels in order to keep the planning of tasks manageable. A number of the teachers interviewed were trying to follow this advice but some found that this was unrealistic and had to differentiate further. Some teachers felt that they had to plan for four, five or even six tasks, or levels of task.

All but three of the class teachers interviewed employed a 'sit and listen and wait' system for the transition from main activity introduction to individual or group activities. In four classes this meant that all children listened to everyone's tasks before being dispersed. The reason given for this was that to send off one group to start work while trying to explain tasks to other children was disruptive. In these classes the introduction to the main activity was kept as short as possible. In eleven classes, groups were dispatched once they had received instructions. This cuts down on the waiting time but means that the children have to be able to work independently. The NNS training materials suggest the use of a 'holding' activity for one year group during this time, but this method was only employed by one KS2 teacher who often split her class into an upper and lower KS2 group for the whole lesson. This reduced the range of ability that had to be catered for, however, holding activities need to be thought up and, once started off, can take longer to finish than required. This can reduce the amount of time available for the direct group teaching and group activity that needs to be carried out. This system also meant that two separate oral and mental starters had to be carried out. One other often sent her most able group away with their task sheet to discuss what they thought the task entailed and to plan their work while waiting for the other groups to be seen to.

All the teachers interviewed, (except for three groups of reception children), carried out direct group teaching with all groups. They all tried to ensure that each group received a fair balance of teacher focus and independent work throughout each week. Many mentioned that, due to the context of the mixed age class, the children in their classes were well use to working independently and that direct group teaching was feasible.

### *The Plenary*

Only half of the teachers reported that they always made sure that they carried out a plenary session and felt that this was an important part of the lesson, using it for the whole range of activities suggested in the NNS training materials. The rest felt that having to involve such a wide age range made the plenary less than useful during some lessons and preferred to deal with misconceptions and assess what was being learnt during the main activity. They preferred to spend the additional time completing main activity tasks. (This could well be due to the fact that more time had to be spent preparing the children for all the different tasks in the first place).

In all but one of the KS2 classes all the children were involved in any plenary that was carried out. In this class the class teacher sometimes carried out the plenary with only those children who had not been part of her focus groups for the main activity. In the KS 1 classes, whole class plenaries took place in only four cases. In five classrooms the T A often took the group that they were supervising (often the reception children) for a separate plenary session. In the KSI classroom that used the 'Back to Back' system, two separate plenaries were held by the class teacher.

One useful strategy employed by one school on occasion was that of 'pair sharing', where children explained to a partner what they had found out, what they had found difficult or easy or which strategies they had used etc. This system was developed as the class teachers at the school had found that the plenary was far too long if every year group fed back in some way and some children were 'switching off' if the plenary was not kept short.

### **Weekly planning**

All the teachers interviewed filled in a weekly maths planning proforma. These varied a little from school to school. Eleven blank proformas were available for study, some of which were based on the example provided by the North Yorkshire Advisory Service, others were more similar to the examples given in the NNS training materials. The way these proformas were adapted for a whole key stage class was discussed and it was found that none were officially adapted to allow for differentiation for year groups in most of their sections. Teachers had to customise their proforma afresh each time they wished to show this. However, all proformas had a space for the main activity tasks for each year/ability group to be specified daily. Some class teachers preferred to write each year group's task in a vertical block, others wrote in horizontal format within in each day's row. It was up to the individual class teacher to decide on the format used.

Four of the proformas included space for oral and mental starter learning outcomes. One school cut these, for each year group, from the sample medium term plans provided in the NNS training materials, and pasted them in three blocks across the top of the proforma. One school provided some space near the top of the proforma to note down the 'Instant Recall' objectives for the week and two others specified one or two outcomes for each day. One teacher (KSI) felt that an important part of the planning stage was to try to ensure that the learning outcomes for each year group were linked so that the oral and mental starter became a coherent whole for the class.

All but four class teachers mentioned that the layout of the supplementary examples of activities in the NNF helped with weekly planning as parallel, differentiated tasks could be easily found for each year group when they were working on related learning outcomes. One KS2 teacher did find the siting of Y3 examples with those for Y1 and 2 inconvenient.

### **Medium term planning**

Teachers differed in their views on whether, or not it was reasonably easy to keep a whole key stage working together on the same topic for most of the time. Some felt that year groups could be kept together for most of the time but almost twice as many reported that, although this was sometimes possible, often different year groups needed to work on completely different topics. If this was the case then these individual groups were given separate inputs during the main activity part of the lesson. All agreed that when the children were working on the same topic teaching was a great deal easier.

Again there were some differences in the proformas for medium term planning. Although all followed the basic format of the blank grids provided in the NNF some used separate A4 portrait style proformas for each year group whilst others used an A3 landscape format with all year group learning outcomes in separate columns. Some put pairs of year groups together using the two year, A4, blank proforma, taken from the grids provided on the CD ROM, produced by the NNS team to help with planning. One of these teachers decided that Y1 and 2 learning outcomes could be fairly well matched in this way. She used a separate A4 portrait style grid for the reception children, trying, as far as possible, to find links to the work to be covered by the Y1 and 2 children. A KS2 teacher, who had a Y 4/5/6 class, used a similar system. During the first two terms the Y 4 learning outcomes were on a separate proforma but in the summer term she decided that it was the Y6 children who needed to be considered separately as work had to be done with them in preparation for the national assessment tests.

Eight used sample 2-year plans from the CD Rom provided by the NNS team. Seven used the paper based, one year sample medium term plans, also provided by the NNS team. The rest had copied, then cut and pasted learning outcomes from the NNF as the saw fit.

### **Teachers' views on the NNS at the end of the first year**

Analysis showed that, when teachers were asked to reflect on the first year of implementing the NNS, 60% of the comments made were positive. 20% of these comments were specifically to do with teaching maths to a

mixed age class. However, 71% of the negative comments made were in this category.

The stretching of the most able children was still a concern for at least two teachers. One other upheld her initial view that having to address the needs of four year groups meant that the oral and mental starter wasted the most able year 6 children's time. One school had devised a monitoring system whereby samples of work from across the ability range were regularly checked by all members of staff, to ensure that such children were being challenged. (This school also organised for teachers to observe each other's maths sessions in order that teaching and learning could be monitored.)

Some KS 1 teachers upheld original views on the inclusion of reception children in the daily maths lesson. Two teachers felt that the oral and mental starter was often too long and entailed too much for reception children to cope with. The plenary was problematic for another, as the Y2 children were not always interested in the work done by the reception children. The mismatch between the NNS and the Early Learning Goals was raised again here. One teacher had devised a system whereby the reception children were 'corralled' in one area of the classroom to allow them the freedom of movement they needed without disturbing the Y1 and 2 children during the main activity. Without this system she felt that these youngest children would have to be too static throughout the lesson.

Regarding general issues, many of the teachers interviewed upheld their initial view that the half-termly topic system was better. The time allocation for some units was not thought to be sufficient and they would have to continue using extra days from other, easier, units or assess and review days. The feeling of these teachers was that the rush through so many topics each term did not allow for full understanding to develop. One teacher nicely illustrated this, when she said *'I feel that the children and I are dipping our toes into a lot of little ponds but not really getting our feet wet'*.

Six teachers upheld their initial view that the daily maths lesson with the whole key stage was problematic, but felt that they had managed to find ways around the problems. One (KS 1) teacher felt that the gap between the new reception intake and the Y2 children became more manageable as the year went on. The majority intended to continue with the organisation set up this year as they felt the system they had developed was working reasonably well. The teacher using the 'back to back' system felt that it was particularly successful and should be commended to other small schools.

Three of the KS 1 teachers thought that the reception children were actually working at a higher level in maths than they might have otherwise reached under the old system. However, as one teacher pointed out, this could be due to the fact that this year's intake was particularly able. More generally, the emphasis on mental maths - welcomed by many initially - was thought by seven teachers to have increased the children's confidence, particularly the less able mathematicians. Many felt happy with the strategy and had positive comments to make about the support the framework provides, the improved ability of the children to spot alternative ways of working out calculations and the chance to link topics.

### **The implications of the findings for schools**

Cockcroft's list of possible problems for small schools, mentioned in the introduction to this report (1982: 106), were given some consideration in the light of the findings. The likelihood of a wide age range in each class was, of course, the case. His point about possible lack of expertise in maths in small schools was borne out, as only one teacher (also a head) had pursued maths to degree level. He felt that there would be a larger burden for staff when preparing schemes of work. In all schools visited there were, at most, three or four teachers. This would have made preparation of a maths scheme more onerous in the past. However, since the introduction of the NNS the task of writing such a scheme of work has been effectively removed. The report also mentioned possible difficulties of monitoring of maths schemes. This is problematic as all the head teachers were found to have class teaching responsibilities. However, one school in particular felt that they were managing to do this effectively. At least two others approached the advisory service for observation and advice. (Others may well have done so but this did not emerge at interview). Finally, Cockcroft cited the possible lack of professional support in small school contexts. However, this did not really apply as, although some of the schools were in isolated positions, all but two of the teachers had attended the training courses provided by the local authority. This not only provided professional support, it also meant that they had received the training direct, without the need for it to be cascaded by the maths co-ordinator and head teacher, as it would have been in larger schools. Presumably this meant that their training was of a similar quality. One teacher did remark that often staff in small rural schools are better prepared for new initiatives as often all members of staff can attend training sessions that are restricted to two or three representatives from each school. The 'Chinese whispers' effect is lessened if everyone hears the message first hand!

All of the KS 1 teachers were well experienced in dealing with the level of organisation involved in teaching a vertically grouped class. The KS2 teachers had had less experience of this situation but had adapted the skills developed organising learning for single aged classes. In view of the fact that it cannot be assumed that all teachers in

small rural schools have a great deal of experience of the difficulties involved with this context, it would seem essential for additional training and advice to be given to such staff. The original NNS training materials were not really written with these people in mind. Although additional materials are now available to help teachers in this context, national monitoring of how small schools are coping would be useful so that good practice could be shared. The implementation of the NNS is being monitored by HMI and by the Ontario Institute, Toronto (2000), but, as yet, small rural schools are not the focus of these studies.

The 'sit and wait' system of managing the transition from main activity introduction to group tasks was felt to be effective in the KS1 classrooms, as often the teacher only had Y1 and 2 children to see to once the T A had removed the reception children. This meant that only one year group would have to wait before becoming the teacher's focus group. Often the input they were listening to served as useful revision for older children or as a taste of things to come for younger children. This would also be the case in the KS2 classrooms. However, because additional support may be less likely here, and as there are four year groups to deal with, this system could mean that children would have to wait a long time before starting on their main task. Teachers need to work within a system they feel most comfortable with and will have to find a balance whereby children receive the direct teaching required without having to sit through unnecessarily long inputs. The holding activity system suggested by the NNS team seems a useful strategy but, as mentioned previously, has its own drawbacks. The 'back to back' system outlined earlier seemed to be one way of solving the problem of children having to sit and wait.

The need to plan more than three tasks for each lesson increases the workload for whole key stage class teachers. This may become easier as teachers get used to the new strategy and build up banks of ideas from previous years. However, for those teachers that do try to follow the three level differentiation advice, it must be borne in mind that the likelihood of the task being well matched to the child is reduced. More than three tasks or levels of task may be unavoidable in the whole key stage context if children are not to be left feeling frustrated because the level of work is too easy or too difficult.

The organisation of the plenary varied from class to class but, in the main, most of the teachers seemed to be having as many problems ensuring that this was carried out effectively as their single age class colleagues. HMI evaluation of the NNP (1998)

found that "the plenary phase was a weak part of many numeracy lessons" (p. 12) Book 1 of the NNS training materials (1999) highlighted the particular challenges of the plenary. One, that "some children will require more feedback than others, who may lose interest unless you take care to involve them;" (p. 116) has real significance in the mixed age class. It must be questioned whether a whole class plenary, which includes three or four different year groups at the same time, can always be managed effectively - especially if the children are working to different learning outcomes. Perhaps whole key stage teachers will need to pursue the system of having a plenary with their non-focus groups whilst leaving the other children to finish their work as independently (and quietly!) as possible.

The need for teachers to customise weekly planning proformas to accommodate whole key stage class planning would suggest that it may be useful for schools to share their individual ideas and develop one or two standard proformas, which would not need customising each week. It may be that such a proforma would be easier to read if year groups were specified in type rather than in teachers' own handwriting.

The point made by the KS2 teacher about the siting of Y3 examples within the NNF is a valid one. It might be in small schools' interest for this point to be made to the NNS team so that future editions could be modified with Y1/2, Y3/4 and Y5/6 examples being parallel. However, the first version would be beneficial for schools where numbers dictated the need for cross key stage organisation.

Medium term planning proformas varied and teachers need to develop their own system so that it works well for them. However, in view of the consensus of opinion that teaching is easier if all children are following the same topic, it would be beneficial if teachers continued to look for links between learning outcomes. Sample plans for mixed age classes are available but the NNS team may need to offer more help here for teachers of whole key stage classes.

The absolute need for extra adults within the key stage one classroom, to help supervise the reception children during maths, has major implications for funding. One school in particular was finding this a great drain on resources. As in any classroom, when there is T A support being used to supervise children's activities, class teachers need to liaise closely with their assistants to ensure that they are fully aware of the learning objectives and the activity which they are to supervise. If the T As are not allocated time to be involved in planning meetings then the class teacher has to communicate these very specific expectations in other ways. If T A support is crucial to the maths session in a whole key stage class then perhaps additional resourcing should be available to ensure that these additional adults are present at planning meetings. Certainly the class teachers interviewed were spending a great deal of time ensuring that these adults were well prepared and debriefed. Perhaps some consideration of this issue should be given when allocating funding to small rural schools.

There are a number of implications, arising from the implementation of the NNS in small rural schools,



regarding reception children. Firstly, reception children in whole key stage classes are likely to be exposed to more advanced levels of counting and number operations in the oral and mental starter than those in single age classes. Teachers of reception classes may well have the same level of expectation of reception children as those of a whole key stage one teacher. However, having to work with Y1 and 2 children at the same time, and allowing reception children to answer questions or carry out activities targeted at the older children, has the same effect as raising teacher expectation for reception children within the mixed age class. This must be good news for the more able children in particular.

Secondly, there was heavy reliance on TA support for the youngest KSI children in the sample schools for at least part of the lesson. Teachers in this context will need to make a conscious effort to ensure that the reception children do not miss out on teacher attention. They need to be the focus for regular direct group teaching rather than be continually supervised by an TA, no matter how competent that individual may be. A number of teachers felt strongly about this and agreed that these children must receive such inputs otherwise their future mathematical progress may be hampered.

Thirdly, many of the KSI teachers raised concerns about the reception children having to be too static and stressed the need to make every effort to ensure that the oral and mental starter was lively and fun. Certainly there seems to be a mismatch of philosophy between the new Foundation Stage approach (DfEE 2000) for the early years and that of the NNS when teachers have to implement the strategy for reception children alongside Y1 and 2 children. This is one area where further thought and more advice are needed for small rural schools.

Comparing teachers' initial thoughts on the NNS and their views at the end of the first year it would seem that, although coping with the NNS within a mixed age context still raises concerns, those concerns are slowly decreasing as teachers are getting used to the strategy and finding ways of dealing with the problems that such context brings.

#### **The implications of the findings for ITT**

Students faced with the prospect of a small rural school placement need to be reassured that an effective daily mathematics lesson, based on the NNF, with a whole key stage, is not impossible. One KS 1 teacher summed up the key to success when she listed the following items that a whole key stage teacher needs when implementing the NNS:

- confidence in maths
- good quality help
- exemplary planning (the teacher must be very familiar with the learning outcomes for the whole key stage, and the Early Learning Goals at KS 1 )
- good resources

The Alexander report (1992) highlighted the importance of secure subject knowledge for effective teaching, and government circular 4/98 (DfEE, 1998) laid out the subject knowledge expected of a newly qualified teacher. Section C of the maths annex is viewed by many primary QTS students as being an unnecessarily long and advanced list of competencies and many voice objections to having to study maths to such a high standard. However, the relevance of such knowledge can now be highlighted for them, with reference to the above.

As a result of the project, students specialising in KS 1 can now be informed that the likelihood of whole KS 1 classes having additional adult support for maths is high. However, they will also have to be made aware of the need to ensure that extra adults working within the classroom are well prepared and debriefed with regard to maths. They must realise that if they are placed in a small rural school, this preparation is paramount. The class teacher will expect TAs to be as well prepared for their supervisory role as they are when the student is not in charge.

Students can also be reassured that, in the sample schools (and, therefore, possibly in other small rural schools), the children were used to working independently. This will help during direct group teaching. However, reception children are often an exception to this and this is no doubt why TA support is targeted here.

Students, like class teachers, will need to be encouraged to look carefully at both the new Foundation Stage documentation, and the reception level of the NNS, to see how they can be reconciled. This would have to be done by anyone likely to be dealing with either single aged reception classes or mixed age key stage one classes, but is all the more important in the latter context, where the older children in the class are obliged to follow the more rigid NNS only.

While on teaching practice, students will be asked to use weekly planning proforma (this is the same as one of the examples provided in the NNS training materials). However, those in a mixed age class will have to customise standard proforma according to the number of year groups they have. Medium term planning will

normally be provided by the schools. However, students must be made aware of the various alternatives they may find in schools and understand the importance of being able to link learning objectives for different year groups so that children can work on the same topic if possible. A good working knowledge of the NNS is essential.

Students will also need to be made aware of the fact that the grouping of children for maths will vary from school to school in mixed age classes. The majority of the class teachers taking part in the project grouped by age, the rest according to ability. Whether this situation is mirrored in all small schools would have to be researched further. Students should be aware of the fact that numeracy consultants advised two of the schools to group the children according to ability and that it seems to be a preferred option by the NNS team. The use of resources within the oral and mental starter and the need to keep this part of the lesson particularly lively and varied when teaching a mixed age class (especially a KSI class) needs to be stressed. These planning issues should form an important part of the maths section of any contextual analysis carried out before the start of a school experience. They are of particular importance in a mixed age class context and should be well researched before embarking on such a teaching practice.

### Conclusion

In conclusion, this project has produced some valuable findings, particularly the knowledge and understanding gained from practicing teachers of how the NNS can be implemented successfully in a whole key stage class. This knowledge can now be shared with a variety of parties including fellow maths education tutors and QTS students, both within the institution funding the project and from other institutions, the teachers who contributed and the Numeracy Task Force itself. Although based on a relatively small sample, many of the details given by the teachers were useful additions to the existing materials provided in the NNS training packs and will, no doubt, go towards helping those having to teach in similar contexts. It is felt that there are two areas, raised by some of the teachers interviewed, which need to be looked at more closely. The first is that of catering for the most able, oldest children in a mixed age class and the second that of reconciling the philosophy of the NNS with that of the Foundation stage curriculum within a whole key stage one class. To this end it is intended that this project be developed further in the future to include a wider range of small schools. This will also allow the gathering of further data on the general organisation of the daily mathematics lesson within this context.

### References

- Alexander, R., Rose, J. & Woodhead, C. (1992) *Curriculum Organisation and Classroom Practice in Primary Schools* London:DES
- Bennett, N., O'Hare, E. & Lee, J. (1983) 'Mixed Age Classes in Primary Schools: a survey of practice' *British Educational Research Journal*, Vol. 9, No.1, pp. 41-56
- Cockroft, W.H. (1982) *Mathematics Counts* London: HMSO Dille (1998) *High Standards, High Status* London: HMSO DfEE (1999)*The National Numeracy Strategy* London: HMSO
- DfEE (1999)*Guide for your professional development: Book 1* London: HMSO
- DfEE (2000) *Early Learning Goals* London: HMSO
- Galton, M. and Patrick, H. (eds.) (1990) *Curriculum Provision in Small Primary Schools* London: Routledge
- Galton, M., Hargreaves, L. & Comber, C. (1998) 'Classroom practice and the National Curriculum in Small Rural Primary Schools' *British Educational Research Journal*, V 01.24, No.1, pp. 43-61
- Hargreaves, L., Galton, M., & Comber, C. (1996) 'The National Curriculum: Can small schools deliver? Confidence and competence levels of teachers in small rural primary schools' *British Educational Research Journal*, V 01.22, No.1, pp. 89-99
- HMI (1978) *Primary Education in England* London: HMSO HMI (1998) *Standards in primary Mathematics* Ofsted
- HMI (1998) *The National Numeracy project - an HMI evaluation* Ofsted
- Mason, D.A. and Good, T.L. (1996) 'Mathematics Instruction in Combination and Single-Grade Classes: An Exploratory Investigation' *Teachers College Record*, Vol. 98, No.2 pp. 236-265

Mason, D.A. and Burns, R.B. (1997) 'Reassessing the Effects of Combination Classes' *Educational Research and Evaluation* Vol. 3, part 1, pp. 1-53

Mortimore, P., Sammons, P., Stoll, L., Lewis, D. & Ecob, R., (1998) *School Matters* (Wells, Somerset: Open Books)

Numeracy Task Force (1998) *Numeracy Matters* DfEE

Numeracy Task Force (1998) *The Implementation of the NNS. The Final Report of the Numeracy Task Force* DfEE

Reynolds, D. and Muijs, D. (1999) 'Numeracy Matters: Contemporary policy issues in the teaching of maths' in Thompson, I (Ed) *Issues in Teaching Numeracy in Primary Schools* OUP

Veenman, S. (1997) 'Combination classes revisited' *Educational Research and Evaluation* Vol. 3, No.3, pp. 262-276

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