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Subject Knowledge and Pedagogical Knowledge

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Introduction

At a simple level it would seem to be self-evident that any teacher of mathematics should in some sense 'know' the material that they are being expected to teach. For instance, it would not seem unreasonable that a teacher working with pupils on a GCSE course should themselves be able to obtain near enough full marks on a typical examination paper. It is often naively assumed that somebody who has studied at least some mathematics at degree level would be able to achieve that with little difficulty, although there may be the occasional topic that is unfamiliar. Again it might be assumed that somebody used to working at a higher level would be able to acquire the necessary knowledge readily. It is very clear to those who work with students aspiring to be secondary teachers that many certainly do not have this level of knowledge in relation to A level mathematics, but there are often very significant gaps and misunderstandings with much more elementary aspects of the subject. This is even more evident amongst students training to be primary teachers. Formal levels of subject qualification are not sufficient to ensure that a potential teacher has an appropriate level of subject knowledge, even when it is viewed in this restricted way.

However, the ability to obtain a mark of 100% on an examination paper, whilst necessary, is far from sufficient for effective teaching. A teacher's ability to prepare effective lessons and to respond perceptively and flexibly to the multitude of difficulties that pupils encounter with mathematics is dependent on their own depth of understanding of the topics involved and their own powers of mathematical thinking, as well as their more general pedagogical skills and understandings. Even in the context of a narrow examination defined curriculum, there is always the need to prepare students for longer term examination goals as well as the immediate goal. However, the aims of a mathematical education are much wider than helping pupils to pass examinations. Subject knowledge which embraces depth of understanding, an ability to think mathematically and subject related pedagogical knowledge, as well as content knowledge at an appropriate level, is vitally important to all who teach mathematics.

What is subject knowledge and why is it important?

The phrase 'subject knowledge' is used in the original version of the Standards - Circular 4/98, DfEE (1998) - which sets out what is expected of students in order to achieve Qualified Teacher Status. At the time of writing these Standards are being revised, but there is little suggestion that there will be any significant change to those sections that refer specifically to subject knowledge, except that subject specific elements will appear in a separate Handbook to be labelled 'non-statutory'. This will offer an interpretation of the statutory elements and therefore indicate what is expected, so that, in practice, the framework of requirements will remain essentially unchanged.

To me the most telling subject knowledge requirement in the Standards is the statement that those to be awarded Qualified Teacher Status must be able to 'cope securely with subject-related questions which pupils raise', DfEE (1998). This is a very demanding, open ended requirement which is not at all easy to assess in any direct and meaningful way, but it certainly suggests a deeper and wider knowledge than is required to get full marks on an examination paper. Given the opportunity some pupils ask very demanding questions about ideas that are well within the school curriculum and all pupils should certainly be encouraged to be curious about mathematics and to probe deeply. Teachers need wide and deep knowledge if they are to respond well, even though the response may often be to ask further questions and to point to ways of finding out or exploring further.

Moreover, teachers need subject knowledge that is linked closely to pedagogical knowledge. For example, an awareness of common misconceptions and ways of looking at them, the importance of forging links and connections between different mathematical ideas and the flexibility that comes from seeing alternative ways of looking at the same idea or problem are all essential for effective teaching.

There is a very wide gulf between a desirable level of subject knowledge and the level of knowledge that most student teachers display either at the start or in many cases at the end of their course. This gulf is brought out very dramatically in Ma Liping's comparative study of the subject knowledge of elementary school teachers in China and the United States of America, Ma (1999). She posed four simple arithmetical problems to a sample of teachers in each country and looked at their responses in terms of how they themselves would solve the problem and how they would approach it with pupils in the classroom. For example, one of the problems was $1\frac{3}{4} \div \frac{1}{2}$. Only 9 out of 21 American teachers could answer the question correctly, whereas all of a sample of 72 Chinese teachers were successful. Moreover the successful American teachers were much less successful than their Chinese counterparts in explaining why the process worked or in finding examples to exemplify the calculation. Ma notes also that the American teachers had spent a longer period in higher education before qualifying compared to those from China, including study of mathematics at a higher level. One cannot help but reflect sadly that teachers in the UK are more likely to reflect the weaknesses of the USA than the strengths of China and, of course, that the performance of Chinese pupils is better than those in the UK and the USA.

Another recent study which relates to this issue is *Effective Teachers of Numeracy*, Askew et al(1997), which investigated the teaching styles of primary school teachers and identified the characteristics of those who were most effective as measured by improvements in pupil performance. The teachers fell into three broad orientations, referred to as 'transmission', 'discovery' and 'connectionist'. Most teachers displayed facets of more than one of these orientations, but usually one tended to be dominant. The 'transmission' orientation is characterised by a traditional 'explain and practise' style and 'discovery' emphasises setting tasks through which pupils discover ideas for themselves. The 'connectionists' were teachers who put a lot of emphasis on drawing out the connections between mathematical ideas and developing understanding through discussion and it was teachers with this as their dominant orientation who were found to be most effective. There appeared to be no connection between the level of the teachers' formal mathematical qualifications and their effectiveness.

Both these studies seem to suggest that high level mathematical qualifications are much less important than the depth of teachers' understanding and their ability to make connections within the mathematics of the school curriculum. Study of mathematics beyond school level might be expected to reinforce understanding of elementary mathematics, but anecdotal evidence suggests that this does not necessarily happen. There is little or no formal research evidence relating to the subject knowledge of secondary school teachers, but it would not be surprising if replications of the two types of study produced similar results.

If the quality of mathematical education is to be improved, the emphasis in the Standards on subject knowledge linked closely to pedagogical knowledge is entirely appropriate for our student teachers. The key issue for all involved in teacher education is how to extend the subject knowledge of our students, who often have a very narrow topic and technique oriented view of the subject with limited understanding of where ideas come from, how they are linked and how they can be applied to solve unstructured problems or to generate proofs. They are largely products of a system which has encouraged a narrow view of the subject and they will themselves go on to reinforce that view in their own teaching if their vision is not extended and their understanding and knowledge enhanced.

How can the problem be tackled?

Formal subject qualifications and narrow testing or auditing procedures are insufficient to determine whether student teachers have acquired this more broadly defined subject knowledge. Assessment needs to be placed in the context of tasks designed to enhance subject knowledge, where advice can be offered to help students extend and deepen their knowledge. Formative assessment is much more productive than summative assessment with student teachers just as it is with pupils. The excellent sequence of three articles by Dylan Wiliam, published in *Equals*, Wiliam (1999/2000), draws attention, in the context of school pupils' learning, to the idea of 'rich questions' as a way of revealing misconceptions and to the virtues of offering comments without marks or grades. The same principles seem to me to apply to the way in which subject knowledge should be approached with those aspiring to Qualified Teacher Status.

One of my approaches to improving my secondary PGCE students' subject knowledge has been to develop a Subject Knowledge Workbook. This consists of a variety of tasks designed to enhance both mathematical and pedagogical knowledge. I describe below some of these tasks, their rationale and the sort of responses that students make, but it is important first to describe how the Workbook is used and how this is linked to the ideas of formative assessment espoused by Wiliam. At intervals during the one year course I ask students to hand in their Workbook completed up to a certain page. The books are returned to them with comments indicating areas where further work is needed, together with occasional hints and suggestions, and praise for some good comments or ideas. I do not give them a mark, although I do indicate by means of a red tick at the bottom of each page when I think that sufficient work has been done on that page's topic. As well as the written comments students are given oral feedback, both individually and as a group, and they can always ask for additional advice from myself, their mentors or anybody else. The Workbooks are a part of the formal course assessment in that they have to be handed in complete with their final assignment and have to be such that each page is worthy of a red tick! They do not in any way contribute to the grade that is awarded for method work.

Whilst the Workbook seems to me only to be scratching at the surface of a very big problem, it is regarded by the students as one of the most valuable things that they do during the university based part of the course. They commonly say that they find it both enjoyable and challenging: 'it makes you think' is a common reaction. Our last Ofsted inspector was very suspicious at first, not least because I argued that the Workbook took the place of a subject audit and was much more valuable because it grasped the problem of doing something about the deficiencies that an audit might reveal, besides attempting to do much more. The final report on Hull's mathematics course, Ofsted (2000) said: 'The university has recently developed a very useful 'Subject Knowledge Workbook', which is designed to enhance trainees' subject knowledge and highlight common errors and misconceptions. Trainees make good use of this workbook and their progress is informally reviewed and well supported by the method tutor who ensures that training is differentiated to build on previous academic experience.' So, there is life without a more conventional audit!

What aspects of mathematics and mathematics learning are particularly significant?

There is a lot that I could say about the areas of difficulty that my students have encountered in working on their Subject Knowledge Workbooks. I will highlight three areas by commenting on questions taken from the Workbook:

- • Explaining standard procedures and finding alternatives;
- • Discussing conceptual difficulties;
- • Solving problems and creating proofs in geometry.

Suggest various ways of helping pupils understand that: $\boxed{\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}}$

It is rare to find students who can readily explain where the 'turn it upside down and multiply' procedure for division of fractions comes from or who can offer any alternative procedures. Their first reaction is often 'I have never been told why it works' or 'I have never been taught any other way of doing it'. As they become attuned to expectations they begin, often painfully, to think about mathematical ideas from first principles by trying to put themselves in the place of a pupil who is challenged to find a way to divide one fraction by another when they have not previously encountered a formal procedure.

Seeing that it means 'How many times does $\frac{3}{4}$ go into $\frac{2}{3}$?' is often a valuable first step and thinking about an estimate for the answer is often useful. Interesting alternatives often emerge besides the more conventional explanations. I give two here:

- • Multiply both fractions by 12: $\frac{2}{3} \div \frac{3}{4} = 8 \div 9 = \frac{8}{9}$.
- • Solve $\frac{3}{4}x = \frac{2}{3}$: $\frac{4}{3} \times \frac{3}{4}x = \frac{2}{3} \times \frac{4}{3} \Rightarrow x = \frac{8}{9}$.

Give several alternative methods to show that: $\boxed{\frac{2}{5} < \frac{3}{7}}$

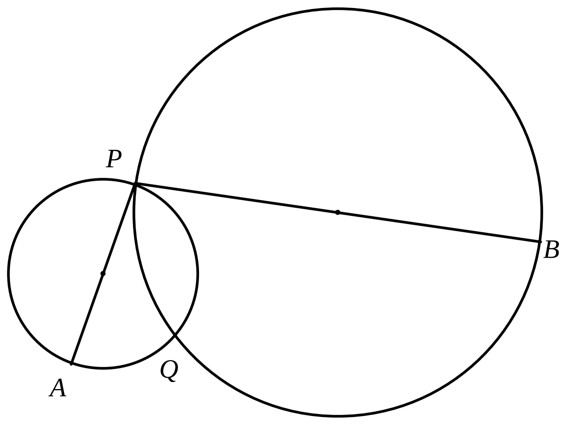
This example has often resulted in incorrect reasoning when students have 'cross multiplied' and obtained $14 < 15$. They then argue that because that is true the original result must hold. This is, of course, false logic, because they have started by assuming that what they are trying to prove is true. Using the same logic I can prove that $1 = 2$, by multiplying both sides by zero and noting that $0 = 0$, which is certainly true. The reasoning is clearly incorrect. The question could be rephrased to ask which fraction is larger, but teachers should be able to reason correctly, so I shall not change it!

- Criticise this statement: a week is 7 days, so $\boxed{w = 7d}$, where $\boxed{w}$ stands for week and $\boxed{d}$ stands for day.
- Comment on the misconception revealed by a pupil whose response to 'simplify $\boxed{p + 2p + 4}$' is $\boxed{3p + 4 = 7p}$

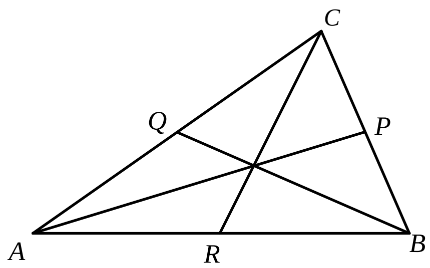
Both these examples illustrate the erroneous use of 'letters as objects', a misconception which I always discuss with students at length when we consider the problems of learning algebra. In both cases, substituting numbers will show that the statements are incorrect, but that is not usually students' immediate response. Indeed, in the first case, they do not always realise that there is anything wrong. Misconceptions of this kind are commonly reinforced by the approaches to algebra in many current school textbooks, a problem that I discuss at length in the second chapter of my book *Learning and Teaching Algebra*, French (2002).

Recent changes in the National Curriculum, DfEE/QCA (1999), have accorded greater importance to aspects of geometrical reasoning and proof, something which many students acknowledge did not feature strongly in their own school experience. They are aware of key theorems, but they often find it very difficult to apply them to problems or use them in proofs. The difficulty with many geometrical problems is to see a key feature of the diagram or to add a line or two which enables a key result to be applied. The two examples from the Workbook which follow illustrate the importance of seeing the right step, which raises the fascinating issue of how appropriate strategies might be acquired.

- Two circles intersect at P and Q. PA and PB are diameters. Prove that the points A, Q and B lie in a straight line.



- Find some alternative proofs that the medians of a triangle meet in a point which divides each median in the ratio 2 to 1.



In the first example, students readily draw in the lines AQ and BQ, but do not always then see that drawing PQ is a key to the solution. A vector solution is the favourite for the second example, but this, although interesting, really is 'using a sledgehammer to crack a nut', because there are various shorter, simpler methods and simplicity is something to encourage. Asking students to look for alternatives is a way of extending their solution strategies. I like to drop hints that relate to questions like this by looking at appropriate strategies when discussing other problems in the weeks before they reach a particular page in the Workbook. Two simple ways of proving the medians property that students often find are;

- showing that the six triangles in the diagram are of equal area and then looking at three triangles sitting on one side of a median;
- ignoring (or deleting) the line CR, drawing in the line PQ and looking at the similar triangles.

What are the ways forward in a wider context?

As I have suggested earlier, the subject knowledge requirements of the Standards are very demanding, but there is a very great latitude in the way that they are being interpreted, particularly in relation to the evidence institutions expect tutors to produce for their own internal purposes or for Ofsted inspectors, who sometimes have a rather narrow content based focus. In both cases we are up against the obsession that the system has at all levels with summative assessment, which runs counter to all the evidence that formative assessment is much more effective in raising standards. We must constantly counter the arguments for more testing by providing evidence that there are other much more effective ways of raising standards, both in teacher education and in school mathematics.

A major difficulty that we face is the problem of time. Students are in school for two thirds of a secondary PGCE course and for a half of a primary PGCE course, where the subject knowledge demands are spread across a whole range of subjects. It is very difficult for the school based time to be used to enhance subject knowledge except in an incidental way, because the daily demands of coping with lesson planning, classroom management and pupil behaviour are inevitably going to be dominant. Self-study has an important part to play, but there is also a vital need for discussion and reflection, both at an individual level and within wider groups, and for access to the expert knowledge which we as PGCE subject tutors surely have. There are many legitimate competing demands for time within a PGCE course. I would suggest that the balance between time spent in school and university is skewed too much towards school at present, but that is a subject for another paper!

Clearly the problem of broadly defined subject knowledge is wider than the needs of those aspiring to Qualified Teacher Status. We know only too well that there is a dire shortage of well qualified teachers and that there are many who are teaching mathematics, whether nominally qualified or not, who would readily admit to having considerable deficiencies in their subject knowledge, as well as many others who have deficiencies, often at a very elementary level, of which they are sadly not aware. There is nothing new about calls for more in-service training for mathematics teachers, but the need for more will continue to be urgent for the foreseeable future. The National Numeracy Strategy and the Key Stage 3 National Strategy for Mathematics have both resulted in a substantial increase in training for teachers, but the emphasis has not primarily been on subject knowledge, although the focus on pedagogy inevitably impinges on the issues discussed in this paper. However, we have little evidence about the effectiveness of different forms of in-service training. It may, for instance, be much more effective for a few teachers to do longer intensive courses, like the 20 day courses for primary teachers a few years ago, than large numbers of teachers doing very short courses.

We can always dream about the golden age that may arrive at some point in the future. We must constantly strive towards that and not lose a vision of how things could be, but we do have to identify our priorities for tomorrow and the next day, by finding ways - often small ways - to improve things whilst working with what we have. I am constantly amazed at the goodwill and enthusiasm of so many student teachers, who often survive in very trying circumstances and still manage to smile. In spite of all the difficulties we can continue to make a difference.

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A Free Offer

I have a limited stock of old copies of the Subject Knowledge Workbook which I will willingly send to anybody on request. I am very happy for anybody to make use of any of the material in the Workbook with their students in whatever form seems appropriate.

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What can, and cannot, be achieved in Mathematics INSET ?

The MEI experience.

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Background

Mathematics in Education and Industry (MEI) was one of the large number of curriculum development bodies, or "projects", formed in the 1960s, mostly in mathematics and the sciences. While most of these have long since ceased to exist, MEI has grown stronger over the intervening years, with a membership consisting almost entirely of classroom teachers. This was in large measure due to our introducing the first modular A level (in any subject), in 1990.

There was immediate and widespread interest in this development, leading us to feel that ours had become a test case course. If we made a mess of it, it would not just be our syllabus that went down but the whole idea of modularity at A Level. With large numbers of schools and colleges adopting our scheme, it was clear that we had to provide training in how to teach and administer it. So, almost by accident, we became major providers of A Level mathematics INSET.

This paper summarises the MEI experience in providing training and advice to a very large number teachers over the twelve years since then, and relates it to the needs of the present situation.

A changing teaching force

Twelve years ago, two themes were of over-riding concern to teachers coming for MEI INSET: how to conduct and assess the various required coursework tasks, and how to stage-manage a modular syllabus. Most of those coming were confident in their knowledge of the subject, and in their ability to teach it. The sessions we provided were well received and met these requirements.

The need for help on the administration was quite short-lived, but anything we could offer relating to the assessment was, and still is, eagerly received. However while such provision is clearly valued by teachers, and valuable to them, we have become increasingly aware that it cannot fully address today's requirements.

This is because the clientele is changing, with a higher proportion of teachers who are not particularly confident in the subject matter themselves. Their needs are different, and much less easily met.

An example is the lady who came to a session on the coursework in our Statistics 2 module. In conversation over lunch, she had the courage to admit that she was herself studying the module at her local College of FE of an evening and then teaching the same material the following day in her school! Clearly she would have benefited from much more substantial help than we could give in a few hours on one day.

We now seem to have reached the situation where it is no longer possible to assume that those who come to INSET can actually do the questions themselves. In the past you could use the technique of getting people to do a question themselves and then discussing round it, the teaching points to be made, what would be deemed important in the assessment and so on. That strategy now carries a real risk of isolating those most in need of help.

This is not meant to be a swipe against today's teachers. Almost without exception, those who come to INSET are keen and want to do a better job of teaching their students. It is just that many of them have less knowledge of mathematics than would have been expected of A Level teachers in the past.

The age profile of mathematics teachers means that this situation is inevitably going to get worse, with a disproportionate number of people coming up to retirement in the next 10 years or so. It seems inevitable that more and more of those teaching mathematics will be learning much of the content of what they are teaching on the job. It needs to be recognised that the pool of people available to be recruited into Initial Teacher Training is not large enough to make up the deficit in the foreseeable future. Consequently many of those delivering mathematics lessons will be people who have started their careers teaching other subjects.

The MEI INSET programme

After a somewhat abrupt beginning, our annual INSET programme soon settled down into a combination of three types of event.

There are a number of training days held at convenient locations around the country. The content is almost entirely assessment orientated. These days are well attended; there is usually quite a high proportion of new or inexperienced teachers.

These are supplemented by a smaller number of specialist days, with a much greater emphasis on the content of particular modules. So far we have only run these days for those modules where the material is unfamiliar to many people: for example Numerical Analysis and Commercial & Industrial Statistics. Usually such days are not well attended, so that it is hard to ensure their economic viability. The exceptions, however, are those covering modules in the decision and discrete mathematics strand, which usually prove popular.

Our third event is an annual three-day conference. Many of the sessions in this are similar to those in the single days but in addition we are able to offer a greater diversity including a fair number that are ICT based, as well as a number of lectures. However the greatest take-up is always for those that are closest to the assessment requirements of the syllabus. Indeed many delegates are under orders from their heads of department to go to just such sessions.

The content of INSET

It follows from what I have just said that much of our provision can be criticised as being narrowly focused on the assessment requirements. The counter-arguments to such criticism raise a number of very important issues which relate to mathematics INSET generally. They focus attention on what is achievable and what is not, and so merit serious consideration by anyone designing an INSET programme.

Teachers come willingly to such days, and their schools and colleges are usually happy to release them to do so, as far as their budgets allow. This is, of course, a major consideration; you achieve nothing unless people come.

A first impression might be that this just shows how examination oriented our education system has become. Undoubtedly there is some truth in that, with school managements wanting to see money spent on INSET translated into better league table positions. It is also the case that all teachers genuinely want to help their students get better results.

However that may well not be the whole story. It is undoubtedly the case that many of today's mathematics teachers feel insecure about the subject; some are, unsurprisingly, reluctant to admit it. Going to INSET based on assessment requirements can seem less threatening, and so more appealing, than risking having one's lack of understanding exposed. So it may well be that many teachers arrive at sessions with titles like "Conducting and assessing Statistics coursework" and "Last summer's Mechanics 1 examination" actually hoping to learn more of the relevant content for themselves, but without needing to admit it to anyone.

Once the tutor running the session recognises that this will be the situation, it is not difficult to plan the content accordingly. In the Pure Mathematics 2 module we have a coursework requirement on the numerical solution of equations. My regular presentation of this includes a run through of the basic mathematics, along with some guidance on how to teach and assess it. About 2% of those who come complain that it was too elementary for them; the other 98% say they found it really helpful.

There are two elements to the assessment, examinations and coursework.

It is possible to dismiss examination questions as being somehow beneath the dignity of those concerned with the build-up of concepts in students' minds. That contrasts with the experience of those involved in setting and revising papers, that a great deal of mathematical thought and care goes into the process. The best questions are works of art, designed to allow candidates to show whether or not they understand the relevant concepts. INSET sessions run by those involved in this process can explain the thinking that has gone into questions and so be genuinely instructive for teachers. That, too, is often the experience of markers attending examiners' standardisation meetings.

AS/A Level Coursework provides a particularly good medium for professional development because it moves the subject beyond the limited problems set in examinations towards those encountered in real life. Often these require a deeper understanding than that required to perform routine techniques. The principles underlying the subject, encapsulated in themes like sampling, mathematical modelling, error analysis and proof, suddenly come to the fore. Sadly the increased time pressure resulting from Curriculum 2000 has forced us to reduce our coursework requirements.

On a number of occasions mathematics graduate teachers have told us that they have learnt more of what mathematics is really about from MEI than they did in their degree courses. So beware of the danger in being too dismissive of assessment based INSET.

Who delivers the INSET ?

With our membership consisting almost entirely of teachers, it is natural for MEI to look to them to provide much of our INSET. At its best this works very well. It is particularly easy for a teacher delegate to relate to a tutor saying "This is how I did it with my Year 12 students last week …".

However that can present problems with sessions that are based on the assessment. Where marks are involved, be it in examinations or coursework, the information given must be consistent with how those marks are actually given. Anyone delivering such INSET really needs to have first hand experience of the assessment, at least as a marker or moderator. That, of course, introduces the danger of the mathematics taking second place to tips on how to secure the most marks on the paper

A curriculum development body like MEI tends to attract many of the most enthusiastic teachers in the country, people whose basic motivation is a love of the subject. It is common for such people also to be involved in the assessment process. All of those employed by the examination board to set or revise our papers would place themselves in this category, as would many of those marking the papers and moderating the coursework.

So there is a pool of suitable people to run our INSET days. However the increasing pressure on teachers' time makes it difficult for them to do so, and in practice a lot of our provision is delivered by our small professional staff ; we typically cover about half of the sessions on our training days. (This does have advantages in terms of continuity both in terms of the content of our sessions and also on a personal level of knowing and being known by many of the teachers coming to our INSET.) There is, however, no problem in finding people keen and eager to run one or two sessions at our conference.

Being able to call on so many people, including a core of tutors, is a luxury that the examination boards do not have to the same extent. Consequently it may well be that some caution needs to be exercised in extrapolating our experience on this point to appropriate provision for the boards' own syllabuses.

INSET or Professional Development

In most of this article I have used the term INSET rather than Professional Development. The provision that I have written about is well described as "training". It is designed to help people with some knowledge of what they are doing to do it better.

However this does not, and cannot, meet the needs of many of those who are now teaching mathematics in our schools. An increasing number of teachers need both to learn the content of at least A Level mathematics themselves, and to understand the philosophy behind it. Such Professional Development needs a totally different sort of provision. It is dishonest to pretend that an odd day or two will suffice.

Perhaps the greatest challenge now facing mathematics in this country is to find ways of turning today's P.E. specialist into tomorrow's mathematics teacher.

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Implementation of the National Numeracy Strategy in Birmingham LEA

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In 1999 the Faculty of Education won a contract to monitor the delivery of the numeracy strategy in Birmingham LEA. This paper presents the results of that monitoring and puts it into context with respect to other monitoring and evaluations of the strategy. The picture painted in Birmingham is a positive one. Aspects of the strategy are being taken on board and implemented with enthusiasm by teachers, received well by pupils and appear to be having a positive effect on standards.

This article is presented with the full approval of Birmingham LEA. The author and the research team wish to thank the Authority, schools and teachers involved for their support and co-operation in this study.

Introduction

In 1999 the Faculty of Education was contracted by Birmingham LEA to carry out independent monitoring and evaluation of the implementation of the Numeracy Strategy in the Authority. The purpose of the monitoring exercise was ‘to help both the LEA and the schools by putting together a clearer picture’ of various issues regarding the Strategies (UCE, 1999).

The purposes of this paper are:

- to present the main findings of this project (the Birmingham Study).
- to make comparisons between the situation in Birmingham, the desired outcomes devised by the Numeracy Task Force and the national picture.
- to highlight issues requiring further attention.

Background: Development of the Strategy.

The Numeracy Task Force.

The Numeracy Task Force was established by David Blunkett, Secretary of State for Education and Employment, in May 1997. Its remit was to develop a strategy to raise standards of numeracy in order for the national numeracy target to be met by 2002. This target is for 75% of 11 year olds to achieve the standards expected for their age in mathematics. In objective terms the target is for 75% of 11 year olds to achieve level 4 in the National Curriculum tests.

In devising recommendations for a strategy the Task Force took a broad view of current research and practice. Their aim was to raise standards. With this in view they looked to build on existing good practice whilst replacing that identified as being less successful or ineffective. Their recommendations were based on strategies already acknowledged as being beneficial in raising the standard of primary mathematics. This was done in the context of learning from the effective practice of other countries as well as that of our own. Reynolds, D. et al. (1998) identified a number of desired outcomes for the strategy along with some general principles on how they should be achieved.

Crucial in the way forward was the *Framework for Teaching Mathematics* (DfEE, 1999). This document gave general advice on aspects of the NNS and provided a recommended teaching programme from Reception through to Year 6. Of particular importance to the class teacher were the four principles on which the approach to teaching advocated by the strategy were based:

- Dedicated mathematics lesson everyday;
- Direct teaching and interactive oral work with the whole class and with groups;
- An emphasis on mental calculation;
- Controlled differentiation, with all pupils engaged in mathematics relating to a common theme; (DfEE, 1999. Sec1 p11)

When evaluating the progress made in Birmingham it is against a selection of the desired outcomes and these general principles that judgments will be considered and recommendations made.

Review of monitoring and evaluation of the National Numeracy Strategy.

In *Watching and Learning* (Earl et al, 2000) the Ontario Institute team, who had been contracted by the DfEE to evaluate the National Literacy and Numeracy Strategies, outlined what they considered strengths and challenges of the NLNS. In considering them it must be taken into account that this was very early days for the NNS. However the report identifies several areas of the design and implementation of the strategy as strengths among these are:

- Leadership:

  The strategy is acknowledged as having strong leadership at the highest level with those leaders having "high credibility among educators (Earl et al, 2000 p. 38).

  This feature is also reported as being evident at regional level.

- Support: The body of support provided both in terms of resources and expertise was acknowledged to have the potential to bring about change.

- Resources: The strategy is considered to be "adequately funded, with funds tied to specific implementation initiatives" (ibid. p. 39). The provision of high quality resources is reported as having helped "clarify the nature of the practices being advocated by the Strategies." (ibid.)

- Responsiveness and adaptability: It was acknowledged that close attention had been paid to feedback from the field.

A number of challenges are highlighted. Of particular interest with respect to this study is the recognition that changing practice is, and will continue to be hard work and that educators at all levels "are likely to experience frustration and tire along the way" (ibid.). Also identified is a need to cope with demands that may not yet be anticipated as well as with those that will change as the implementation of the strategy proceeds. Clearly putting ideas into practice is a complex activity but what is more important and more complex is "not only to establish large scale reform, but also to sustain it" (ibid). This of course must be the target if desired outcomes of the Task Force are to be achieved.

In their report, HMI (2000) acknowledged qualified progress with the implementation of the Strategy nationally.

> *The NNS, through the three-part daily mathematics lesson, is having a profound effect on the way that mathematics is taught in primary schools The Strategy has made a good start but there is more to be done if it is to achieve its full potential. (p. 1)*

Amongst the positive aspects identified are:

- Mathematics lessons much more sharply focused on clear learning objectives.
- Considerably more direct interactive teaching…
- the priority given to the development of pupils' oral and mental skills…(p. 1)

However concerns were expressed at inconsistency in the effectiveness of the delivery of aspects of the three-part lesson:

> *There are weaknesses in the teaching of at least one of the parts of the daily mathematics lesson in around a quarter of lessons (p. 1)*

The mental and oral starter was identified consistently as being the most effective part of lessons while the plenary was considered to be the weakest. Timing was also highlighted as an issue with a significant number of lessons taking longer than the prescribed time. There was concern at the effects of this on other important aspects of the curriculum.

Another concern highlighted was the quality of subject knowledge of those delivering the Strategy at the chalk face:

> *there are important aspects of the teaching of mathematics with which many teachers are not yet secure. (p. 1)*

This pattern of qualified success is evident throughout the report. Other aspects will be discussed in direct comparison with the results of the Birmingham Study.

British Market Research Board (BRMB)/Centre for Better Teaching (CfBT) Poll

The annual poll of primary headteachers, commissioned by CfBT and conducted by BMRB, provided encouraging support for the Strategy. The key findings indicated that:

- 98% of heads support the Numeracy Strategy;
- 91% said that the Strategy had improved the quality of mathematics teaching a great deal or a lot;
- 92% were confident that the Strategy would raise standards;
- 82% said that the daily mathematics lesson had made teachers more confident teaching mathematics.

Summary of monitoring

There are several features common to each of these evaluations:

- The Strategy is widely accepted by heads and teachers
- There is a perception that the strategy has and will continue to raise standards
- There are still some important aspects of the strategy that require attention at classroom level.

The strategy is still at an early stage of implementation. It has so far gone well but the real challenge to continued progress is the concern “that we are likely to experience frustration and tire along the way.”

Monitoring Numeracy in Birmingham

In 1999 the Faculty of Education was contracted to carry out independent monitoring and evaluation of the National Numeracy and Literacy Strategies for the Birmingham LEA. The LEA senior advisors responsible for the strategy selected 34 of the LEA’s schools for involvement in the Project. They described the schools as ‘a representative sample ’that had been ‘identified by the Research and Statistics Department, reflecting the spread of schools in the STRAND structure outlined in the EDP and including “Light Touch” schools’ (Birmingham LEA, 1999).

The purpose of the monitoring exercise was ‘to help both the LEA and the schools by putting together a clearer picture’ of various issues regarding the Strategies (UCE, 1999). The issues included:

- The process of implementation of the Strategies in schools.
- The quality of teaching of the two Strategies.
- The effectiveness of the schools’ own systems for monitoring and evaluation.
- The impact of the consultants’ work and the LEA’s training programme.
- Trends in pupil attainment and progress.
- The impact of partnership with parents.
- The impact of initiatives such as homework and summer schools.
- The impact of resources and the learning environment.

A Steering Group for the Project was set up, with representatives of schools, the LEA and UCE, and chaired by the Senior Primary Adviser.

The two Strategies were investigated and evaluated separately. For the Numeracy Strategy qualitative data was collected from schools by a team of numeracy tutors from the Faculty of Education. Schools were visited three times between February and July 2000. Each visit was typically a day-long and two classes class were observed. Classes were selected from Reception and Year 4. Feedback and discussion sessions of approximately 30 minutes with each teacher followed. In addition there were structured interviews with headteachers, co-ordinators, class teachers and pupils. In all, the Numeracy Evaluation comprised the following:

- 102 days of data collection

- 186 lessons observed
- 34 headteacher interviews
- 32 co-ordinator interviews
- 65 teacher interviews
- 300 (approx.) pupil interviews (5/6 pupils per class)

The data-collection instruments

Observation of teaching

A set of indicators which covered the structures of the Numeracy lesson was used, against which observers recorded evidence of the quality of teaching and/or learning in each lesson. Their observations were discussed with each teacher after the lesson and teachers' responses noted.

In addition to the observation schedule, after each observed lesson teachers were asked 'Clarification Questions', which elicited their responses on various background aspects of lesson preparation and teaching. Teachers were asked about the extent to which they were following the Strategies in their planning and teaching. The responses have been integrated into the analysis of the interviews.

Interview questions

A common core of questions was put to the headteachers, the co-ordinators and the Reception and Year 4 teachers. In addition, a number of questions specific to their roles were put to the heads, co-ordinators and class teachers respectively. Interviews lasted for an average of one hour each. In a few cases it was not possible to carry out interviews, and the schedule questions were given to participants to complete and send to the Faculty. Information collected from the different participants in observations and interviews was treated in strict confidence and not discussed with others.

Pupil interviews

Semi-structured interview questions were used to gather pupils' responses regarding their perceptions of the Strategies. Pupils were interviewed in groups of 5/6 for about 10/15 minutes each group.

Data preparation and analysis

A Microsoft Access database was created and all the data collected through observations and interviews was entered for systematic preparation and analysis.

Observation and interview data were categorised into themes/topics under each indicator and then percentages calculated to indicate strength of evidence or opinion. The interview schedule was not supplemented by probes, prompts and follow up questions, as this would have made the already extensive and intensive schedule unmanageably long.

Selected points for discussion.

A positive picture of the implementation of the Strategy in Birmingham was revealed by the monitoring project. The main points together with issues for discussion are presented below:

Reception

Observations showed very many positive features of the teaching and learning of numeracy in Reception classes.

- All teachers were using the Strategy Framework for planning. The majority of teachers felt that implementing the Strategy had improved their teaching in terms of planning, structure, and in providing a clearer focus. Greater use of interactive teaching methods was also identified as an outcome of the Strategy.
- Nearly all lessons were using the three-part structure with a high proportion of direct teaching. Pupils were given opportunity to work in a variety of ways and were enthusiastic. There was a wide range of content and activity reflecting the range within the Strategy Framework.
- Over the three visits to schools, plenary sessions addressed an increasing range of issues including misconceptions and difficulties, reflection of learning and identification of what needed to be remembered.

There were no areas of serious concern. However, there are a number of issues for discussion:

Pupils' Explanations

Some teachers still used mainly closed questions, with few opportunities for pupils to give explanations or apply reasoning. Teachers sometimes felt that encouraging extended answers would slow the pace, and some felt that this approach was inappropriate for Reception pupils. Open-ended questions don't have to involve lengthy explanations. Some Reception lessons provided excellent opportunities for pupils to justify and give explanations without losing pace or focus.

Problem Solving

Few lessons observed involved aspects of problem solving. This aspect is a strand within the Strategy Framework for Reception and perhaps is addressed solely within the lessons specified as such. Should this aspect remain as a separate skill or should it be addressed, when appropriate, throughout pupil's learning? It has been identified as an area of concern at primary level, nationally, in the HMI evaluation. (HMI, 2000)

Pace and Timing

There are a significant minority of Reception teachers who feel that the timing and expected pace of the lessons are inappropriate. Are these seen as inflexible? Perhaps flexibility needs to be discussed. What is appropriate pace?

The expectation of the Strategy is that by the end of Reception year, pupils and teachers are working within the format adopted by Key Stages 1 and 2, although it is suggested that timing is, to some degree, flexible.

Lesson Structure

On the basis of the lessons observed, teachers are teaching numeracy to the whole of Reception class at the same time. Some teachers felt that the lesson structure made it difficult to cater for all children's needs. Can extra adult support be used to provide more effectively for the range of needs? Or, are there ways of working which move *towards* the Strategy lesson structure which teachers would find more helpful?

The plenary

Some pupils found it hard to concentrate throughout this part of the lesson. Is it possible to address the various aspects promoted by the Strategy and maintain interest?

Year Four

There is a good range of evidence showing very many positive features of teaching and learning numeracy in Year 4.

- Teachers felt that planning and structure associated with the Strategy were important factors in improving their own teaching. Also, some Y4 teachers identified questioning, demonstration and broader subject knowledge as ways in which their teaching had improved.
- The three-part lesson was clearly established, with a brisk pace observed in most lessons. Lessons observed featured a high proportion of direct teaching. Pupils were given opportunities to approach their work in a

variety of ways and were generally enthusiastic.

- Pupils were made aware of what they were going to learn during lessons and appropriate references were made in plenary sessions. Difficulties and misconceptions were addressed in an increasing number of lessons.

As with Reception Age Phase there were no serious areas of concern. However, there are a number of issues for discussion:

Teaching difficult topics

Although some teachers listed ‘broadening subject knowledge’ as a result of the Strategy there remain some teachers who still find certain aspects difficult to teach.

Most teachers gave clear demonstrations and explanations but there were lessons observed where explanations were confusing and demonstrations unclear.

Pupils’ explanations

There were some lessons where pupils were not given opportunity to provide explanations or where single word answers were always expected. Very little use was made of ‘talking partners’ or other strategies that might encourage pupils to give fuller answers.

Differentiation

Where schools have set for maths, some teachers felt that it was unnecessary to provide any differentiation within the set. In these circumstances there was still a wide range of ability in the set. Is setting alone enough to ensure appropriate tasks for the pupils?

Main part of lesson

In over a third of the lessons observed, the teacher didn’t work with focus groups at all during the lesson. In many of these cases pupils worked individually on a set task, whilst the teacher supervised. The issue of teaching during this part of the lesson was raised in the interim report on the NNS. How can teachers effectively teach a group and ensure that the rest of the class are on task and coping?

Dealing with errors

Sometimes errors were identified but not effectively dealt with. Perhaps some teachers find it difficult to know how to use errors as teaching points.

Whole school Issues.

- Discussions with head teachers, numeracy co-ordinators and class teachers indicated general agreement about the positive value of the Strategy.
- Most people involved in the audit thought it had been useful.
- Many people felt that the Strategy was central to the school development plan.
- Co-ordinators were involved in range of support activities that were positively regarded by class teachers.
- Many head teachers felt that LEA training had been very effective and feedback had taken various forms.
- Schools reported that a variety of school self-evaluation strategies were in place and that evidence of a positive effect on standards was emerging.
- Almost all people who had experienced LEA training and consultant support were positive about its effect in school.

- Schools had increased their resources, both purchased and school-produced, and it was felt that evidence of a positive impact on standards was emerging.

There were no areas of serious concern arising from the general points but there are a number of issues for discussion:

Effect on other curriculum areas

Although most respondents thought that the Strategy had a positive effect on other curriculum areas, some felt that it had resulted in a reduction in time available for other subjects. Do some schools feel under pressure to commit more than an hour per day for maths? Is timetabling an issue?

Target setting

Target setting varies. There is a mixture of individual, group and class target setting with different schools opting for different combinations. This may not present a problem but the process didn't always appear clear.

School Action Plan

Many class teachers had no involvement with the school's action plan for maths and seemed unaware of its usefulness. Should all staff have some involvement?

Further development

Co-ordinators identified the following areas as those they felt needed further development: the plenary; grouping; independent learning; pupils articulating their ideas.

Head teachers, numeracy co-ordinators and class teachers identified two common factors which they felt would help to raise standards: a greater number of qualified classroom assistants and further staff training to include shared observations of teaching.

Leading Maths Teachers

Although half of Y4 teachers had observed a Leading Maths teacher, only one in four Reception teachers had. Why is this the case? How can more use be made of this type of expertise?

ICT

Little use is made of ICT to support numeracy lessons, less in Y4 than in Reception. The new NNS ICT pack might help but maybe teachers feel that it is difficult to incorporate within the lesson structure. Historically, ICT has not been used when teaching groups or whole class.

SEN Pupils

Most SEN pupils were included in sessions but in a significant minority of cases they were withdrawn. Is practice changing? How are decisions made about inclusion?

Desired Outcomes and the Wider Context

Birmingham's performance in relation to the Task Force's desired outcomes is now considered. The discussion focuses mainly on those desired outcomes that have a direct effect on the mathematical experience of pupils. Comments referring to the performance of schools in Birmingham are taken from the summary of numeracy observations in the Birmingham Study.

Schools in Birmingham seem to have adopted defined, dedicated, daily mathematics lessons that feature the recommended three-part structure. The lessons have a strong focus on numeracy and involve a high proportion of interactive direct teaching. Many schools have attempted to extend the learning time for mathematics beyond the daily lesson by setting regular homework and trying to involve parents in a partnership to improve their children's numeracy. Where parental support was strong - about one in three schools- heads suggested that there was an improvement in pupils' enthusiasm and sometimes in their attainment. However in about a third of schools teachers reported that homework had little impact because of a poor response by parents.

Desired outcomes

- Primary schools give defined teaching time to mathematics, with daily lessons, a high proportion of which are devoted to numeracy.
- Primary schools extend learning time though out of class activities and homework.
- A greater emphasis is given in the curriculum to oral and mental work to secure the foundations of numeracy, before written methods are introduced.
- All children have the opportunity to take part regularly in oral and mental work that develops their calculation strategies and recall skills.
- Teachers have a secure subject knowledge of mathematics that is relevant to the primary curriculum and to pupils' later development.

Reynolds, D. et al (1998)

A small number of schools made after-school provision of clubs that involved some mathematical input. Where provision was made support from parents and children was considered to be good. Almost all schools reported no provision of summer schools with a mathematical content. However, as with the national picture, booster classes were seen as an important feature in the raising of standards.

> *Booster classes have been welcomed by schools, and have contributed to improving the achievement of pupils on the borderline between Levels 3 and 4.* *HMI (200) p. 3*

The emphasis on mental and oral work is being addressed in Birmingham Schools. The vehicle for this has been largely, but not entirely, the mental and oral starter. An interesting comment from one of the observers was that on occasion it was difficult to spot the transition to the main part of the lesson. This would seem to be due to the interactive mental and oral approach extending into the main part of the lesson.

Many teachers in Reception used a variety of short oral and mental activities. Nearly all observed sessions included some form of appropriate counting while nearly half addressed aspects of vocabulary and involved some form of mental imagery. The situation in Year 4 was similar. In the majority of sessions the teacher led a variety of oral and mental activities involving mental recall, mental calculations or activities using number operations and relationships. However, in reflecting on difficulties in the mental and oral session more than a quarter of teachers indicated that they had experienced problems with catering for all abilities and keeping all pupils fully involved.

This positive view of mental and oral work, and in particular the mental and oral starter, is also referred to by HMI (2000) who found this to be the most effective part of the daily mathematics lesson. However HMI identify well-directed questioning as a strong feature in mental and oral starters with most teachers requiring more than single word answers and expecting children to explain their thinking. The Birmingham study revealed some teachers giving children only few opportunities to explain their reasoning often relying heavily on closed questions. However opportunities for explanation became more frequent over the three phases of the study.

Teacher's subject knowledge remains an area for development with some teachers finding some aspects of mathematics difficult to teach. However an encouraging sign is that a number of teachers and headteachers identified broadening subject knowledge during the implementation of the strategy. It is not clear from the study what aspect of the strategy caused this effect. There are a number of possibilities:

- The general emphasis on mathematics in general, and the identification of clear learning objectives in particular, may have caused teachers to focus more of their attention on really understanding the mathematics that they are delivering.
- Cascade of knowledge from co-ordinators and other teachers attending five-day course and other training.
- Delivery of the Professional Development materials during school INSET sessions.

The majority of headteachers and co-ordinators indicated that they had used the Professional Development materials extensively. However more than half of teachers reported no use of them outside of INSET sessions. Some were not aware of the range or even the existence of such materials. Data from the BMRB survey indicates that the highest frequency of use in INSET of any session from Professional Development Books 3 and 4 was 63%. This was the session on entitled "Developing mental strategies in Key Stage 2". This would tend to suggest that on a national level there are many teachers who have not had access of any sort to the training material in these books. This raises the question of how the Strategy's approach to calculation, solving word problems, using a calculator, addressing special needs in the mathematics lesson and other important areas is being communicated to teachers who have not had the benefit of INSET involving this material from Professional Development Books 3 and 4.

The situation in Birmingham seems consistent with the national picture where weakness of subject knowledge for some important areas of mathematics is a key concern of the HMI report. The need to develop aspects of teacher's subject knowledge is currently being addressed in Birmingham (and nationally) by extending the opportunity to attend the five-day course to an increasing number of teachers. This is a course of action recommended in the HMI report:

> *There remain many teachers who would benefit from attending the five-day courses; in particular, further training is required to address weaknesses in the mathematical knowledge and understanding of teachers.* *(HMI 2000 p. 3)*

It was noted in the Birmingham Study that throughout most lessons observed there was an interactive approach with teachers engaged with the whole class at some stage in virtually all lessons. Lessons were generally appropriately paced and included clear demonstrations and explanations although there were a small but significant number of lessons where this clarity was not evident.

The focus of the lesson was usually made clear to the children. Mostly this was done verbally but often such explanation was accompanied by written objectives displayed on a dedicated part of the board. On occasion pupils were questioned about their understanding of the objectives. Links were sometimes made with the objectives of previous lessons. However a distinction perhaps needs to be made between telling children what they are going to learn rather than what they are going to do.

The majority of lessons observed included appropriately differentiated tasks. These tasks were always based on a common theme and organised in a manageable way. In the lessons where differentiated tasks were not offered it was often the case that pupils were set for mathematics. In such cases teachers felt that further differentiation was unnecessary. What is not clear is whether these teachers would provide differentiated tasks should the need become obvious. The decision not to provide differentiation within mathematics sets as a matter of 'policy' is perhaps one that should be challenged.

Desired Outcomes

- More time in mathematics lessons is devoted to interaction between teachers and pupils about mathematics, especially in interactions with the whole class, and also in groups
- Teachers are clear about learning objectives and progression in relation to pupils' knowledge skills and understanding in mathematics, and can share this information with pupils and parents.
- Teachers know how to illustrate, demonstrate and explain mathematical concepts, offering models and contexts from which the key ideas can be extracted.
- Teachers provide appropriately demanding work for pupils, with limited differentiation around common work to all pupils in one class.
- Teachers establish appropriate links between different topics in mathematics, and between mathematics and other subjects.
- Less time in mathematics lessons is spent working and trouble-shooting with individuals, and in using questions that do not challenge children to think.
- Teachers are knowledgeable about the forms of classroom organisation that are most effective in improving standards of numeracy, and know when it is appropriate to use each particular form.

Reynolds, D. et al (1998)

About a third of teachers reported that they found differentiation difficult. Some indicated that the issue was one of finding suitable activities for groups while others found it very difficult to work with a focus group while making sure that independent groups were engaged with appropriately demanding work. The availability of additional adults in Reception classes tended to ease this problem but in Year 4 in over a third of lessons the teacher did not work with a focus group at all.

It is not clear from the observations what degree of variation in approach to classroom organisation was employed by teachers. It appeared that where there was differentiated work for groups the grouping from lesson to lesson, and from unit to unit, remained unchanged. It is also not clear how often in such classes that all children would be given the same task to do in the main part of the lesson.

Desired Outcomes

- All teachers have access to key resources for the classroom and individual pupils, and use them effectively to teach mathematics.
- Teachers are well-informed about ICT that can enhance the teaching and learning of mathematics, and are confident and competent in using it.
- The National Grid for Learning provides an up to date, accessible and stimulating source of ideas for the classroom and information about good practice for teachers. Reynolds, D. et al (1998)

The Birmingham study indicated that a good range of appropriate resources was made available to children in about two-thirds of lessons. Where resources were used children were observed to be making good use of them. A majority of schools reported that the implementation of the Strategy had demanded an increase in resources for mathematics. Some schools said that the demand for additional resources was substantial. The majority of teachers thought that the use of additional resources was making a positive contribution to improved standards citing enriched understanding; pupil motivation and confidence; and improved teacher knowledge as areas of improvement. Only about a quarter of teachers said that more resources were required with even fewer indicating this as a priority.

The provision and utilization of resources in Birmingham Schools reflects at a local level the ‘strength’ identified by the Ontario Team. It was true of many observations in the Birmingham Study that new, high quality resources were helping to “clarify the nature of the practices being advocated by the Strategies”. (Earl et Al, 2000 p. 39)

Appropriate use of ICT in the numeracy lesson in Birmingham Schools seems underdeveloped with about two-thirds of teachers indicating low useage and little use being observed during the study. The only cases of ICT being used when teaching groups or the whole class were in the context of a computer suite. Many schools did report plans to improve the provision of appropriate computer hardware and software. In many cases this was to be with the development of a computer suite. What is not clear is the way in which the new hardware and software is to be used to support numeracy. Computer suites seem to be only a part of the answer. One teacher explained that her class were allocated an hour a fortnight in the computer suite which didn’t allow them much opportunity to use ICT to develop skills in numeracy. Perhaps more imaginative approaches to using ICT need to be developed.

The picture painted by HMI is not dissimilar. They suggest that schools recognised the role of ICT in developing numeracy but that there was much variation in practice. Many schools were reported to have “too little software relating to mathematics or needed help to match it to Strategy objectives” (HMI 2000 p. 17). The new NNS ICT pack may help schools in making appropriate use of ICT. However at the time of the BMRB poll less than a half of schools had received a pack and only about an eighth of schools had used it. Effective use of ICT to support the teaching and learning of numeracy seems to be a difficult ball to get rolling. Addressing the reasons for this both in Birmingham and on a national level would seem to be a priority.

Conclusion

This paper has only discussed a few aspects of the implementation of the Strategy in Birmingham in any detail. However the general situation tends to be positive. Teachers are embracing the Strategy and feel well supported by the LEA’s Numeracy Team. The increased confidence and enthusiasm displayed by teachers seems to be transferring to the pupils, the majority of whom expressed the view that they enjoyed mathematics lessons. There is a general view that the Strategy is having a positive effect on standards of numeracy displayed by primary pupils.

There are of course features that need further development. These include:

- Teachers’ subject knowledge.
- Wider use of the Professional Development Materials.
- Appropriate use of ICT.
- Approaches to Problem Solving.

Good progress towards the desired outcomes devised by the Task Force has been made by schools in Birmingham. They have made an effective start, but, in the words of the Ontario team, must now take steps to ensure that they don’t “experience frustration and tire along the way.”

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Graphical Knowledge Display – Mind Mapping and Concept Mapping as Efficient Tools in Mathematics Education

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In didactical discussion there is a widespread consensus that mathematics should be experienced by students as a network of interrelated concepts and procedures rather than a collection of isolated rules and facts. This experience may be supported by representing mathematical knowledge graphically in the form of networks.

In this paper, two special graphical representations of mathematical networks, mind maps and concept maps, are presented. Both are means to show ideas and concepts connected with a topic. Their suitability as a pedagogical tool for mathematics education is considered and the possible applications of mind mapping and concept mapping in mathematics education together with their advantages and limits are discussed. It turns out that both, mind mapping and concept mapping, may be efficient tools to improve mathematics achievement.

1. Introduction

Mathematical knowledge has the character of a network, as mathematical objects, i.e. concepts, definitions, theorems, proofs, algorithms, rules, theories, are interrelated but also connected with components of the external world. Accordingly, there is a widespread consensus in the didactical discussion that mathematics should be experienced by students in its interrelatedness (see e.g. NCTM Yearbook 1995, Preface, or NCTM Principles and Standards for School Mathematics 2000, p.64). The importance of this notion also becomes apparent in the recent PISA–Study, where interconnections and common ideas are central elements (OECD, 1999, p.48).

One means to experience the network character of mathematics is by visualising it. Two methods especially suited for representing graphically a mathematical network around a topic are mind mapping and concept mapping. These two techniques are presented below; their suitability as a pedagogical tool for mathematics education is considered and the possible applications of mind mapping and concept mapping in mathematics education together with their advantages and limits are discussed.

2. Mind mapping

2.1 Background

Mind mapping was firstly developed by Tony Buzan, a mathematician, psychologist and brain researcher, as a special technique for taking notes as briefly as possible whilst being interesting to the eye as possible. Since then, mind mapping turned out to be usable in many different ways other than just simple note taking. Mind maps have, among other things, been used in education, but despite their usefulness (see 2.2) are surprisingly rarely used in mathematics.

The method of mind mapping takes into account that the two halves of the human brain are performing different tasks. While the left side is mainly responsible for logic, words, arithmetic, linearity, sequences, analysis, lists, the right side of the brain mainly performs tasks like multidimensionality, imagination, emotion, colour, rhythm, shapes, geometry, synthesis. Mind mapping uses both sides of the brain (Buzan, 1976), letting them work together and thus increases productivity and memory retention. This is accomplished by representing logical structures using an artistic spatial image that the individual creates. Thus mind mapping connects imagination with structure and pictures with logic (Svantesson, 1992, p. 44; Beyer, 1996).

2.2 Rules for making mind maps

Mind maps are hierarchically structured. They are produced following the rules given below (see e.g. Beyer, 1993; T. Buzan & B. Buzan, 1993; Hemmerich et al., 1994; Hugl, 1995, p. 182; Svantesson, 1992, p. 55-56):

- Use a large sheet of paper without lines in landscape format.
- Place the topic of the mind map in the centre of the paper. (The topic of the mind map should be displayed in an eye-catching way, preferably by a coloured image. If a picture does not seem appropriate, the topic should be named by a well-chosen keyword.)
- From the topic draw a main branch for each of the main ideas linked to the topic. Write keywords denoting the main ideas directly on the lines. Use printed letters. (The order of the branches is not important. If a special order is needed for understanding the topics, the branches may be

numbered or ordered clockwise. If possible, only one word per line, preferably a noun, should be written down. As 90% of the words in texts are unnecessary, using a few meaningful keywords will be sufficient to remember the entire context.)

- Starting from the main branches you may draw further lines (sub-branches) for secondary ideas (sub-topics) and so on. The order follows the principle: from the abstract to the concrete, from the general to the special.
- Use colours when drawing a mind map.
- Add images, sketches, symbols, such as little arrows, geometric figures, exclamation marks or question marks, as well as self-defined symbols to your mind map.

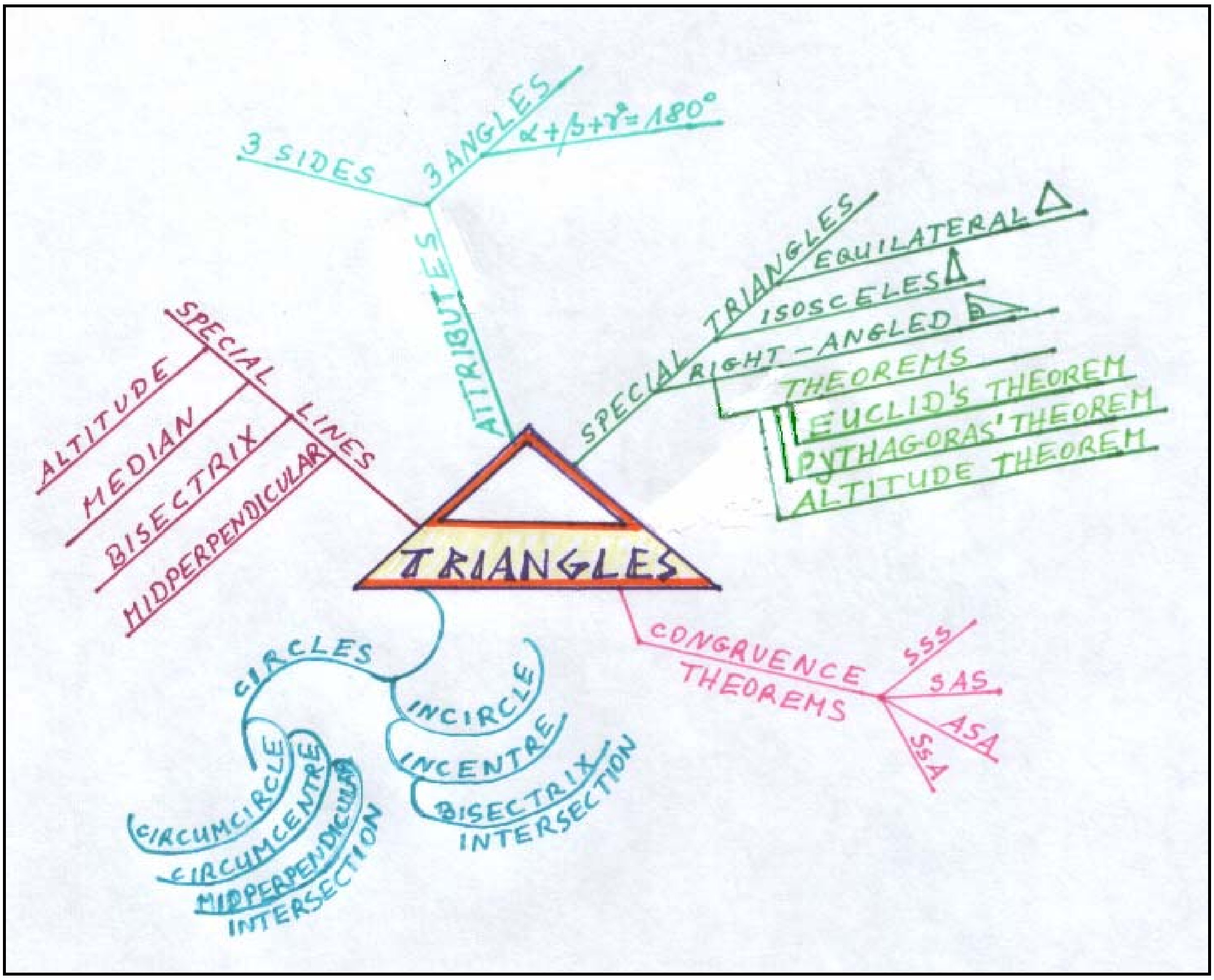

Figure 1: Mind map on the topic of triangles

2.3 Mathematical mind maps

Both, the structure of a mind map and the technique of mind mapping emphasise the usefulness of mathematical issues as topics for mind maps (Brinkmann, 2000, 2001b, 2002, in press).

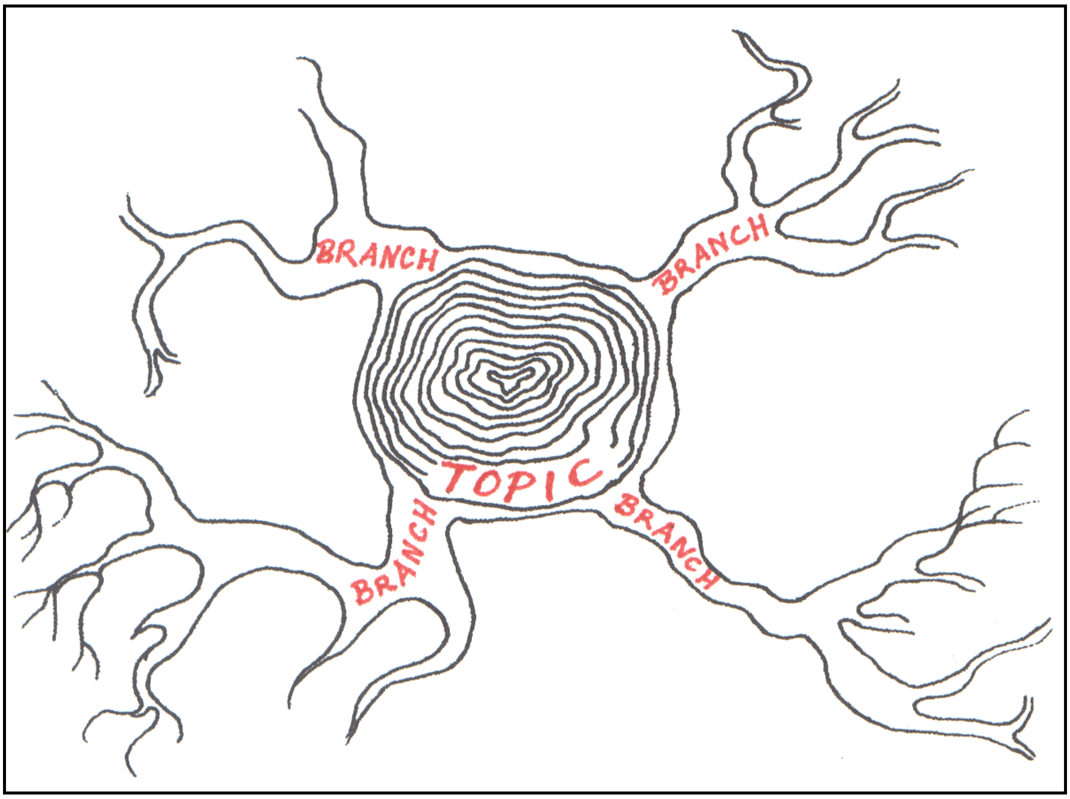

Figure 2: Structure of a mind map

The structure of a mind map resembles a tree seen from the top (figure 2): from the trunk in the middle, representing the topic of the mind map, the lines for the ideas linked to the topic branch off like tree branches. Thus a mind map is structured similarly to mathematics: "Mathematics is often depicted as a mighty tree with its roots, trunk, branches, and twigs labelled according to certain sub disciplines. It is a tree that grows in time" (Davis & Hersh, 1981, p. 18). Relations between mathematical objects may thus be visualised by mind maps in a structured way that corresponds to the structure in mathematics (Brinkmann 2000, 2001a).

The special technique of mind mapping, which uses both sides of the brain and has them working together, is of benefit to mathematical thinking, which takes place in both sides of the brain. The left hemisphere is better suited for analytic deduction and arithmetic, the right hemisphere for spatial tasks, e.g. geometry. The constant emphasis in mathematical education on rules and algorithms which are usually sequential may prevent the development of creativity and spatial ability (Pehkonen, 1997). Thus "the balance between logic and creativity is very important. If one places too much emphasis on logical deduction, creativity will be reduced. What one wins in logic will be lost in creativity and vice versa" (Pehkonen, 1997; see also Kirckhoff, 1992, p. 2). Accordingly, Davis and Hersh (1981, p. 316) "suggest that in mathematics it would be better for the contributions of the two halves of the brain to cooperate, complement, and enhance each other, rather than for them to conflict and interfere."

2.4 Uses of mind mapping in mathematics education

Some of the most important uses mind mapping may have in mathematics education, are listed below.

- ***Mind maps help to organise information.***

The hierarchical structure of a mind map conforms to the general assumption that the cognitive representation of knowledge is hierarchically structured (Tergan, 1986). Mathematical knowledge may thus be organised in a mind map according to this knowledge’s mental representation. A clear and concise overview of the connectedness of mathematical objects around a topic is enabled. Moreover, this is supported by the use of colours and pictures.

In addition, mind mapping supports the natural thinking process, which goes on randomly and in a non-linear way. As mind maps have an open structure, one may just let one's thoughts flow; every produced idea may be integrated in the mind map by relating it to already recorded ideas, and this with virtually no mental effort.

- ***Mind maps can be used as a memory aid.***

Each mind map has a unique appearance and a strong visual appeal. Thus information may be memorised and recalled faster, the learning process is speeded up and information becomes long living.

- ***Mind maps can be of help to repetition and summary.***

At the end of a teaching unit the subject matter of the treated topic can be repeated and structured by composing a mind map; this mind map then serves as a good memorisable summary.

- ***A mind map may summarise the ideas of several students.***

A mind map may grow as the common task of an entire class: The teacher might write the topic in the middle of the chalkboard and ask the students what main ideas they connect with it. For each idea the teacher draws a main branch of the mind map. Further on, students are asked to tell all other ideas they link to these main ones. Due to the open structure of a mind map, each single contribution can be integrated. The complete mind map should be redrawn by each student in his or her own personal style.

- ***Mind maps help meaningfully connect new information with given knowledge.***

New information can be integrated into an existing mind map and related to previously learned concepts. Such an activity with students has to be initialised by the teacher, who has the overview of already created mind maps and of how new concepts fit to old topics.

- ***New concepts may be introduced by mind maps.***

Entrekin (1992) reports that she used mind maps to introduce new concepts in mathematics classes. The new concept "is written on the chalkboard or transparency. As the concept evolves in later lessons, the teacher may add additional components and form an extended mind map. This visual representation serves to help students relate unknown concepts to known concepts."

- ***Mind maps let cognitive structures of students become visible.***

Mind maps drawn by students provide information about the students' knowledge. In broad outline, a learner's knowledge structure gets visible by means of mind maps for both the teacher and the learner.

  - The student develops an awareness of his or her own knowledge organisation.

This process might be enhanced by having the students construct mind maps in small groups. The students the have to discuss the concepts to be used and the connections to be drawn.

  - Wrong connections in a students' knowledge become visible and can be corrected by the teacher. It is recommended to first ask the student why the (wrong) connection was drawn; the explanation given by the student might bring more insight into the underlying cognitive structure than the simple and reduced representation in the map.
  - The students' growth in the understanding of a topic can be checked when asking them to create both a pre- and a post-unit mind map (Hemmerich et al., 1994). The teacher might see e.g. if supplementary concepts are linked to the topic, in a meaningful way.

- ***Mind maps foster creativity.***

Everybody may develop a personal style for mind mapping. Mind maps may have different forms and shapes, different colours, symbols or images. Artistic arrangements are not only allowed but desired as advantageous. This leads to a gain in creativity and moreover gives great pleasure. The fostering of creativity has a positive effect on mathematical achievement. It is common experience that in schools where emphasis is placed on creative activities such as working on arts, music or literature, the students are also better in mathematics (Svantesson, 1992, p. 26).

- ***Mind maps may show the connections between mathematics and the "rest of the world".***

As a mind map is open to the addition of any idea someone associates with the main topic, non-mathematical concepts may also be connected with a mathematical object (see figure 3). Thus it becomes obvious that mathematics is not an isolated subject but is related to the most different areas of the "rest of the world" (Brinkmann, 1998, 2001c).

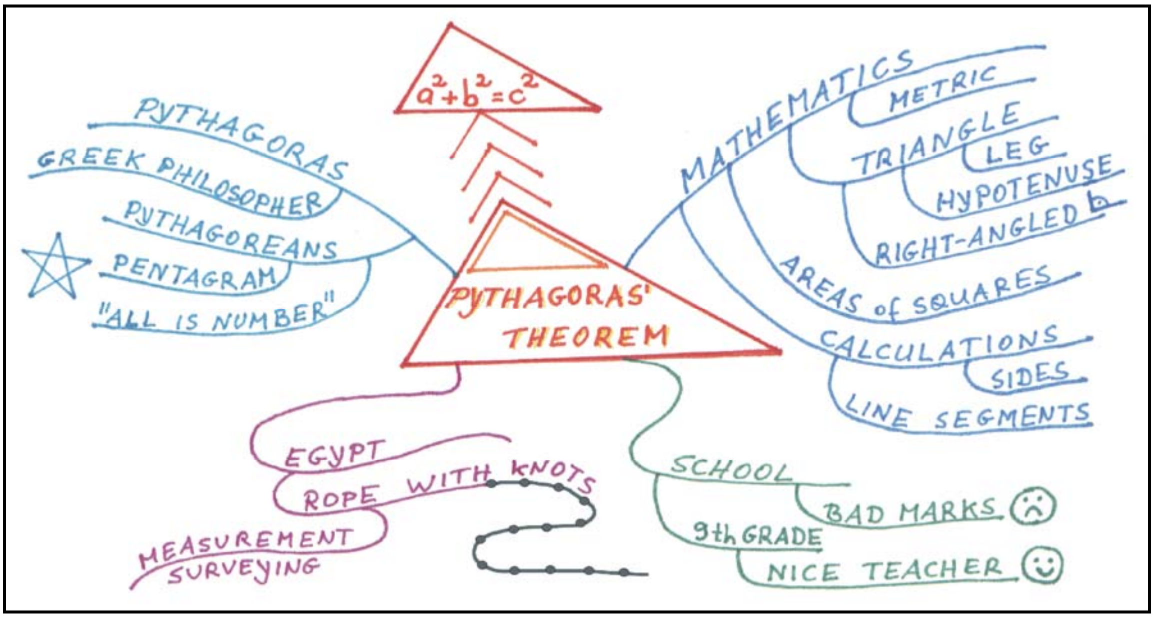

Figure 3: Mind map on the topic of the theorem of Pythagoras.

2.5 Limitations

In spite of its well-structured and ordered contents a mind map may sometimes, appear confusing. Mind maps are very individual graphic representations. As different people have different associations with the same topic they also draw different mind maps. The correct grasp of the relations represented in a mind map affords the right associations to the used key words. Hence, any mind map that someone wants to use should be drawn by that individual or group to which the individual belongs.

In a mind map, each main branch builds up a complex whole with its sub branches. Connections between the single aspects are not drawn in order to increase the clarity of the map. Thus, the relations in the mind map are probably incomplete.

3. Concept mapping

3.1 Background

Concept maps were first introduced by Novak as a research tool, showing in a special graphical way the concepts related to a given topic together with their interrelations. The method of concept mapping "has been developed specifically to tap into a learner's cognitive structure and to externalise … what the learner already knows" (Novak & Govin, 1984, p. 40), according to Ausubel's statement: "The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (Ausubel et. al., 1980).

Although the primary intention was to use concept mapping in research, it was found this to be a useful tool in a variety of applications, including helping students to "learn how to learn" (Novak & Govin, 1984; Novak, 1990, 1996). Consequently, concept mapping has been used also as an educational tool, above all in science, especially in biology. The instruction and use of concept mapping in science is now well documented, but less comprehensively so in mathematics (Malone & Dekkers, 1984, p. 225; Hasemann & Mansfield, 1995, p. 47).

3.2 Rules for making concept maps

Concept maps are similar to mind maps. They are hierarchically structured, according to the assumption that the cognitive representation of knowledge is hierarchically structured (Tergan, 1986). A concept map is constructed according the following rules (see e.g. Novak & Govin, 1984).

- Use a large sheet of paper for your concept map.
- Position the topic at the head of the map.
- Arrange the other concepts beneath it on several levels, the more inclusive, general, abstract concepts at the top, the more specific, concrete concepts lower down. (It is helpful to transfer first these concepts to small pieces of paper and arrange these on the different hierarchy levels you see. There may be more than one valid way in ranking the concepts, depending on how you interpret the relationships between ideas.)
- If possible, arrange the concepts so that ideas go directly under ideas that they are related to. (Often this is not possible because ideas relate to several other concepts.)
- Note beneath the last row some examples to the concepts situated here.
- Draw lines from upper concepts to lower concepts that they are related to; do the same for any related concepts that are on the same level. (You may decide to rearrange the concepts during this stage; sometimes two or three

reconstructions are needed to show a good representation of the meaning as you understand it.)

- On the connecting lines, write words or phrases that explain the relationship of the concepts. (This is the most important and most difficult step! You may continue to rearrange the concepts to make the relationships easier to visualise.)
- Sometimes it is useful to apply arrows on linking lines to point out that the relationship expressed by the linking word(s) and concepts is primarily in one direction.
- Beneath the last row, put examples to the concepts situated here and connect the examples with the concepts they belong with. As linking words write a phrase like "for example".
- Copy the results of the above steps onto a single sheet of paper.
- Draw borders around the concepts. Do not draw borders around the examples.

3.3 Uses of concept mapping in mathematics education

Concept maps have been found to be useful in a variety of applications, in the teaching of the different sciences but also of mathematics at all levels ranging from primary school to senior high school. Concept maps can be used for example in the following situations (Novak & Govin, 1984; Novak, 1990, 1996; Malone & Dekkers, 1984):

- ***Concept maps help to organise information on a topic.***

In order to be useful, knowledge must be organised so as to facilitate understanding and problem-solving ability. A concept map organises knowledge into categories and sub-categories so that it can be easily remembered and retrieved.

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- ***Concept maps facilitate meaningful learning, they aid in organising and understanding new subject matter.***
- ***Concept maps are a powerful tool for identifying students' knowledge structures, especially also misconceptions or alternative conceptions.***

This helps the teacher to plan effective lessons by taking into account what a learner already knows. Students themselves gain awareness of their own knowledge organisation. Possible wrong connections in a student's knowledge become visible to the teacher and can be corrected by him/her.

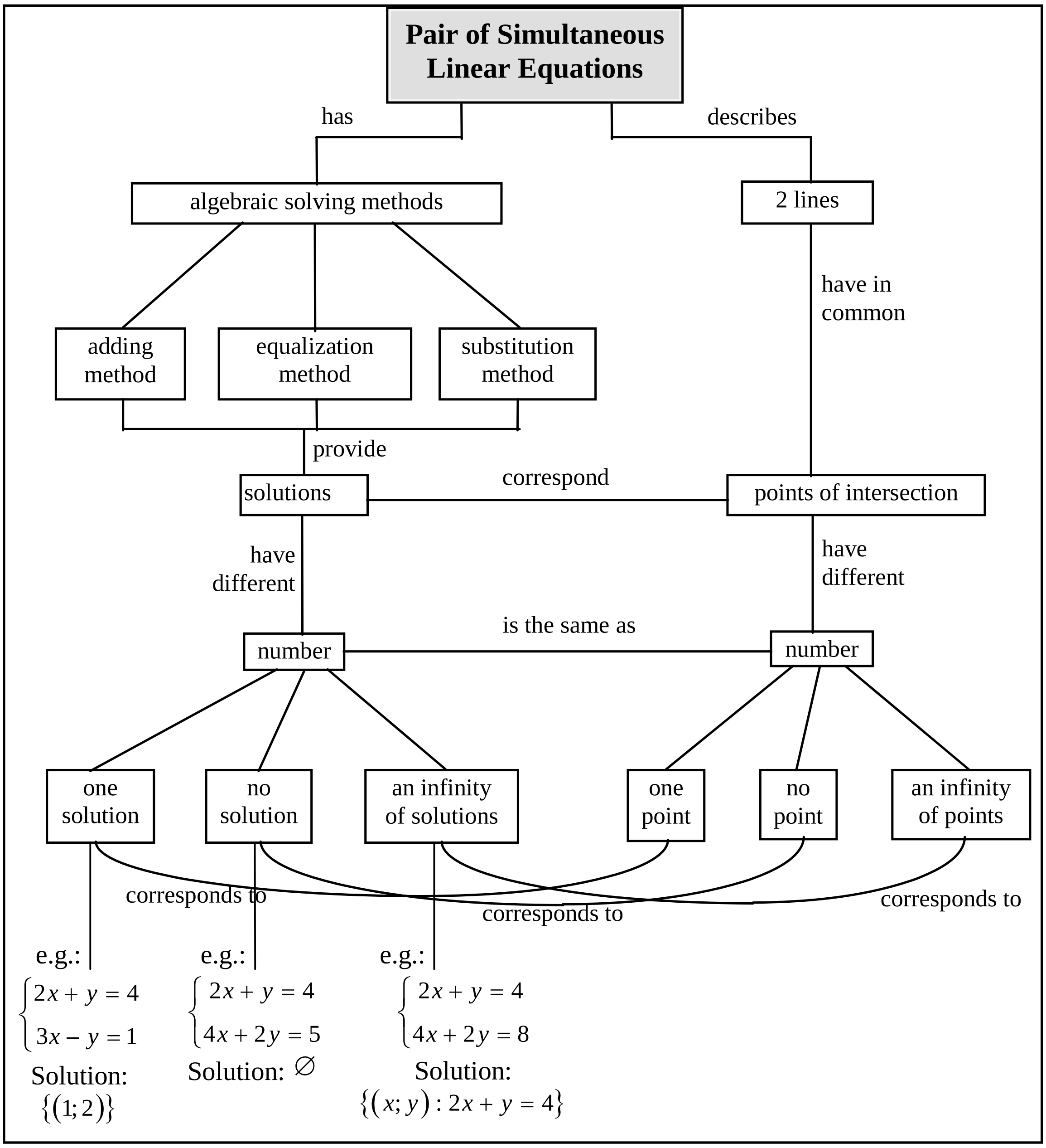

Figure 4: Concept map on the topic of linear equations

- ***Concept maps help to train the brain.***
- ***Concept maps may serve as a memory aid.***

As a concept map is a graph, a pictorial representation, it may be grasped at once, and due to its unique appearance committed well to one's memory and recalled faster.

- ***Concept maps may be used for revision of a topic.***

At the end of a topic a concept map can be constructed as repetition and in order to get a lasting and well-organised overview of a topic.

- ***Concept maps can be used for the design of instructional materials.***

Teachers found that concept maps were useful tools for organising a lecture or an entire curriculum. Moreover, they were not only aided in planning instruction, but also their own understanding of the subject matter was increased (Novak, 1996).

- ***Concept mapping may improve attitudes towards mathematics.***

By means of concept maps, an individual's mathematical knowledge may gain more structure and clarity and the individual's viewpoint on mathematics may become more positive. Furthermore, concept maps enable students through their visualisation to realise that mathematics is not a collection of isolated rules and facts but a network of ideas in which each idea is connected to several others. The authors of the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) "contend that the establishment of connections among mathematical concepts enables students to appreciate the power and beauty of the subject" (Hodgson, 1995, p. 13). Thus concept mapping may contribute to a change of an individual's beliefs on mathematics giving them a more positive emotional loading.

3.4 Limitations

It has to be considered that the method of concept mapping can be used only if one has become familiar with it. Moreover, the time that it takes to construct a concept map has to be allowed for.

In contrast to mind maps, the concepts within a concept map are linked by lines whenever they are related in some way, moreover, every single relationship is described by linking words written on the linking lines. Thus, a concept map provides much more information on a topic than a mind map, but it has not got that open structure allowing any new idea one might associate to the topic to be added easily. In addition, a concept map does not allow the same display of creativity as does a mind map.

4. Final remarks

The methods of mind mapping and concept mapping were not invented as educational tools, but it was found that these methods are useful in a variety of applications in teaching and learning processes. Yet, up to now, mind mapping and concept mapping have been rarely used in mathematics education.

However, reports about first experiences are very positive. Entrekin (1992), for example, states about mind mapping: "I found mind mapping to be an effective and delightful pedagogical tool".

The feedback of teachers that took part in further education events which I offered on the topic of mind mapping and concept mapping in mathematics is full of enthusiasm throughout. Teachers reported that students who were not good in mathematics particularly benefited from these educational tools. These students often first realised connections between mathematical concepts while producing a map. Further on, they told their teachers that only after having drawn a map they could “see” the structure of the respective mathematical knowledge. The graphical display helped the students to organise their knowledge.

Of course, depending on the pursued goals, teachers have to decide which of the two methods they particularly want to use in their lesson. The various positive learning effects that can be expected by means of both mind mapping and concept mapping ought to result in an enhanced usage of these methods in mathematics education.

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Innovations in Mathematics, Science and Technology Teaching (IMST²). Initial Outcome of a Nation-wide Initiative for Upper Secondary Schools in Austria

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Following the poor results of Austrian high school students in the TIMSS achievement test, a research project was set up in which the results were analysed and additional investigations into the situation of mathematics and science teaching were started. As a consequence, the initiative IMST² was launched to support teachers' efforts in raising the quality of learning and teaching in mathematics and science. In the school year 2000-01, 126 Austrian schools participated in total with about one quarter collaborating more intensively with the IMST²-team and documenting one or more innovations at their school. The concept, initial experiences and findings of IMST² are presented and discussed here.

1 Preliminary remarks

Whereas in the last few decades many countries launched reform initiatives in mathematics and science instruction, similar systematic steps in Austria did not really happen. There is a big gap between intended and implemented instruction, in particular with regard to upper secondary schools. Although the promotion of understanding, problem solving, independent learning, etc. and the use of manifold forms of instruction and didactic approaches are regarded as important, in reality, teacher-centred instruction and application of routines dominate. At the same time, Austria does have a variety of dedicated teachers and innovative teaching approaches across the country, supported by some promising initiatives. Nevertheless, there is no adequate nation-wide support system for mathematics and science teaching that would promote systematic professional communication among teachers, and among teacher educators, as well as between these two groups. As is now clear, the educational system needed an external and publicity-driven impulse which appeared in the form of the Third International Mathematics and Science Study (TIMSS).

Whereas the results concerning the primary and the middle school were rather promising, the poor results of the Austrian high school students (grades 9 to 12 or 13), in particular with regard to the TIMSS advanced mathematics and physics achievement test shocked the public. The ranking lists showed Austria as the last (advanced mathematics) and the last but one (advanced physics) of 16 nations (see e.g. Mullis et al. 1998, 129 and 189). As a reaction, the responsible ministry launched the research project “Innovations in Mathematics and Science Teaching" (IMST) in the year 1998. The task was to analyse the situation and to work out suggestions for the further development of mathematics and science teaching in Austria. The university project team started from the assumption that international comparative studies like TIMSS are good starting points for analyses of the national situation. However, due to differences in the general cultural, societal and economic conditions that lead to different curricula or to typical regional patterns of instructional practices (see e.g. Cogan & Schmidt 1999) etc., the comparability is limited. Each test constructs its own “test reality” (with specific attainment goals) which differs from the various “teaching realities” in different countries. Achievement tests alone might give some insights into differences in students’ achievement, however, they do not contribute to our knowledge on the process of learning and teaching and, very importantly, to its further development. Therefore, the project team decided not only to analyse the results of TIMSS but also to carry out additional analyses. The research project IMST (1998-1999) aimed at contributing to the following tasks (see e.g. Krainer 2002): analysis of TIMSS-items and achievement results, suggestions for using the available TIMSS-items, contributions to an analysis of the state of Austrian upper secondary mathematics and science teaching, brief description of exemplary reform initiatives in other countries, and suggestions for consequences on the basis of the national and international analyses.

2 Results of the IMST research project on the status quo of mathematics and science teaching at upper secondary schools in Austria

Below the main findings of the IMST project are summarised (for further details see e.g. Krainer et al. 2000):

- The absolute ranking lists based on the TIMSS-achievement results are questionable because they do not take into account the specific sample of students that were involved in the test. For example, Austria – unlike most countries – sent many students to the advanced mathematics and physics achievement test who had not been taught these subjects in that school year. Other (more fair) comparisons (such as the TOP 5% or 10% students' achievements) show slightly better results, nevertheless, the picture remains disappointing.

- In all the TIMSS tests, literacy and advanced as well as mathematics and science, Austria is among those nations with the biggest achievement differences between boys and girls.

- Again from the TIMSS tests, Austrian (and German) students show poor results in particular with regard to items which refer to higher levels of thinking (see e.g. the analysis concerning mathematics literacy in Baumert et al. 1998).

- In their response to the item in the TIMSS-questionnaire concerning reasoning tasks in lessons , less than a third of Austrian students felt that they are involved in reasoning tasks in most or every mathematics lesson(s), resulting in the last but one place in the international ranking of 16 nations. In physics the figures were half and last place (see e.g. Mullis et al. 1998, 165 and 221).

- Interestingly, Austrian (and German) students who feel that they are asked to do reasoning tasks every mathematics lesson – on average – do not have significantly better test results than those who feel they do reasoning tasks in some lessons. The involvement of students seems to be done less effectively than in other countries. This seems to be an outcome of a special kind of teacher-centred instruction in German speaking countries. This "fragend-entwickelnder Unterricht" aims at leading a whole class to the intended goal of the lesson through posing (mostly "small step") questions to the students. However, interaction studies have shown for a long time (see e.g. Voigt 1984) that during the process more and more students cannot or will not follow, but even a few students might give the teacher the feeling of successful teaching. In the long run, such a method leads students to question whether their active reasoning really has an impact on the process and the outcome of instruction.

- The answers to a written questionnaire by Austrian teachers, teacher educators and representatives of the education authorities supported the results from the TIMSS-data. For example, teachers were predominantly seen as dedicated and as having a lot of pedagogical and didactic autonomy. However, this autonomy is sometimes restricted by themselves or by general conditions and therefore often not passed on to the students. The analysis further showed that students' active involvement in the teaching and learning process is seen as a major weakness of mathematics instruction in upper secondary schools.

- An analysis of web sites at schools in Carinthia (southern part of Austria) showed that schools aim at convincing the public with regard to the quality of their work with a variety of initiatives. However, mathematics and science initiatives are extremely rare, whereas information technology and (predominantly English) language initiatives seem to attract much energy from students, teachers and principals.

- This concern has been underlined in a workshop with principals at upper secondary schools who pointed out that mathematics and science teachers in general don't belong to the "powerful" groups of teachers. This has a magnifying impact on many questions, for example, whether a school decides to set a focal point on mathematics and science teaching.

- Mathematics education and in particular science education are poorly anchored at Austrian universities. In chemistry education, for example, no university has a university professor for that scientific domain. Teacher education is dominated by subject experts, the collaboration with educational sciences and schools is – with exception of a few cases – underdeveloped. A competence centre like the Freudenthal Institute at the University of Utrecht in The Netherlands or the Institute for Science Education at the University of Kiel in Germany does not exist.

- The picture with regard to documented innovations in mathematics and science teaching was ambivalent. On the one hand, it was astonishing how many creative initiatives were carried out by individuals, groups or institutions. On the other hand, it was irritating to see how unlinked these activities were, and that a networking structure was missing. This impression is repeated when looking at the whole educational system (two different pre-service teacher education systems that are nearly unconnected, a variety of different kinds of schools with corresponding administrative bodies in the ministry and the institutions for in-service education, etc.). This shows a picture of a „fragmentary educational system" with people from schools, teacher education institutions, administration, etc. which form a loosely-coupled, self-reproducing system of lone fighters. The consequence is a high level of (individual) autonomy and action, however, less reflection and networking (see e.g. Krainer 2001).

Thus in the IMST analyses a complex picture of diverse problematic influences on status and quality of mathematics and science teaching has emerged. It was the background for the IMST-team to suggest the launch of a long-term nation-wide initiative IMST² – Innovations in Mathematics, Science and Technology Teaching involving the subjects biology, chemistry, mathematics and physics. The addition of "Technology" in the project title is to express the fundamental importance of technologies for mathematics and science teaching. The four-year initiative, starting with a pilot-project IMST² in the school year 2000-01, is being financed by the Federal Ministry of Education, Science and Culture.

In the following, the initiative's goals, tasks and intervention assumptions are briefly described.

3 Innovations in Mathematics, Science and Technology Teaching (IMST²) – a nation-wide initiative

The long-term goals of the IMST² initiative are:

- Better basic education – higher quality of understanding, problem solving, reasoning and reflection
- Bigger variety of teaching and learning styles – creativity, independence, gender sensible teaching and learning, supported by new media and technology
- More, better designed forms of professional exchange of experiences among teachers, contributing also to the further development of the whole school
- Setting up and further developing a network that supports carrying out and evaluating innovations, and for communicating these in various forms to a wider public
- Improved "image" – more favourable perceptions and expectations with regard to mathematics and science in schools and society

In the pilot-project 2000-01 the main tasks were to work out a detailed master-plan for the continuation of the initiative and to start supporting innovations at schools. In the following three years the support of innovations at schools will be continued and the establishment of a support system will be started.

Four priority programmes (S1 – S4) have been established with the following reasoning:

- Basic education (S1): The unclear expectations concerning qualifications, knowledge and contents that students need when leaving secondary school. The four S1-teams (biology, chemistry, mathematics and physics) support initiatives at schools that reflect such expectations and they aim at working out (interdisciplinarily interconnected) concepts for basic education at the upper secondary level for the four subjects. These concepts – intended to be generated by theoretical considerations and by practical experiences from the collaboration with schools and thus negotiated by a wider form – are expected to be a key element for a support system for mathematics and science teaching. It is assumed that teachers' clearer view on the importance of goals and content might raise the quality of learning and teaching.

- School development (S2): The relatively low status of the subjects biology, chemistry, mathematics and physics at schools, in comparison to their importance in society and the economy, might lead, in times of greater autonomy of schools, to a situation where, in general, these subjects are left behind when schools change their profile. The S2-team supports schools that set a focal point on mathematics and science teaching and tries to establish a network of such schools. In parallel, and using the practical experiences, it aims at working out a concept that reflects the initiation, support and evaluation of school development processes that (partially) focus on the enhancement of mathematics and science teaching. This concept is also to be supposed as an important element of a future support system. It is assumed that organisational development (often underestimated in subject didactics) – when fairly linked with classroom development – makes a crucial contribution to the quality of learning and teaching.

- Teaching and learning processes (S3): The dominance of relatively passive forms of learning, not sufficiently taking into account the individual needs of students in general, and the low interest and the poor results of Austrian girls in the TIMSS-achievement test in particular. The S3-team both supports innovations at schools focusing on situation-appropriate teaching and learning processes and aims at working out a concept for generating, analysing and evaluating such processes. Such a concept, supplemented by material like a CD with video-clips of real teaching that is intended to be used in pre- and in-service teacher education, should support teachers' growth in planning and reflecting on their own teaching. It is assumed that such an increased competence has a deep impact on teaching and learning processes.

- Practice-oriented research (S4): The lack of well-developed practice-relevant research and development in mathematics education and in science education in particular. The S4-team initiates, finances and supports teams of school teachers or university teacher educators (or mixed teams) who carry out investigations into their own teaching (action research) or classical research projects. Following the IMST analyses, the promotion of students' independent learning is seen as a major goal, hence the projects focus on that issue. The team also aims at working out a concept for the promotion of subject-didactic research and culture. Through raising teachers' and teacher educators' interest and competence in practice-relevant research, the network of researchers in mathematics and science education is expected to grow, both in quality and quantity. A stronger mathematics and science education, where theoreticians and practitioners collaborate more intensively, is expected to be a fundamental part of a support system for school practice.

This shows that each of the four priority programme teams has two important – closely interconnected – tasks: firstly, to support innovations at schools (and in

S4 also in teacher education) and secondly, to work out concepts that help better to plan, describe and understand such innovations.

Innovations are the key feature of the way of IMST² towards establishing a nation-wide support system. The corresponding basic assumptions behind this intervention into the educational system are:

- Starting from strengths: Innovations are initiated, supported and made visible, thus motivating others to join in, stimulating “attraction” instead of generating “pressure”.
- Innovations are not regarded as singular events that replace an ineffective practice but as continuous processes that lead to a natural further development of practice.
- Participation is voluntary, teachers and schools have the ownership of their innovations.
- There is no “best practice” which might be defined by an external authority. For each learning and teaching different approaches to “good practice” exist. Innovations are planned steps towards a “good practice”.
- Through innovations and reflections teachers construct their own professional growth (likewise the students are seen as active learners).
- Writing down the experiences in a systematic way means a second cycle of reflection and opens the opportunity for more people to learn from those experiences.
- The dissemination of innovations passes along personal relationships and experiences.
- One powerful strategy for spreading innovations to a whole system is to initiate regional networks and to promote their communication with other networks.

Another important feature of IMST² is the emphasis on supporting teams of teachers from one school. The background for that approach is the experience that working with single teachers from different schools may often cause considerable progress for individual teachers but does not necessarily have any impact on other teachers in their school (see e.g. Loucks-Horsley 1998; Borasi, Fonzi, Smith, & Rose 1999; Krainer 2001). If professional communication among teachers is not an important feature of the culture of a school, innovations by individual teachers remain limited to their own heads and classrooms. Even a pair of colleagues co-operating successfully might not be enough as a critical mass. Of great importance is the support of the principal. In IMST², therefore, the teams of priority programmes sign contracts with teams of teachers, and these documents which define the goals and content of the collaboration are also signed by the principal.

It is taken for granted that schools have different starting points concerning interests, resources, time, etc. IMST² schools can therefore choose their intensity of participation (within one school year), getting the status of an information school, contact school, collaboration school or focus school. For example, to become a contact school, one subject or interdisciplinary team of teachers has to collaborate in IMST². To become a focus school, two teams in one school collaborate in one of the priority programmes and a steering group is involved in the further development of mathematics and science teaching at that school.

The role of team members working with schools is to support the teachers' struggle for professional growth, to generate new knowledge about this supporting process and about teacher's growth, and to apply this new knowledge in forthcoming support processes.

Evaluation is an integral part of the IMST² initiative whereby three different functions have been defined:

- -The process-oriented evaluation should generate in a continuous feedback process steering knowledge for the project management and the project teams in order to further develop the internal structures and processes. Sample instruments are interviews with team members on their view on the strengths, weaknesses, opportunities and threats of the project or feedback by an advisory board (consisting of representatives from theory and practice).
- The outcome-oriented evaluation should work out the impact of the project at different levels of the educational system (students, teachers, schools, teacher education institutes, etc.). Sample instruments are case studies about teachers' professional growth or questionnaires for schools (e.g. assessing the clarity of the project goals).
- The knowledge-oriented evaluation should generate new theoretical and practical knowledge that will form a basis for improving support to innovations at schools.

4 Initial outcome of the initiative IMST² (pilot-year 2000-01)

The project started at a time where public discussions about teachers' work and other topics of education policy led to a rather passive behaviour by several Austrian schools. Furthermore, when the 582 upper secondary schools got the first information about the project (beginning November 2000), the school year was nearly two months old. This meant that many schools had started a lot of other activities. Nevertheless, 22% of all target schools expressed an interest in participating in IMST². During the school year 2000-01, 32 focus and collaboration schools and two university teacher education teams were supported, carrying out 38 innovations and research projects (36 at schools, 2 at universities). Given the fact that it needed some time for the participants to become familiar with the IMST² approach, to develop first plans for activities, and to coordinate their plans with other teachers and the principal, not much time remained neither for carrying out and reflecting on the innovations nor for the opportunities of the project team to support the teachers' activities.

Results of a questionnaire

In February 2001, a questionnaire was sent to 86 contact, collaboration and focus schools in order to get a preliminary feedback (see e.g. Specht in IFF 2001). 63 questionnaires (73%) were sent back and showed a representative distribution concerning the four priority programmes. It was not surprising to see that the decisions for collaborating in IMST² were mostly taken by single persons (52% teachers, 12% principals). In no case was the decision made during an official school meeting, neither in a teacher conference nor in a school partnership forum. This reflects our experience that only few schools have established subject-related teams and fora where teachers regularly meet and share experiences. The responses to another question (previous forms of collaboration among teachers) show a similar picture: only 30% of schools reported that they had already systematic collaboration among their subject group or in an interdisciplinary context; 59% regarded the collaboration as informal, and 11% even felt that there was no subject-related collaboration among colleagues at their school at all. Both results support our view that Austrian teachers to a large extent stay as lone fighters in their schools. Considering the four dimensions of professional practice – action, reflection, autonomy and networking (see e.g. Krainer 2001), there is much individual autonomy and much action, but less reflection and networking among teachers. This underlines both the necessity and the big challenge of IMST² to work with teams of teachers (and not individuals) in order to contribute to the establishment of a culture of professional communication and collaboration among teachers. 31% of the schools reported that they did not carry out previous or recent initiatives for the further development of mathematics or science teaching, thus taking IMST² as the first opportunity to jointly share experiences and get external support. This means that the project reaches a considerable amount of teachers that had not been involved in joint activities concerning mathematics and science teaching at their school so far. The schools' reasons for participating in IMST² are predominately pedagogical and intrinsic in origin: "raising students' interest and understanding", "further developing the culture of teaching and assessing", and "improving students' achievements" were the most commonly named motives, whereas for example "proposal by the principal" was ranked last.

Examples of innovations at schools

Four innovations at IMST²-schools that teachers planned and carried out during the school year 2000-01 are briefly sketched. They all relate to the use of technology in mathematics teaching and each stems from one of the four priority programmes:

S1 Basic education: The project "Promoting talented students in mathematics teaching" at a Higher Vocational School (HAK) supports grade 10-students' work on the topics "interpolation" and "regression". The students work almost independently in pairs using Mathematica and MathSchoolHelp.

S2 School development: The project "Mathematics with Derive and Excel in a notebook class" at a Grammar School (Gymnasium) takes advantage of the fact that all students of a grade 9 class have received a notebook. The students work on topics like "linear quadratic functions" and "circumcentre of a triangle".

S3 Teaching and learning processes: Following a variety of preparatory initiatives, the project "How do CAS and intelligent calculators change mathematics teaching?" at a Higher Technical School (HTL) will investigate school-leavers' beliefs concerning the impact of CAS, with a particular focus on the gender aspect.

S4 Practice Research: Within the project "Trigonometry" at a Grammar School (Gymnasium) grade 10-students independently work in a sequence of "stations" with different learning activities. At one station the students (using a TI 85 or TI 92) were supported by a student of grade 12.

Some sample teachers' comments from their written reports on the use of technology are:

> - *The use of technology promotes a variety of learning styles ("... makes math less dry.")*
>
> - *It promotes independent and active learning (Students like this kind of work and it gives them a lot of freedom and space for initiative.).*
>
> - *It means new roles for students and teachers (The teacher becomes a facilitator for his/her students on several levels: e.g. in the case of problems with hard- and software as well as with the students'*

independent study of mathematical problems. This situation significantly enhanced the relationship between students and teachers.

- It is labour-intensive and generates high expectations on the teacher ("The students expect that we know an answer to every question and a solution to every problem, and besides we should help each student quite individually ...").

- The challenge of the use of technology is to find a good balance between the learning of high/low achievers and girls/boys because technology tends to widen the gap ("Low-achievers have a smaller chance than in traditional lessons to regain lost ground through hard work and practice.").

It might be argued that such results are not new at all and can be read in several publications. However, whether a specific teacher really finds in such research reports the viable support with regard to his or her context and situation is questionable. It is the basic assumption of IMST² that teachers through starting from their own questions, investigating relevant aspects of their practice, collaborating with other teachers at their own school, getting support from teacher educators, and writing down their findings, have a better chance to construct the own local knowledge they need to meet the challenges of their practice.

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