Contents

The Advisory Committee on Mathematics Education

Mission Statement

The Advisory Committee on Mathematics Education (ACME) is an independent committee established by the Joint Mathematical Council of the UK and the Royal Society with the explicit support of all major mathematics organisations. ACME is funded by the Gatsby Charitable Foundation and acts as a single voice for the mathematical community, seeking to improve the quality of mathematics education in schools and colleges. Its role is to advise Government on issues such as the curriculum, assessment and the supply and training of mathematics teachers.

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Professor Sir Christopher Llewellyn Smith FRS (Chairman), Sir Christopher is the current Provost of University College London and was previously the Director of CERN;

Chris Belsom is Head of Mathematics at Ampleforth College;

Annie Gammon is Key Stage 3 Numeracy Co-ordinator for Tower Hamlets, previously Head of Maths in London comprehensive school;

Professor Celia Hoyles is Professor of Mathematics Education at the Institute of Education, University of London, Chair: Joint Mathematical Council of UK;

Professor Chris Robson is Professor of Pure Mathematics at University of Leeds;

Dr Sue Sanders is Senior Lecturer at University of Wales Swansea, Honorary Secretary: Joint Mathematical Council of UK.

ACME email address is: acme@royalsoc.ac.uk

Planning for an Extended Teaching Repertoire

Jim D N Smith

Sheffield Hallam University

This article reports on the design and evaluation of lesson planning guidance. Guidelines were developed specifically to support and encourage student teachers to use a variety of pupil-activities in their lessons. This was addressed by making the pupil-activity the building block from which lesson plans were constructed, and providing a framework called PART, which involved a consideration of Presentation, Activity, Reflection and Transition. PART planning met with high levels of student uptake and sustained use throughout the initial training year. The article describes some modifications made to PART by students as they gained in experience, and reflects upon the original intention of broadening student teachers' repertoires.

Introduction

This article reports on a small part of a larger, ongoing, action research project (e.g. Smith 1999, 2001) within a situated social constructivist theoretical framework (Richardson, 1997, p.4) to develop our secondary mathematics ITT course. Our student teachers come from a wide variety of backgrounds and then proceed through one of three alternative university programmes. The final year is common to each of these routes and is called our Professional Year. This is where the vast majority of the school-based training occurs.

Recent changes in Initial Teacher Training (ITT) in the United Kingdom have led to more emphasis being placed upon the school - based component of teacher education (DfE, 1992) and the introduction of competencies and standards that student teachers must acquire by the end of their course (DfEE, 1997). There is an expectation that student teachers will learn the majority of their craft from experienced teachers and from carrying out classroom teaching practice. My overall study, of which this is a part, is concerned with what student teachers of secondary mathematics are actually learning within this partnership about teaching approaches and how to improve that learning process so as to improve practice.

> *The more we come to know about student teachers' experiences, how their practice in the classroom develops and the factors that impinge upon this development, the more we are able to construct models or theories of professional growth that will be able to shape the construction of future courses, inform the training and induction of teachers and serve as guides for action for teacher educators, dealing with the complex task of helping student teachers to learn the practice of teaching. (Calderhead & Shorrock, 1997, p.9)*

There is no easy answer to the question of how to improve practice (Richardson, 1990, p.13). The notion of improving practice is inevitably value - laden, in that what counts as an improvement to one person might be considered detrimental by another. For this reason, there is a need to be explicit about my own views and values. However, this is not the only reason.

According to Carr and Kemmis (1986), a significant characteristic of action research is that it is a problem solving activity. In my view, this depends on a value judgement about what is seen to be problematic. The particular *problem* arises from the value judgement and this value judgement in effect motivates the nature of the initial intervention and subsequent actions. What is problematic to one person may not be problematic to another. Because value judgements are powerful in motivating research and they strongly influence the decisions made in the course of research, it seems to me that the value basis must be made explicit.

The value judgement adopted in my research was that improving practice in the student teaching of secondary mathematics was synonymous with increasing the variety of appropriate learning experiences that pupils are to engage upon. I use the term *restricted teaching style* to refer to teaching consisting solely of explanation, examples and exercises. By contrast, using more flattering terminology, I refer to the use of a multiplicity of varied approaches, which actively engage students in constructing their own understanding, as an *extended teaching repertoire*.

No doubt, this value judgement calls for further clarification and justification, but this would be a distraction from the focus on lesson planning. I have written elsewhere on such matters in more detail (e.g. Smith, 1986, 1981, and 1996), but for now ask the reader to accept the value basis and to examine how this is enacted in terms of lesson planning.

Lesson Planning Guidance

Our previous lesson planning guidance for student teachers was reviewed and replaced in line with the value basis outlined above. In so doing, the intention was to make active learning experiences integral to the planning process, and to build upon early course themes of class management, pupil involvement and motivation. This was done by using the active learning experience as a basic building block from which lesson plans would be constructed. A lesson plan was then to be viewed as a sequence of active learning experiences, each carefully chosen to contribute to pupils' learning.

> *A lesson plan is a carefully constructed sequence of planned learning activities taking place within the context of a scheme of work. It is assumed that the lesson will take place under the leadership of an enthusiastic teacher working in an interactive style and that a variety of planned learning activities will occur during the lesson. Handbook, Professional Year Mathematics, 1999.*

The expectation was that student teachers would use a variety of active-learning-experiences to achieve the desired learning whilst providing a medium for motivating pupils, thereby reducing the need for control and increasing the chances of teacher survival. Furlong & Maynard's (1995, p. 97- 100) different ordering of these same factors suggests that, without deliberate intervention, a variety of teaching approaches is only arrived at late in the course.

Each active learning experience was referred to as a lesson part, and the acronym PART was used to represent the planning associated with each pupil-activity. PART stood for Presentation, Activity, Reflection and Transition., see figure 1.

The main bulk of the lesson consists of a series of linked parts. Many lessons will consist of three PARTS, but other numbers of PARTS can be used.

PART = Presentation + Activity + Reflection + Transition

The purposes of the **Presentation** are:

- to gain pupil motivation through involvement
- to negotiate a shared understanding of the activity
- to give clear instructions on starting the activity ...

The purposes of the **Activity** are:

- for pupils to respond to an appropriate challenge
- to help pupils to construct their own understanding of concepts
- to help pupils develop their own problem solving skills
- to provide consolidation ...

The purposes of **Reflection** are:

- to emphasise key points, language and conventions
- to relate the activity to other aspects of mathematics
- to identify the power of new skills, concepts or ICT...

The purpose of the **Transition** is to manage an efficient transfer from one part of the lesson to the next with the minimum of disruption.

Extracts from Handbook, Professional Year Mathematics, 1999.

figure 1

The written guidance was accompanied by exemplification of the process of planning, example lesson plans, practical tasks, practice of planning lessons and receiving comments on these from University tutors. To make sure that all students made use of the PART lesson planning guidance, a formal academic assignment *required* all students to make use of the approach. The written guidance was made available to school-based mentors through our subject handbook, and mentors were able to read for themselves how students were taught to structure their lesson plans.

It remained to be seen how well students would use the planning format, whether they would persist in using it once the hurdle of compulsory assessment was passed, and whether PART-based lesson planning would encourage a broader repertoire of active-learning-experiences. This was evaluated in the subsequent monitoring phase of action research, which took place over a two-year period.

Evaluating Lesson Planning Guidance: 1999/2000

The intention was to monitor issues of uptake, continuation, and impact on practice of the PART planning guidelines. Students might not use the guidance at all, or might not interpret the guidance as intended. There might be unforeseen and unwanted aspects of introducing PART planning. A specific concern was that PART could equally well be used for a diet of explanations and examples (P), exercises (A), answers (R), gaining attention for the next explanation (T). Since PART did not necessarily involve a variety of pupil activities, there was a need to monitor what happened in practice.

Actions: 1999/2000

The approach to data collection was inherited from the larger action research project, of which this was a part. Quantitative data was obtained from questionnaires and surveys of the entire cohort. Qualitative data was obtained from a more in-depth study, using a variety of techniques (including lesson observation), with a small sample of students.

The sample group was a purposive sample, not random. The sample was selected from their responses to an Initial Beliefs Questionnaire, administered in September 1999, to give a broad range of initial views on teaching, learning and mathematics.The sample students' teaching practice files were examined to check for evidence of PART-planning, interviews and lesson observations were conducted with this group. Corroborating evidence was available from other aspects of the research project, in the form of mentor questionnaires, written lesson observations and other documentation.

To obtain a complementary, quantitative perspective across the cohort, I included questions in an end-of-year questionnaire asking about the use of PART.

Observations: 1999/2000

In the final review session at the end of the year, the 20 students who were present in the 1PGCE group were asked to describe the extent of their individual use of PART in planning lessons. In order to encourage honest responses, this was in an anonymously completed questionnaire.

Six claimed to have used PART for every lesson, for example:

> *I use PART planning in all my lesson planning. I always have a presentation and an activity. I tend to use the reflection to check answers before usually repeating PART again during the second half of the lesson.*

Four students claimed to use PART in most of their lesson planning, for example:

> *Used in most lessons, linked to school approach in terms of a numeracy topic to start, written learning objectives on the board, reflection on learning objectives at the end.*

Four students claimed to be thinking about PART, but had internalised these steps, for example:

> *I used this for my early lesson plans, but found it eventually time-consuming and difficult to follow efficiently when under pressure. I found it useful early on in teaching for me to think of planning in this way - something I believe has stayed with me.*

This *internalisation* fits in with the model of student teachers' skill acquisition suggested by adaptive cognitive theory in Winitzky and Kauchak, (1997, p.69-71).

Two students had rejected the model, and adopted the tradition of explanation, examples, and exercises, for example:

> *I try to write objectives on board, do presentation, pupils practice and then I draw conclusions.*

Four students did not respond to the question.

When I visited Alan (all student names are pseudonyms) in his second placement school in April 2000, PART was still in evidence in his teaching practice file. He had used PART for planning most lessons, linked to the school approach of a numeracy objective at the end of lesson. However, PART was reducing in scale as the academic year progressed, and showing some internalisation of the process. Alan said that the Reflection was particularly important for him as it reminded him to emphasise the value of the learning objectives and to try to monitor pupils' progress towards the learning objectives. He said that sharing progress with pupils would help to develop a sense of achievement for both them and him. On the advice of his mentor in the second placement school, Alan was using shorter activities and more PARTs in a lesson. This was intended to maintain the pace of the lesson and pupils' attention.

I had chosen to work with Alan because his business background had given him a concern with setting and meeting objectives, and it was interesting that Alan had maintained a concern with objectives since the start of the course. He was asked to identify the most important advice that he had *received* from teachers about planning for teaching, and replied:

> *Be very specific about learning objectives and reviewing same at the end of lessons.*

However, in a corroborating questionnaire, class teachers had been asked what the most important piece of advice they had *given* Alan about planning for teaching:

> *Plan to include differentiation, despite a group may already be setted. Teacher 1.*
>
> *Picking the right level. Teacher 2.*
>
> *Flexibility to cope with possible changes of direction in the lesson, based on pupil response or progress. Teacher 3.*

This interesting mismatch had not occurred with the other students. Alan had evidently held very strongly to his initial beliefs about objectives, possibly to the extent of not receiving other messages. This might be an example of *strategic compliance*, (Lacey, 1977, p.72). However, he had been able to incorporate a concern for active-learning-experiences into his thinking, assimilating PART planning and other advice into his own mode of thought. As evidenced in his file, and in feedback from lesson observations, Alan had included a range of active learning experiences in his teaching.

> (Space permits no more than indicative titles or descriptions. Alan had used Frogs, Circle Diagrams, Multink fractions, Number machines, Fizz Buzz, Fairground games, Olympic Record trends, Flip it, Tessellations, Equable Shapes, Building tallest tower from straws, Geoboard Squares, Cuisenaire, pegboards, logo, human turtles, discovering angle properties, discovery methods for fraction multiplication, Face North, Triangles on dotty grids, Quadrilaterals on dotty grids, Decimal games, Fraction Walls, Folding strings to represent percentages, unit cubes to work on volumes, construction of cardiod. Some of these come from "Getting Started" (Smith, 1996c)).

Turning now to consider another student, I conducted a review of Bob's lesson plans (on 01.12.99) which showed the complete abandonment of PART at an early stage of the course. I was concerned that he might be moving very quickly towards his initial preference for a didactic approach (as expressed in his response to the Initial Beliefs Questionnaire). However, with little concern for strategic compliance, he explained that:

It was too time consuming. A lesson plan and evaluation took up four sides of A4.

During a subsequent review of lesson plans, conducted on 06.04.00 I found that Bob had developed his own approach to planning. He began with an idea for the lesson, developed this idea into a rough lesson plan and then into a detailed lesson plan using colour coding, of blue text for instructions for pupils, green for planned board work and red text for planned speech. Bob had incorporated a range of active learning experiences into his lesson planning.

(Including Travel Graphs, Pythagoras investigation, Tree Diagrams, Logo, cutting triangle corners off, Rangoli patterns, balloon model for positive/negative numbers, Circle Diagrams, Pond Borders, Trial and improvement activities, Sieve of Eratosthenes, Two primes, Summing two consecutive triangle numbers, Necklaces, Fish Pies, Main/lane).

Clearly, Bob's abandonment of PART had not been associated with abandoning a range of pupil activities.

I had chosen to work with Carol because her initial beliefs questionnaire had suggested that she was more concerned with the human relationships involved in teaching, and less concerned about the mathematics. In December 1999, I looked at Carol's teaching practice file, and found that her lesson plans all made use of PART framework, although not all sections were spelt out in detail. By April 2000, Carol too had begun to internalise some of the processes:

I use presentation, activity, reflection but I don't really need to plan the transition in advance any more.

There was little variety in the learning experiences of Carol's lessons (including routine practice, investigative work on trial and improvement, graphical calculators). Carol did not appear to prioritise variety in her teaching.

I chose to work with Di because her responses to the initial beliefs questionnaire suggested that she had a social constructivist view of teaching and learning. This was a rare phenomenon, and I was interested to see how Di's ideals would be modified by classroom practice. Di had made continuous use of PART, but found that the Reflections stage unhelpful when doing a routine practice exercise. There had been an extended repertoire of active learning experiences in Di's teaching.

(For example the 24 game, Make me say 20 game, Making Equations puzzles, Think of a number puzzles, Trial and improvement methods, use of real scales to model equation solving, many number bond games, an investigation into the difference between consecutive square numbers, Sheep Fence, crossnumbers, countdown activity, spreadsheet usage was incorporated occasionally, nets practical, maxbox, probability practical, draw your own island, Matchsticks, Volumes and Face North).

Reflections: 1999/2000

Since an early assessed piece of work had been set requiring the use of PART planning, it was certain that *all* students had done some PART planning. It would appear that the great majority of students had continued with it, in one form or another, throughout the year. Since the initial compulsion to use PART had passed, the continued use of PART was optional. Student teachers had been advised of alternative planning approaches by other university staff and by school staff, removing PART as the sole model for planning, yet PART persisted in most cases. I concluded that students valued this approach and found it a useful lesson-planning tool. However, I believed that further study was needed to determine the effect of PART in increasing the student teachers' repertoires of active learning experiences.

Cognitive theory (Winitzky and Kauchak, 1997, p.69-71) would predict the gradual internalisation of the PART planning process, and assimilation into more complex planning schemes. It also suggested that for many students, PART was likely to continue to be of influence in the longer term.

A possible sharpening of the definition of the "Activity" component was considered, perhaps making a constructivist view of learning more apparent, (as in Anderson & Piazza, 1996). An alternative descriptor for an active-learning-experience was that of an *active-construction-experience*. However, after some consideration this was not adopted because the former expression appeared likely to be more immediately understandable to student teachers.

Evaluating Lesson Planning Guidance: 2000/2001

The data collection, analysis and selection strategies used with each group of respondents continued as before, except that there was a new cohort of students to work with.

Observations: 2000/2001

In April 2001, I asked the thirteen one-year PGCE students to describe the extent of their use of PART in lesson planning. At this stage two students claimed not to be using PART at all, having developed their own approach modelled on the placement school advice in one case and on a three-part lesson (starter, main & plenary) in another case.

Eight students were continuing to use PART, but had made some adaptation to it. Four of these were omitting the Transition, as they felt they could now manage these on an impromptu basis. The other four were simply using PART less formally than before. The remaining three students claimed to be using PART for all of their lesson planning.

Once again, I discussed lesson planning with four sample students, observed their teaching and assessed their use of PART-planning by examining their teaching practice files for each placement.

In her first placement, Emma was still using PART and finding it very helpful, but the end product was "too bulky to use in lessons and was sometimes summarised to provide a working document". By the second placement PART was still in evidence in every lesson plan, but very much condensed, especially in the Transition which had often been omitted. There were some very short lesson plans, particularly where the pupil-text was used. Emma had used a variety of pupil activities in her teaching.

> (Including Diagonals in polygons, Brackets, Trapezium numbers, Noughts and crosses, 3d polyominoes, Matchsticks, Multilink, Braille, Face North, Measure triangles intro to trigonometry, Excel, Equable shapes, Game of death, an investigation into factor sums, Snowflakes).

I asked Emma if she felt under pressure to teach in a restricted or extended teaching style, but Emma felt that she had "Not been influenced either way." Emma had only been encouraged to use the departmental scheme of work and pupil texts, but had chosen to look for ideas from a variety of sources, and made up some of her own activities for pupils.

In his first placement, Fred had moved to informal use of PART:

> *I started with PART-planning. I found it difficult for me to follow in the lesson, so I adopted bullet points instead, once I had got over initial transition problems. PART is still used mentally, but it is not strictly followed on the page. Fred, interview, 1/12/00*

By the time of the second placement, PART was not recognisable in his written lesson plans, but Fred claimed that PART was always borne in mind although it was not strictly adhered to in his text. He claimed to think carefully about the presentation and activity in advance of the lesson, but to be impromptu with the reflection and transition aspects. Fred's lessons habitually began with a review of work that pupils had carried out in the previous lesson. Fred had a tendency to use the pupil text for routine practice exercises, but to supplement this with his own examples and explanations. There was little evidence of an extended teaching repertoire. Fred claimed that he had been encouraged to adopt what he called the "very didactic style" of his placement schools. He was not encouraged to research for lesson ideas, and did not often do so.

In Gail's first placement, PART was very much still in use, and was set out for ease-of-use on her word processor. Gail commented that she did not need to think about all the different aspects of PART:

I have never put transitions in, I just deal with them in the classroom. I don't put speeches in any more, I just focus on what I'm going to do, as I am more confident now. Gail, interview, 7/12/00

By the second placement, Gail had changed to Presentation, Activity, Reflection and *Time*. In the 'time' section, Gail would record how many minutes she expected each activity to take. Transitions were still not planned, although these proved to be a weakness in the observed lesson and had been criticised by her mentor, a class teacher and myself.

In her first placement Gail had remained largely content with the restricted style, but in her second placement had been encouraged to use a wide variety of sources for lesson ideas. Gail then incorporated a range of pupil activities into her teaching.

(Including Equable Shapes, Snowflakes, Drawing and describing shapes, Battleships, Practical task with Y9 measuring and estimating, many Numeracy activities, probability practical, Origami polygons).

Gail felt that her class teachers encouraged conformity with their own practice:

Unintentionally, they try to get you to model yourself on them - human nature! Gail, interview, 06/04/01

In Hal's first placement, he used a two-stage planning process, beginning with PART:

The vast majority of my lessons are written using PART-planning. However there is a tendency to produce a complex plan which then is simplified before I use it in the classroom. Hal, interview, 30/11/01

Hal had used some variety of approaches in his first placement (including pendulums, an investigation of square, cube and Quartic Roots, Bracelets, Pond borders, an investigation of the factors of large numbers). I noted that Hal did not write aims and objectives for his lessons very often. This was identified by the mentor and myself as contributing to Hal's lack of clarity in communicating with pupils.

Hal felt that he had some choice of approach:

I'm always encouraged to find my own style but watch other teachers and learn lots of strategies & techniques that you can adopt and adapt in a way that suits me. Hal, interview, 06/04/01.

In his second placement, Hal became aware of the fact that he was struggling to cope with the demands of teaching, and he responded by planning more carefully:

My lesson plans now are extremely thorough (they've been heavily criticised by my head of department for being so). They do fall in line with PART. I usually write something on transition but I don't bother to subhead it. The final reflection is called plenary. Hal, questionnaire response, 06/04/01

Hal felt free to adopt his own teaching style, but he felt restricted in terms of pupil activity:

I was consistently advised to use set text books and, would you believe it, not to bother with the National Curriculum at Key Stage 4, but to use the syllabus. Interview, 06/04/01.

Reflections: 2000/2001

I had discovered that Emma was not particularly keen to try a wide variety of activities in the face of opposing advice from her mentor. However, it remained the case that Emma had used a broad repertoire of active learning experiences with her pupils. She had generally done this when not being observed by her mentor.

Neither placement school had offered Fred the target of developing non-routine activities. The result was that Fred became a competent, restricted-style teacher working largely with explanation, examples and exercises. My view was that PART had been ineffective in helping Fred to challenge the restricted style, and his lesson planning had eventually become simplified because Fred did not see the need for more complex planning. My own observations of Fred's lessons supported this view, as a clear focus on procedures and reliance on pupil texts and worksheets had been in evidence. This confirmed that it was better to view PART-planning as an enabling tool. PART could only support an extended teaching repertoire, rather than being a means of *ensuring* that students develop a wide range of active learning experiences. PART planning could not achieve much in the face of opposition from school staff, or reluctance from student teachers.

Whereas the teaching that I had observed in Gail's first placement had been an excellent demonstration of a restricted teaching style, her second placement lessons were far more adventurous. I interpreted this development as Gail having established basic teaching routines before trying more unorthodox strategies. There was also more encouragement in the second school to look for varied approaches. In any event, it was clear that Gail was developing a good repertoire of varied active learning experiences for pupils, and becoming less reliant upon detailed written plans. PART, particularly PAR, had provided an adaptable framework to support Gail's development.

I believed that Hal was convinced about the value of an extended teaching repertoire. He had made good use of PART-planning but Hal lacked the management skills to sustain either the restricted or the extended teaching styles. On reflection, I wondered whether Hal had been led by university staff, the course aims and the university ethos to accept a model of teaching that was not personally achievable in the time available. Hal may have relied too much upon motivating pupils through active learning experiences and not enough on himself as pupil-manager. Had we focused from the start on developing Hal as a restricted-style teacher, then he might well have met this limited target. As it was, Hal left the course in April 2001 and took up a career in computing.

Conclusions

In terms of student acceptance, PART had been a success. Throughout the monitoring period of two years, there had been a high level of take-up of PART. For the great majority of students, PART had continued to provide a useful lesson planning framework well beyond the compulsory period and despite it being only one of several methods of lesson-planning to which student were introduced. For the majority of these students, the use of PART gradually became less formal over the year. In particular the management of transitions tended to be assimilated and was not necessarily written down or thought through in advance. However, I felt that a consideration of transition was an essential step in learning how to manage lessons (as evidenced by Gail) and that the Transition component should remain.

PART was provided as lesson planning *guidance*, and could never compel students to use a wide teaching repertoire of pupil activities. It was still possible to use PART for planning to teach in the restricted style (e.g. Fred and Carol). However, it was a framework that could support student teachers in providing a wide range of pupil activities, and one that encouraged and enabled student teachers to use a wider repertoire of approaches.

I concluded that PART was an effective *contribution* to the aim of broadening student teachers' repertoires, and that it worked best when complemented by support and encouragement from mentors.

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Primary Student-Teachers Teaching Numeracy

Derek Haylock

University of East Anglia

This article offers a checklist of some characteristics of effective teaching of numeracy that I suggest to my primary PGCE student-teachers. It is written as a set of questions that student-teachers can ask themselves from time to time in reviewing their mathematics teaching. This is then followed by some examples from observations of student-teachers teaching mathematics related to a selection of these questions.

All primary teachers are expected to become effective teachers of numeracy. There is a very short time frame for the many non-specialists to gain this expertise alongside the many other demands on their teaching abilities in their pre-service courses. To help in their development as mathematics teachers, I have found it useful to offer students a set of questions related to the characteristics of effective teaching of numeracy I hold to be important. These questions can be used by students as a way of reviewing their mathematics teaching. Some of these characteristics are then highlighted by examples from observations of student teachers.

Figure 1 offers the questions related to planning and figure 2 offers those concerned with teaching.

Planning

- Do I base my teaching of number on clearly-focused objectives that are made explicit to the pupils?
- Do I plan opportunities to emphasise and clarify key mathematical concepts, principles, properties and relationships, and the associated language?
- Do I plan teaching to counteract common errors and misconceptions in number?
- Do I plan effectively for progression in number for all pupils, including the most able and the lowest-attaining?
- Do I plan the oral/mental starter to promote learning of specific skills and strategies and not just to rehearse existing learning?
- Do I structure my teaching sessions to ensure that as many pupils as possible get substantial teacher-input and opportunity to talk about number?
- Do I plan pupils' contributions to the plenary and tell the pupils what these will be in advance?
- Do I ensure that pupils have regular and substantial opportunity to practise and consolidate numerical skills and processes, both written and mental?
- Do I aim to promote confidence with number through its integration into data-handling, problem-solving and investigative work?
- Do I incorporate calculators sensibly, efficiently and effectively into number work (Year 3 onwards) to promote understanding and confidence with number?
- Do I make good use of IT, textbooks and other resources for teaching number, in a way that is not scheme-driven?

figure 1

Teaching

- Do I use whole-class sessions to explain and to question pupils about mathematical processes, concepts and relationships with confidence and clarity?
- Do I vary the level of questioning effectively to involve all pupils across the range of mathematical ability?
- Do I use small-group activities effectively for teacher-input and to promote pupil-talk and interaction about mathematical properties, processes and relationships?
- Do I motivate pupils to be excited by mathematical processes, properties and patterns?
- Do I emphasise mathematical language, both vocabulary and key language-structures?
- Do I help pupils to make connections between language, concrete experiences, symbols and pictures?
- Do I encourage and develop pupils' informal strategies for calculations regularly and frequently, making key mental strategies explicit?
- Do I teach pupils how to use images that support mental strategies for calculations, such as empty number-lines, hundred-squares?
- Do I teach written methods of calculating clearly with an emphasis on understanding the process, rather than simply learning a recipe?
- Do I exploit unplanned opportunities to discuss mathematical ideas and processes?
- Do I relate the pupils’ mathematical work to the real world?

figure 2

The following sections offer examples of my observations of student-teachers relating to:

- making learning objectives explicit
- making key language explicit
- signalling expectations for the plenary
- differentiated questioning in whole class sessions
- using assessment information in planning subsequent lessons
- responding to unplanned opportunities

- using images that support mental strategies for calculations
- relating mathematics to real life situations

Making Learning Objectives Explicit

In a numeracy lesson a teacher will probably have one main learning objective for the oral/mental starter and another for the main activity. It is good practice to make these explicit to the pupils, using language they can understand. This helps pupils to focus on what it is that they are supposed to be learning and what they will have to demonstrate to you as the teacher that they can do at the end of the session.

> *Working with a Year 2 class, Claire has permanently up on the wall a colourful picture of a target with an arrow through the bull's-eye. She begins the oral and mental starter with some practice in counting backwards and forwards in 5s and then she pins onto the picture the target for the next bit of the session: 'Use 20 as a stepping-stone for addition'. Reference is made to this objective as children are taught how to calculate 17 + 5 on a number-line by thinking of it as 17 + 3 + 2. Other examples follow. Ten minutes later she asks the children to tell her what they have been learning to do, hence reinforcing the objective. She then introduces the main activity by pinning up the target for this section of the lesson: 'Use a balance to decide which objects are lighter or heavier than a kilogram'. There follows a brief discussion of the target, ensuring the pupils understand the key words involved (balance, lighter than, heavier than, kilogram), before the teacher outlines the practical activities the groups will be doing. In the plenary at the end of the lesson, each group is asked to show one object and explain how they decided whether it was heavier or lighter than a kilogram. The teacher then asks the class to tell her what they have learned to do today and whether they have hit the target.*

Making Key Language Explicit

Pupils have to learn the specific vocabulary of mathematics. The National Numeracy Strategy puts a particularly strong emphasis on this and teachers are now provided with clear guidance about the key language to be developed in each year group in primary schools (see: DfEE, 1999, *Mathematical Vocabulary*). Pupils generally enjoy learning technical language, so student-teachers need not be fearful of introducing terminology such as 'pictogram' in Year 1, or 'multiplication' in Year 2, or 'hemisphere' in Year 3, or 'quadrilateral' in Year 4, or 'divisor' in Year 5, or 'icosahedron' in Year 6 – provided, of course, that they themselves understand the words and use them accurately and appropriately!

It is important to note that there are many mathematical terms that are used in everyday language, often very loosely and sometimes with different meanings. For example, pupils will first associate the word 'volume' with a button on the TV remote control, not with the amount of three-dimensional space occupied by a solid object. 'Difference' can mean any kind of difference in everyday language and can be applied in mathematics to any kind of measurement comparison; but when we ask, 'What is the difference between 5 and 9?' we are focusing on a very specific mathematical process, that of subtraction. Similarly, 'similar' applied to shapes in mathematics has a much more precise meaning than it does in everyday language. A sugar cube need not actually be a cube. 'Just a minute' is rarely just a minute. The village square is rarely a square. And what can 'mean' mean, apart from a measure of central tendency in a set of numerical data? (See also: Cockburn, 1998.)

It is very helpful for pupils if the key language used in a mathematics lesson is made overt and displayed clearly (for example, on flash cards). In the best lesson-plans the key language to be developed will have been identified in advance. If there are differences in the ways in which particular words are used in everyday life and in mathematics, then discuss these differences with the pupils – and challenge them to find other examples!

> *Karen is teaching a Year 1 class. She displays the key language for the session, on individual flash-cards: 'taller, shorter, tallest, shortest'. She gets the class to read the words together. A group of three children is asked to stand up. The teacher, each time holding up the appropriate flash-card, asks questions requiring comparisons between their heights; such as, 'Who is <u>taller</u>, John or Sam?'. For each answer, the class repeat together the correct statement; for example, 'Sam is <u>taller</u> than John.'*

In addition to vocabulary, there are characteristic language patterns associated with statements about number-relationships that should be made explicit and taught to pupils. These would include statements having such formats as: ' is more than '; ' is less than ', ' shared equally between is each'; ' sets of is altogether'. These can be written out in this form, displayed, and then referred to in teaching, to assist pupils in learning to connect the symbols of mathematics with the corresponding language patterns.

> *Geetha is planning to teach her Year 6 class how to share out a sum of money in a given ratio; for example, to share £24 in the ratio 3:5. At the start of the session she puts on display and talks about the key vocabulary to be used: 'share, ratio, total'. She then displays the language pattern to be used, in this form: 'Share £ between A and B in the ratio : ' and discusses with the class what this might mean.*

Teachers also have to be careful to use language in a way that supports mathematical understanding – and to avoid careless use of language that can actually mislead. For example, to talk about ‘borrowing a ten’ when doing a subtraction calculation by decomposition is misleading; to talk about ‘exchanging a ten for ten ones’ provides a better match between the language used and the mathematical process. Similarly, to ask the question, ‘Is this a square or a rectangle?’ carelessly obscures the fact that a square is a special kind of rectangle; more accurate would be, ‘Is this a rectangle?’ followed by ‘Now, is this rectangle a square?’ To say, ‘Six take away negative three’ when discussing ‘6 – (-3)’ is totally baffling; but to ask, ‘How much more than negative three is six’, with the support of an appropriate number-line diagram, removes the mystery and promotes understanding.

Using whole-class sessions to question pupils about mathematical concepts

To establish understanding of any concept the teacher should present pupils with examplars that embody the concept and non-examplars that help to refine it. Here is a good illustration of a student-teacher using a non-exemplar to promote pupils' understanding through questioning.

Rachel is teaching a Year 6 class and introducing the main part of the lesson with some interactive questioning about circles. There is a circle already drawn on the board. One of the questions she asks the class is: "What is the diameter of a circle?" A pupil responds: "It's a line going across a circle from one side to the other." Rachel immediately turns to the board and draws a line across the circle, but not passing through the centre. "So is that a diameter?" she asks. This non-exemplar immediately helps the pupil to sharpen up their 'definition' of what is a diameter, by telling the teacher that it has to pass through the centre of the circle.

Signalling Expectations for the Plenary

The plenary session in a numeracy hour can be one of the most stimulating components. It can also be used in a predictable and unimaginative way, with the teacher not having planned this part creatively; for example, simply asking the group to tell the class what they have been doing, or just working through one of the questions the pupils have been doing on the board. Here is a student-teacher using the plenary well with a Year 4 class.

John's objectives for the main activity are that pupils should know a half, a quarter, three-quarters and a tenth of a litre in millilitres, and be able to recognise whether the capacity of a container is about a litre, half a litre, a quarter of a litre, three-quarters of a litre, or a tenth of a litre. This involves mainly practical work with water, containers and measuring jars, with some recording on a prepared table. The set of containers given to each of the groups of pupils are all fairly easy to predict. During the practical work John takes to each group one additional container that is really very difficult to estimate: such as a tall thin tube, a squat circular container and a distorted vase. He tells them to measure its capacity in millilitres and that they will be asked in the plenary to challenge the other groups to estimate whether it is about a tenth, a quarter, a half, three-quarters or one litre. For the plenary session the teacher first ensures in good time that the groups clear up and dry their tables, with the containers arranged neatly in the centre. There is then a five-minute discussion about what the class have been learning, highlighting again the key language used and the facts they have been learning: half a litre = 500 ml, and so on. Each group is then asked to hold up their special container and to challenge the rest of the class to estimate its approximate capacity. There is some brief discussion about why a particular container might be misleading. The teacher then explains to the pupils, with a few recall questions, that in tomorrow's lesson they will be doing something very similar about kilograms and grams. Next he gives the pupils a task for homework: with a parent's permission, to try to find one item at home that is sold as a litre, one as half a litre, one as a quarter of a litre, one as three-quarters of a litre, and one as a tenth of a litre. If any of these are empty to wash them out and to bring them in to school tomorrow for a display.

Differentiated Questioning in Whole Class Sessions

Student-teachers may recognise the value of interactive direct teaching of mathematics with a whole class but they often get concerned about how effective this can be as a teaching approach with a class in which there is a wide range of ability. The teaching skill to be developed here is differentiated questioning around one topic. This skill is particularly important in teaching mathematics, where the range of competencies is often most marked and the issue of differentiation is therefore most acute. Here's an example of this being done well.

Rita, working with a Year 3 class, is leading a whole-class question-and-answer session about fractions. Her objectives for this part of the lesson are that pupils should develop their skills in calculating mentally a half of a given number and to connect this with their knowledge of doubles.

With most of the children her questions focus on finding a half of number up to 40, getting pupils to explain their thinking. For example, a pupil calculates half of 28 by partitioning it into 20 and 8, to get 10 + 4 = 14. She asks another, more able pupil, if they know what is half of 30, and how could they use this to find half of 28. The teacher then follows this by asking a less confident child what is double 14. The connection between the halving and doubling is discussed. Each question about halving then provides an opportunity for a less confident child to handle the corresponding question about doubling. At one point the teacher directs a question to one particularly able child asking her to find half of 128 – and to explain to the others how she did it. She then asks her least able pupil to put up all his fingers and then to put one hand behind his back. Her questioning leads this pupil to recognise that the 5 left on view is half of the 10. Very skilfully, the teacher is ensuring that all the pupils are involved and able to participate, using her knowledge of the children to match carefully some of her questions to the different levels of mathematical ability within her class.

Using Assessment Information in Planning Subsequent Lessons

The Numeracy Strategy framework is often implemented in schools in such a way that the class will spend a few days on a particular topic for the main teaching activity and then will be required to move on to a new topic. Student-teachers, conscientiously making their ongoing assessments of pupils' learning through feedback from question-and-answer sessions and their marking of the pupils' written work, are then sometimes puzzled about what to do when this assessment information indicates that various pupils need further help in achieving the learning that had been intended. The NNS framework suggests that important topics will be revisited each term – but that will seem like a long time to wait for the pupils and no help for the student-teacher who will probably not be there next term and who has to demonstrate now that their assessment is informing their planning. One approach is to try to pick up some of these problems in the oral-mental starters. Here's an example.

Having spent three days in which they developed their skills in ordering numbers up to 1000 and rounding numbers to the nearest 10 or 100, a Y3 class is then expected to move on to addition and subtraction calculations. Matthew, a student-teacher, is disappointed that quite a number of the pupils have made numerous errors on a written task requiring the rounding of numbers to the nearest 100. He wants to find time to revisit this process, making better use of the number-line to enable pupils to understand it. So he builds some further teaching and discussion of this process into the oral-mental starter for Monday, Tuesday and Wednesday of the following week's numeracy lessons. By *the end of these three days, the informal feedback from the question-and-answer sessions indicates that the pupils are now much more confident with the ideas involved.*

Responding to Unplanned Opportunities

One of the characteristics of a really good teacher of numeracy is the confidence to respond to opportunities that arise from pupils' ideas or contributions that you had not planned. Here's an example where a student-teacher did not do this – and thereby disappointed his tutor!

In an oral/mental session with a Year 4 class Andy is using questioning very effectively with the whole class to explore mentally questions such as '5 x = 35'. He makes good use of the 'tell us how you did it' approach, drawing on the pupils' knowledge of the multiplication tables. Then one girl, Clare, gives the correct answer to '6 x = 54' and explains how she did it: 'Well, if it was 10 sixes that would be 60, but 54 is 6 less than that, so it must be 9.' The teacher is clearly impressed and rightly congratulates the pupil – but then just moves on to the next question in her lesson plan.

A really confident mathematics teacher would seize this opportunity to explore this approach with the rest of the class. For example, asking: 'How could we use Clare's idea to find the missing number in 3 x = 27? In 8 x = 72? In 4 x = 32?'

Using Images that Support Mental Strategies for Calculations

Research into the development of children's mental strategies for calculations (Klein et al, 1998) has shown how the 'empty number-line' can provide pupils with an effective image to support their manipulation of numbers. An empty number-line is used to represent the relative positions of numbers, without concern for the actual scale. Teachers regularly using this approach in discussing calculations with pupils have found that many pupils by the end of Year 4 are able to handle confidently the addition and subtraction of numbers with up to three digits by non-standard, informal methods. This is especially effective with those calculations that conventionally cause so many problems when done by standard vertical layout methods, such as subtractions involving zeros in the first number. Here's an observation from a Year 4 class working with a confident student-teacher.

In her plenary session Kate draws an empty number-line on the board, writes '302 – 197' and asks one girl to come out to the front to demonstrate to the class how she worked this out. The girl marks 197 and 302 on the line and then uses 200 and 300 as stepping-stones to find *the difference (3 + 100 + 2 = 105), (figure 3). The teacher discusses the picture with the class, specifically emphasising the language 'stepping-stones' and 'difference' to underline the method used. She then writes '402 – 197' on the board and asks which numbers could be used as stepping-stones for this calculation. The children respond quickly and enthusiastically (using 200 and 400 as stepping-stones, the answer is 3 + 200 + 2 = 205). Another number-line is drawn and another child demonstrates the calculation. The teacher follows this up with '502 – 197', '602 – 197', '702 – 197', which most of the children are now able to calculate mentally.*

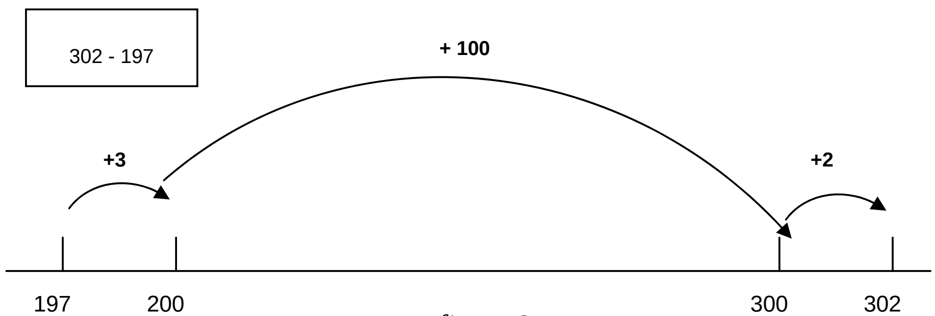

figure 3

Relating Mathematics to Real Life Situations

A good teacher of mathematics will always seek to ensure that pupils connect what they do in school with their everyday experience and the real world outside of the classroom. Mathematics is applied in almost every sphere of life and it is poor teaching if pupils rarely get to see the point of what they are learning in school.

Mike is a student-teacher working with a Year 4 class on reading the time of day from an analogue clock, using a.m. and p.m. notation. The school's mathematics scheme has worksheets involving drawing hands on clock faces. Mike has decided not to use these because he thinks the activity is totally unreal. Instead he takes the classroom clock down from the wall and runs a question-and-answer session with the clock as a visual aid, using questions related to the events of the children's day, such as: What time is this? What will we be doing when the clock shows this time and we are at school? What will you be doing when the clock shows this time and you are not at school? Each pupil is then given a piece of paper with a time written on it in am/pm notation and told that if during the rest of the day they present the piece of paper to the teacher when the clock says that time he will give them a team point. Somehow or other all the pupils manage to succeed with this task!

To conclude

PGCE student-teachers have very little time to develop as effective teachers. As mathematics educators we can offer frameworks to enable them to analyse their practice in planning and execution to scaffold their development. By sharing the frameworks we use and examples which clarify interpretations of being "effective", we may further the debate and our understanding of how student-teachers can become effective teachers of numeracy.

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Working with students on questioning to promote mathematical thinking

Anne Watson

University of Oxford

In this paper I unpick how I work with PGCE secondary mathematics students on questioning. Some structures and strategies are suggested, and some examples of students' growing awareness of the issues are given.

On not knowing how I work with students

I was recently asked "How do you work with students on questioning?" and my reaction was "I have no idea". On reflection, it was the question I had found confusing, not my practice. I do not have an answer to 'how' because my ways of working on questioning are fluid, always changing, sometimes explicit and sometimes implicit. I do not run a session called 'questioning'; nor am I always aware when students might perceive me as working on questioning; nor do I have any particular routines or strategies on which I rely to involve students in thinking about questioning.

Let me explain, lest some passing inspector misunderstands. This does not imply that I do not work on questioning with my students … nor that they finish the PGCE course without developing a range of questions, question-styles, and ways to question questioning. The problem with 'how' is that such work is integrated into many different kinds of teaching/learning situation and I cannot be explicit about that of which I am not aware; I cannot even be explicit about some of which I am aware. And when I am aware of what I think the students are working on, they may be thinking about something else. Does one need a session entitled 'questioning' in order to convince them they have worked on it? What is more important to me is that I work on questioning, keeping the issue at the forefront of my mind, so that students hear about questioning, and work with examples of different kinds of question.

Knowing how I work on questioning

I constantly and explicitly work on mathematical questions and questioning, so teaching students about questioning runs parallel to whatever I am learning at the time. During the two years leading up to the publication of Questions and Prompts for Mathematical Thinking (Watson and Mason, 1998) my own learning curve about questioning was very steep, and students inevitably heard, or heard about, the latest bunch of thoughts I had. Either I used particular forms of question in my work with them, or would give them examples of questions which can be asked in classrooms about which I had only just become articulate through my own work and writing. Maybe they did not always spot that I might be offering them questions which were generic in form and could be adapted to other situations; maybe they thought of my questions as something to answer rather than structures and strategies to be thought about.

Some deliberate teaching acts

In the first week or so of our course we do a lot of mathematics with students, working in open-ended ways, using models of whole-class discussion which might be new to them. We sometimes pause and point out what we are doing; this is often about the dynamics of questioning:

- varying wait-time, before and after answers;
- using open questions (but see below);
- not commenting on answers but asking for more;
- bringing in other people;
- collecting a range of responses on the board;
- seeking agreement, alternatives or dissent;
- using, or not using ‘hands-up’;
- using names to get particular people to answer;
- remaining silent until something else is said.

We have to maintain a balance between saying too much, so that such tactics become a list of things to do, and saying too little so that they are not noticed.

Early in the course I make explicit reference to one kind of question, and give session time to its creation. Students talk enthusiastically about how they have helped students in other teachers’ classrooms when they have asked for help, but less is said about other kinds of interaction. I ask them to create an ‘interrupting question’, one they can use to approach any student in the classroom who appears to be comfortably working, but which allows the learner and the student to discuss some mathematics. Students suggest possible questions. The context is usually that learners are working through textbook exercises. Students tend to agree that questions like: "Which has been the easiest so far?" or "Can you show me how you did number 8?" work rather better than "How are you getting on" or "Everything alright?

Interrupting devices do not need to be interrogative of course. Prompts such as "Show me how you did number 8" or "I really want to see number 8 because people are using all sorts of methods" have similar effects. Giving session time to this exercise of question creation shows the value of carefully crafting questions, and also draws attention to the use and recognition of generic question-types. It also suggests that teachers have a role to play during textbook exercises for all learners, not just those in difficulties, and opens an avenue for assessment to inform teaching. However, I am not going to say more in this paper about using questioning for assessment purposes, nor about questions which have solely a motivational or disciplinary purpose.

Open and closed: an unhelpful dichotomy

Sometimes I stop the flow of a session and ask students about the type of questions I have been asking. Usually they comment on my use of open and closed questions, but these are distinctions I now find unhelpful. At the time of the introduction of the first National Curriculum, the accompanying Non-Statutory Guidance (NCC, 1989) included a chart showing typical closed questions and how they might be adapted to be open-ended. Becker and Shimada (1997) report how a group of Japanese teachers researching the development of open-ended approaches to mathematics teaching eventually wanted to abandon the notion. Many of the strategies they devised were not open-ended, more open-started and open-middled. As teachers with a curriculum to teach they wanted some idea of what mathematics their learners would eventually understand, but recognised that learners needed to have freedom to find their own ways through problems in order to make sense of the underlying concepts, techniques or ideas.

An open question is usually taken to mean one with several answers, to which many learners can contribute, but contrast these two open questions:

> *If the answer is 4, what could the question be?*

> *I want you to make up three questions to which the answer is 4, and each question must come from a different topic we have studied this term.*

The first question is wide open, and is likely to generate low level arithmetical operations using small whole numbers. The second is more constrained and denies the possibility of sticking with simple operations; learners are forced to think beyond the obvious. Both are open, with the advantages of open questions, but one is more likely to involve grappling with concepts than the other.

Compare the first open question to this closed question:

> *What number is a square number, and is also the number of sides of a shape which can be made by sticking two congruent triangles together edge-to-edge?*

The latter question is closed, but encourages engagement with concepts. The almost Orwellian mantra "open-good; closed-bad' is clearly misleading.

How can I help students get beyond the open/closed dichotomy? I do not want to directly contradict what they are reading … to be reading anything about teaching mathematics is a good thing (reading-good; not-reading-bad!). I have to recognise that it took me years of experience, playing with my own mathematics and thinking about teaching and learning to reject the dichotomy myself.

Genuine and pseudo: a helpful dichotomy

So I use other authors with gratitude. Janet Ainley wrote, in 1987, an accessible article about the types of questions which she saw teachers using in which open/closed (as linguistic descriptions) are edged out in favour of categories which relate to the teacher's intentions. In particular, she distinguishes between genuine questions and those which are used for control purposes, or which are versions of "tell me what I already know" or "guess what the teacher is thinking". When the students and I begin to talk in detail about lesson planning they have already thought about the needs of adolescents and the social life of classrooms in the generic component of their course. It is easy, therefore, for them to see the affective difference between being asked a genuine question such as "I wonder how you worked that out" or "What can you tell me about this diagram?" than pseudo-questions such as "What was the best way to work this out?" or "What should we get from this diagram?"

By this stage they have had two opportunities to work explicitly on questions, and in each case the shift has been towards encouraging learners to articulate what is really going on for them and away from giving one answer which matches the teacher's … or mine.

Telling

In a recent course review, some students asked for more on questioning. In previous years in response to such requests I used to give out a paper about class discussion in which I described how one can more easily discuss genuine questions in which there is room for doubt and confusion than questions which present no problems to be resolved. By 'problems' here I mean conceptual difficulties; I am not using the term in its North American sense of a mathematical question which goes beyond practice of algorithms. As I give the paper out I am painfully aware that handouts are not a good medium for work on questioning, although there are parts of the text which invite conjecture and interaction. There are always some students who will not read it, some who will read it but get little from it, and others who will read it and use some of the ideas in their own way, possibly leading to developing their own ideas later. I have been aware that (a) being given a text is not a substitute for working on something oneself and (b) the paper can never express my current thinking, because I am always actively working on questioning.

Cognitive challenge

My own knowledge of questioning does not, in general, come from reading, but from observing others and working on mathematics regularly in ways which help me recognise a cognitively challenging prompt when I hear one or see one. By 'cognitively challenging', I mean that my structuring of mathematical knowledge is challenged by the phrasing and direction of the question, and some conceptual reorganisation may have to take place. For example, the question "Find a function which is symmetrical about $x = 4$, non-differentiable at only two points and continuous everywhere" is considerably challenging to the notions of "continuous and not differentiable" which come directly into my mind.

A question which requires me to work backwards through a very familiar algorithm could make a cognitive challenge. For example, learners would find it much harder to reconstruct a quadratic equation, given some roots in the form of surds, than to find roots from an equation, given the formula.

A further kind of cognitive challenge is to connect two different representations of the same concept (Dickinson, 2001) and ask learners to describe how to get from one to another. For example, how would you get from a cumulative frequency curve to its related frequency polygon?

Teachers face cognitive challenges when they try to construct good teaching examples … and this gives a way into working with students on questioning. For example, it is common for teachers to give learners pattern sequences which, when analysed, can be expressed as formulae of order one or two. Asking the question the other way, as teachers do when they construct such questions, is much harder. Try constructing a pattern sequence in two dimensions for which the formula is $n^3 - n$. A cognitive challenge indeed! No wonder teachers rely on textbooks to do this kind of question creation.

Working on question creation not only challenges teachers but also gives models for providing challenging questions for their learners, but my ability to pose these challenges cannot be passed to students like a package. Some may not see the power in questions that I see as full of potential; they may not perceive challenge similarly. For some students 'challenge' means moving learners onwards towards harder and harder mathematics rather than, as I see it, working more deeply and structurally with what is currently being studied.

Strategies for working with students

One strategy is to use a wide range of questions consciously in my sessions with students, drawing attention to them and their effects in terms of intrigue, exploration and understanding, going beyond open/closed classification by asking "what else can be said?" "what other variations are there in my questions?"

Another is to work on questioning when I observe their teaching, but my observations are rare. Working on questions before observed lessons could be useful; can learners be asked to conjecture before tackling a task? How would conjecturing aid motivation and interest? Can they pose their own questions because of their conjectures?

Discussing the effectiveness of, and alternatives to, questioning after a lesson can be taken as criticism even when a lesson has been, on the whole, good. Another approach would be to discuss questioning with mentors, in the belief that the articulation of purposes and strategies in such a forum makes it easier to raise the subject with students later.

But all of these approaches pre-suppose that I, and the mentors, have knowledge of how to question which we need to pass on to students. Yet some of the ideas on which I currently work have been derived from watching students teach. In a forthcoming paper (Watson & Mason, forthcoming) we describe getting learners to generate the raw material for a lesson, both as a motivating strategy but also to encourage them to reflect on and transform their conceptual understanding. Some of the incidents we use come from students' lessons. For example, Ed asked everyone to make the equation $x = 2$ more complex, effectively hiding the value of x in ever more complex versions by "doing the same thing to both sides", including getting x to appear on both sides. He then pooled the results and asked everyone to "undo" the equations. This was an idea adapted from a number of sources, but to make it work Ed had to recognise its power, adapt it to his purposes, and understand the demands it would make enough to have confidence in the learners' response. Jonny spent a lesson asking learners to devise their own diagrams to show how the perimeter of a rectangle changed as one of the dimensions changed, and to choose the best method. Eventually they re-invented graph-plotting for themselves as the 'best' way to do it. All he had to do was point out that it is conventional to put the independent variable on the horizontal axis. He trusted them to reach the endpoint he had in mind, he knew that genuine problem-solving of this kind was a very good way to learn, and he believed the lesson was well-spent as it helped them understand the way we plot graphs.

My development of ranges of questions comes from my own experience of doing mathematics. Thinking about processes. Observing how I had to wringle and wrangle my knowledge into new contortions to answer new types of question. Watching other teachers. Identifying question-types which conformed to my beliefs about what learners can do, supported by discussions with friends, students and teachers in sessions, workshops and articles. In other words, questioning skills develop through observation, reading, use, reflective thought and awareness, supported by others. Access to others' strategies and experiences is only a small part of this process, more is learnt by trying one question and evaluating its effects than by reading a list of ten types in a book.

Structuring question-posing

In Questions and Prompts (Watson and Mason, 1998) we record a huge range of generic challenging questions, but we also give a structure for developing one's own questions. The structure is based on the apparently simple idea that learning involves acting in a certain way on a particular aspect of knowledge. At the end of a lesson, we hope that what learners knew to start with, and what was presented in the lesson, will have changed in some way as a result of mental activity. So to make appropriate mental activity likely, we can ask learners to do characteristically mathematical actions, such as sorting, classifying, defining, varying etc. to characteristically mathematical statements such as techniques, theorems, definitions, representations etc. with the aim of increasing their understanding. Typically I ask students to select a curriculum topic, and then we choose a statement and an action at random from lists[1]. So if they choose straight line graphs, and the 'statement-type' is representation, and the 'action' is sorting, we might end up with a question like "Sort some equations of straight-line graphs into those which have positive, negative, zero or infinite gradients". It does not really matter if the question they end up with has little to do with what was chosen at random, what matters is that the structure provokes them into discussing what could be asked.

1 For those with access to this book there is a grid provided at the end to make this choice easy.

There is room for creativity and also room for further thought. For example, if we are supposed to be teaching addition of fractions (the question ‘why teach this at all?’ has been asked for over fifty years and not yet convincingly answered) we can ask:

- What is a useful representation of a fraction?
- Do I wish to treat it as a number, a notation, a division, a ratio?
- Do I wish to treat ‘addition’ as an object, being a noun, or would it be more helpful to think of ‘add’ as a verb?

If we treat fractions as numbers (for why else would we be adding them?) then how can I help learners transform their knowledge of the properties of fractions usefully so that addition is more obvious? How can I help learners explore the properties of addition, or the behaviour of adding, so they understand how it relates to fractions? Clearly I have to provide an environment in which they can transform and explore.

To transform knowledge learners have to have images or representations which can be changed, a sense of a range of application which can be extended, and a structure in which new links can be added and existing links ruptured if necessary. To explore, learners have to have a collection of objects to make sense of, to discriminate between, to observe patterns within, to conjecture about. If I offer too much variety, they may be confused; if I offer too little they will have nothing to discern. What aspects of addition of fractions can I offer, and what can I ask learners to do with them, which will promote change to their current understandings by using their power to spot patterns, recognise sameness and difference, conjecture and test?

I am not going to offer my answers to these questions. The answers take the form of questions to be posed to learners … indeed they may vary for different circumstances. My articulation of these views has recently been influenced by the work of Marton (for example, Marton and Booth, 1997)

Tensions about telling

The tension is between telling them my own ideas and letting them develop their own, while I am passionate (of course) about my own current ideas. What is particularly hard is holding back from pushing my ideas under my students’ noses, while being aware that there are few written sources to help them. Naming and articulating a variety of questions in a book does not make them usable, but at least it makes ideas available to others.

Other ways to structure question posing can be found in Brown and Walters (1983), Becker and Shimada (1997), Clarke and Sullivan (1991) Prestage and Perks (2001), Watson (2000), Mason, (2000). All of these offer generic advice and structures for question development far beyond the Non-Statutory Guidance (1998) open/closed classification. While Clarke and Sullivan focus on motivational, social, organisational and discursive aspects of questioning, all the others draw heavily on mathematical structures to devise questions. At the root is recognition of:

- what can vary and what would it be useful to vary?
- what distinctions can be made and what distinctions would it be useful to make?
- what relationships can be conjectured and what (inverse) relationships would it be useful to conjecture?
- what concepts can be constructed and what concepts would it be useful to construct?
- what constraints might learners bring to the work, and what constraints would enrich their work?

To pose questions which are mathematically challenging, you have to understand mathematical challenge. To pose questions which enable learners to construct useful mathematical understandings, you have to think about the construction process and give them the tools and scaffolds with which to create.

Working on questioning through analysis and critique of textbooks

Towards the end of their first term, we ask students to take a critical look at textbooks and include ideas about working creatively with them as part of an assignment. Their responses and comments reveal much about their views of what makes a good question.

The session started by talking about some research by Runesson (1999) in which she shows how two teachers who co-planned to teach parallel classes taught lessons which were totally different in terms of what was offered to the learners. In one lesson, learners were given a method (divide by five and multiply by the numerator) and then asked to find 1/5 of 40, 3/5 of 40, 3/5 of 60 and so on. The type of question and the method of solution did not vary, nor did the denominator. Learners who were paying attention to what changed and what stayed the same (what else could they be doing?) would have been thinking mainly about multiplying by small whole numbers. In the other lesson, learners created several different ways to calculate 3/7 of 56. In this lesson, learners focused on methods and the meaning of 3/7 as an operator.

One student, Camilla, wanted an example of an approach to simultaneous equations which encouraged students to focus on meaning rather than manipulating coefficients. Keeping the form of the equations the same and only varying the numbers would, obviously, focus attention on the numbers. Therefore, what is required are two equations of different forms (such as $ax + by = c$; $y = dx$) so that the learner will focus on the meaning, or at least the structural representation, rather than just the coefficients.

Alan reported on a lesson about straight-line graphs in which he wanted to provide a mental warm-up. Using the structure $mx + c = y$ he devised a game splitting the class into three parts who had to supply a number, a multiplier and an addend respectively. Thus he was focusing them mentally on the algebraic structure he wanted them to work with.

Earlier in the week, Rachel had focused on her use of open questions, but after the lesson she said: “I think my questions were too open, they need to have something to talk about at the end, I should have closed them down”.

After this introductory discussion, students analysed textbooks to see what the exercises and examples offer in terms of focus and generality. In general they were critical of exercises which were only providing practice, and they emphasised the role of the teacher in helping students generalise from the repeated application of techniques. For example, a follow-up to an exercise involving multiplying brackets to get quadratics could be a question about expressing the results algebraically, particularly looking at questions of the type $(ax + b)(ax + b)=$.

One of the textbook pages was about finding areas of shapes made up of various rectangles. A student picked out an example in which learners were to find the area between two squares, one of which is totally inside the other. This was a spatial representation of the difference between two squares (both squares cry out to be rotated and jiggled about!), and students agreed this was more appropriate as a starting question than tucked away among other less interesting shapes.

In these comments and discussion I believe I saw and heard that some students, too, are working on questioning.

Summary

I work on questioning, and I work on questioning with students, who pose mathematical questions for themselves, from which they construct mathematical challenges for their pupils, who may eventually ask mathematical questions for themselves. It is a delicate thread of expectation and I am still learning how to braid it rather than break it.

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Continuing Professional Development: Teachers Learning to Research.

Pat Perks and Stephanie Prestage

University of Birmingham

This article describes the change from offering an M.Ed. degree course to an M.Phil. taught research degree as the basis for continuing professional development in mathematics education.

The need for change

As with many universities, Birmingham developed a modular scheme for its taught masters course in education, something which is now in place for all of its undergraduate courses. An M.Ed. could be gained by a full-time or part-time routes by taking six modules and writing a dissertation.

Pat did her course at the time of change, with the first year following the old scheme, doing two, year-long mathematics courses, and changing to the new by doing one-and-a-half modules in other bits. The contrast was quite stark. The year long courses offered development and were tailored to the needs and interests of the students and the assignments became progressively more academic as we learned the relevant skills.

In mathematics education, under the modular system, we offered four modules – two in the afternoon, two in the evenings, so that full-time students could take all four and part-timers could do two each year. The modules lasted for twelve weeks lasting three hours a week. The five to eight slot in the evening getting harder and harder as winter set in.

As we taught these courses, we enjoyed them – it is great teaching experienced teachers who want to know – but the balance between in-service and working on the 'academic' was difficult to manage. The full-time students were usually from overseas and their approach to discussion was very different from the local teachers. Sometimes teacher would only choose one module to complete their set, perfectly acceptable in this new system, but difficult to manage in terms of academic growth.

Our own degrees had short courses – so why did modules cause a difficulty? The issue appears to be that within those short courses, very little choice was allowed. A few courses were elective, but on the whole those teaching them knew what courses their students had taken and what was coming next. It was not long before we faced problems with the pick and mix system. Some students were coming to a module having done three previous maths modules, whilst for others this could be the first module. Their needs were different. This showed up in a major way with assignments. In the year-long course assignments had built up from the practical to the more theoretical, academic essays during the course of the year. In the modular system, the assignment was the same, regardless of previous experience. The solution was to change the front cover – to inform those marking of the differences between the students. A pragmatic solution – inspired in its way – at least someone was considering development not finished product, but it did not look at the educational implications.

The numbers dropped – running a module for four or five students becomes hard work the fourth time you do it. Because the different maths tutors taught the different courses, we compounded the problem, we offered no continuity. We no longer seemed to be offering a masters in mathematics education, more some disjoint offerings. Fewer and fewer teachers did a dissertation based on mathematics teaching. Other curriculum subjects had similar difficulties, it was information technology, management and special needs which survived (for many reasons).

Although this happened after modularisation, to assume this was the only cause for the loss of students is too simplistic. But for us, what was happening was the teaching was lacking development, we could do it and not challenge ourselves too much – so the enjoyment went and with it our need to bring in the students.

For some time we did not consider the issue much further. The place of the M.Ed. was considered in the school alongside the development of a module for newly qualified teachers (NQTs) and the proposal of a module which could be taken in the PGCE year. It was intended that both of these modules could be credited towards a masters degree.

Stephanie worked on an NQT course and what was most obvious was the way it appealed to our ex-students who were working in the local area. What they most wanted from the NQT course was to work on their subject disciplines with tutors they had worked with before. This was the aspect which made them return over the year. To work with others on their subject teaching and place their continued learning within the wider perspective seemed to be easier if you began with the subject. They commented that too often the courses offered dealt with classroom management issues across the subjects, when it is planning for the subject which can create or prevent many problems.

So what next? The M.Ed. did not suit us – to teach six modules was too great a commitment – and it was uneconomic to have a course with only a few teachers on it if the tutors did not gain something as well. By this time we had both finished our doctorates and were beginning to find personal ways of understanding the research community. We rarely recruited teachers to research degrees – so we were not building a strong foundation for the link between theory and practice. Could this be the area we needed to look at developing?

The taught research degree model

The M.Phil.(B) was a possible answer – our school had a one-year full or two-year part-time research degree. It is a taught degree in that students have to complete three modules as well as a dissertation. The modules have to be research based. This was made more possible because the university has distance education materials which we could use for two of the research modules. We could develop a taught module based on small research activities in mathematics for the third (we already had a "Special Studies in Mathematics Education" as a level M module). By teaching all three modules in parallel, with the focus for the generic support material being directed to the mathematics classroom, we could offer a coherent mathematics based introduction to research. The students would enrol for the whole course – the modules were fixed. We could work in an organised, integrated and appropriate way for all.

There was a problem – it is a lot of work to achieve in two years – the timetable, September to July, does not fit the busy teacher's schedule. To encourage teachers to finish, the timescale needs to be realistic. Could it be a three-year course - at the same price as the two-year course? The model was designed to allow the writing of assignments to link with the breaks in teaching, see table 1. For the mathematics module a portfolio was planned, short pieces of research, with a final synthesis, for research 1, the assignment was based on research articles in mathematics education and research 2 was based on methods used in the portfolio, each equivalent to 4000 words). Eventually, the course was approved, including a dissertation of up to 25 000 words.

Table 1: the pattern of assignment work

	Year 1		Year 2	Year 3
	Mathematics	Research	Mathematics/Research	
Spring Term	Portfolio begins Short activities	Research reading in mathematics education (1) research methods (2)	Piloting reading on area of interest + methods	
Easter	Portfolio summaries	Writing on one article	First draft Research 2	Analysing findings
Summer Term	Portfolio Longer activity		Data Collection	Drafting dissertation
Summer		First Draft Research 1	Complete Research 2	First Draft
Autumn Term			Collect data	revision
Xmas	Portfolio completed	Complete Research 1		Dissertation complete

The teaching was done on Saturdays to allow people to travel further than the local area. The first course began in February 1999 – we only recruited seven students – but we only advertised to our ex-PGCE students. This was a comfortable beginning, they were used to us and we could begin to work on extending our own thinking as well as theirs.

Only five students have completed their three years – but everyone has gained promotion and that has created more challenges for some than others. The dissertations still have to be completed, but the work is ongoing – M.Phil. degrees allow for ‘writing-up’ status. More exciting, however, is that two of the students are working on transfer to Ph.D. Two more Ph.D. students than we would have expected.

Recruiting to this particular course has been overtaken. More recently we have been able to use the M.Phil. format for bids to the Best Practice Research Scholarship (BPRS) scheme run by the DfES and a scheme with our local education authority, the first year can be considered separately; the 60 credits from the taught modules can be used for a post-graduate certificate. Last year two cohorts began – with 50% wishing to continue into the second year to complete a dissertation. There are still problems with the completion of written work – they have done it, but some teachers hate to go beyond the first draft; a c.p.d. problem for us, but the energy created was wonderful.

The teaching model has remained the same, in each session we work on literature, research methods and personal research activities. The assignments work to aspects which will be useful for the final dissertation. In every session, some research articles are discussed, both for method and for finding information relevant to mathematics education. In between sessions, the teachers do a small piece of research, such as interviewing a couple of pupils or videoing part of the lesson. This then leads to discussion of the advantages and disadvantages of the method and how this can change data collection and its consequent analysis and the literature of methods. The final stages for the mathematics module develops to a small pilot project, which we hope will feed into the research for the dissertation.

This approach has been helped by the recent emphasis on developing the teacher as researcher. In 1996 the Teacher Training Agency (TTA) launched the "Teacher Research Grant Pilot Scheme". BPRS was launched with Estelle Morris asking Higher Education:

> *to support teachers using research processes to investigate their classroom practices as a valuable tool for building knowledge and understanding about raising standards of teaching and learning …… ensuring high quality research* *(Estelle Morris, 2000)*

For the beginning teacher, Circular 4/98 (DfEE, 1998) expects that every NQT will demonstrate

> *understand the need to take responsibility for their own professional development and to keep up to date with research and pedagogy in the subjects they teach (p 16)*

The course has had immediate impact on the teachers. In a session where one group was discussing the interviews with pupils, one of our ex-students said

> *They said things I was not expecting. I know they (pointing to the tutors) told us on our course and gave us articles to read which made similar statements, but you didn't really believe it. Doing this ourselves and finding out what the other teachers did has brought it home in a very different way.*

In a discussion after the teachers had been set the task to investigate the evidence available from pupils' written records after a lesson, comments included

> *It was really useful to spend the time looking at this work in a different way, we don't have enough time to do this usually, so it has been valuable.*
>
> *It made me realise that I wasn't giving the pupils any real opportunity to write in a worthwhile way.*

Looking at short sections of video, time appears different, and once initial apologies are over, the group can work on the teaching/learning. The teachers have to be supportive of each other and this helps with the analysing of articles, one can be analytical without being negative. Looking at video has helped the teachers to identify what evidence is available as well as the advantages and disadvantages of the medium.

Throughout the discussions the students have to work hard on the difference between talking as a teacher and talking as a researcher. The teacher always wants to make things better, the researcher wants to find evidence to respond to the research question. There is also the expectation by those brought up with a strong science background that only the scientific method is proper research, but here the teacher role can be a useful contrast. The work on research literature has sometimes been difficult, but the teachers are developing criticism without negativity and are beginning to find those area which help to inform their own thinking. After only a couple of sessions, one teacher, who claimed to have been very sceptical, said:

> *I want to continue to a dissertation, this is making me think and look at my classroom in ways I have not expected.*

The model of working can cater for different content interests; we have had a group of mathematicians working with a group of science teachers, where we could have many of the discussions in common. Research methods, working on understanding the ‘isms’ have lead to lively debates illustrated by experience. The teacher-as-researcher is much easier to develop than we ever thought possible – as long as they have time, to reflect, change their minds, catch-up and as long as they have the support of the tutors’ interest and enthusiasm.

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