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The Changing Beliefs of Trainee Mathematics Teachers

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This on-going study concentrates on the changing beliefs of trainee mathematics teachers as they progress through a Postgraduate Certificate of Education course. It discusses the relationship between trainees' professed beliefs and their actions in the classroom as observed by the university tutor during their first placement and suggests reasons for the apparent disparity. Some recommendations are given for future development both for trainee teachers and for some mathematics teachers within the partnership. These will be elaborated in subsequent stages of this research.

Introduction

Recent reform documents within the UK, for example, Cockcroft (1982) and Ahmed (1987), have suggested that the 'best' mathematics classrooms are those in which pupils are actively involved in constructing their own mathematical knowledge. Within such a framework, mathematics teachers are seen as providing situations and opportunities in which children can develop their own mathematical thinking, rather than as transmitters of knowledge. Pupils are believed to develop their mathematical knowledge and understanding by internalising and elaborating their own mathematical ideas through interaction with both their classmates and their mathematics teachers. Practical advice and reflections upon successful teaching which adheres to such a framework is plentiful in journals such as *Mathematics Teaching* and *Mathematics in School* and in books and articles written primarily for beginning teachers, for example, Watson (1994), Ollerton (1994).

Researching Trainees' Beliefs about Mathematics Teaching This small on-going study investigates trainee teachers' changing beliefs about effective mathematics teaching during their one-year PGCE course, and further, whether those professed beliefs are translated into observed actions in the classroom. At the time of writing, the 17 secondary mathematics teacher-trainees studied are almost at the end of the second institution-based element of the course and are about to embark upon their second block placement. Trainee teachers following one-year PGCE courses during the current academic year 1995-96 will have sat GCSE Mathematics in the academic year 1989-90, assuming that they followed the 'traditional' pattern of a 'no-gap' education in which they went straight from 'A' levels onto a three-year undergraduate degree course. GCSE Mathematics did not have a compulsory coursework element until 1991. Those departments which opted to include coursework in their GCSE assessments prior to 1991 were considered to be the more forward-thinking mathematics departments, where teachers viewed mathematics as an active process on behalf of the learner; seeing themselves more as facilitators rather than transmitters.

Despite the change in direction of the assessment procedures, it is still the case that, although 59% of students in the current Mathematics PGCE group joined the course immediately after graduating, the whole group's accounts and memories of the mathematics teaching they received at school are almost identical. Each student has vivid memories of their own mathematics lessons being teacher-led, consisting of little other than teacher exposition followed by pupil practice of closed questions which became progressively more difficult the further one worked through the textbook exercise. Only two trainee teachers (12%) had any experience of working investigatively within mathematics and that was limited to GCSE coursework tasks which were relatively structured.

It was to be expected that all the 41% of trainees who are 'late entrants' to teaching, the majority coming to teaching as a second career, would have deep-seated preconceptions that learning mathematics is about taking on board facts, skills and procedures presented by the teacher and about practising these facts, skills and procedures through answering a series of questions which become progressively more difficult. These trainee teachers have never been part of a mathematical community in which their conjectures are analysed, tested or challenged. Consequently it is quite reasonable that they believe that effective mathematics teaching involves telling or showing children what to do (Ernest, 1989; Haggarty, 1995). Thus, at the beginning of the PGCE course, it was not surprising that 82% of the trainee mathematics teachers listed 'ability to impart knowledge' as a major factor in their description of what constitutes an effective teacher of mathematics. Interestingly, the three trainees who formed the remaining 18% are all late entrants to the profession who have small children.

As with all one-year PGCE courses, ours is a partnership in which trainee teachers spend a total of 12 weeks based at the university and 24 weeks in schools. At the beginning of the course, trainees were asked to write about 'What is an effective teacher of secondary mathematics?' All their responses could be classified under the headings: classroom management, pupil motivation, teacher communication, relationships with pupils and subject knowledge. Not one trainee mentioned listening to pupils or encouraging an active approach to learning. In view of their own mathematical backgrounds, all these responses were expected.

For the first six weeks of the course, trainees spend three days at the university and two days in their first placement school. Mathematics curriculum sessions at the university follow the same topic headings as the professional development strand of the course until the first block placement in November. A central tenet of our course is that trainee teachers are considered as active learners who are expected and encouraged to think, act and reflect upon their experiences. They are not provided with 'recipes' for good practice, but encouraged to develop, at their own pace, into effective teachers by asking questions about the teaching and learning situations in which they work, both at the university and in schools; investigating their own teaching and feeding their results back into their subsequent work. Thus, university-based sessions are almost always workshop or seminar-based and involve feedback from students about their work in school during that week. These sessions aim to challenge trainees' assumptions that effective teaching is mainly concerned with transmitting knowledge and skills to audiences who have varying degrees of competence at reproducing those skills. Trainees are constantly encouraged to broaden their perceptions of effective teaching and learning.

In November, the trainees began a five-week block placement in their first placement school. After that block placement trainees spent a week back at the university during which they were asked to repeat the exercise describing their beliefs about what constitutes an effective teacher of secondary mathematics, without having access to the beliefs they wrote about in September.

Having just spent five weeks in school full-time, their major concerns were classroom management and personal survival, typically for student-teachers at this early stage of their training, as shown by Haggarty (1995). It was therefore expected that these would be the major factors in the trainees' accounts. Whilst it is certainly true that most of these concentrated heavily on classroom management, particularly in terms of discipline and control, lesson planning, flexibility and effective differentiation, five of the group (29%) talked of the importance of listening to their pupils, two mentioned catering for the different ways in which pupils prefer to learn, and one wrote about the importance of 'encouraging children to discuss and talk about their Mathematics'. It seemed that several trainees were beginning to acknowledge the importance of perceiving pupils as active participants in the learning process. Some trainees seemed to have begun to re-think the perceptions of teaching and learning which they had held in September.

This written data, collected just before the Christmas vacation, was encouraging. What I had observed in my visits to the partnership schools to see the trainees teach during their first placement was less encouraging in terms of my witnessing strategies other than trainee exposition followed by pupil practice. Lesson plans talked of 'discussion' at the beginning and possibly the end of the lesson, which seemed to be a euphemism for trainee-led question and answer sessions, usually employed as a review of the previous lesson content, after which the pupils were 'taught' some new knowledge and then asked either to complete a worksheet or to do an exercise from the textbook.

Either the trainees' perceptions and beliefs had not changed by the time I visited them and observed them teaching, or they were finding it difficult or impossible to put these new beliefs into practice. From my experience of working with them in university-based session, it seems that, for many, the latter was the case. Only one trainee, during the lessons I observed, showed any real evidence of involving her pupils as active learners with opportunities for each to construct their own mathematical knowledge. This trainee is one of the three who had not listed 'ability to impart knowledge' as a factor in describing an effective mathematics teacher at the beginning of the course. Her first placement school was one in which the head of department, who is also the mentor, is sympathetic to a constructivist model of learning.

At the time of writing, (February 1996), trainees are just finishing their last period of university-based learning and are about to begin their second block placement at a different school. Recently, they all

completed a short, 12-item questionnaire designed to elicit their current beliefs and perceptions about mathematics teaching and learning. The questionnaire was designed to show whether the trainees had developed a constructivist view of mathematics teaching during the course (McDiarmid, 1990). The trainees responded to each item on a five-point Likert scale ranging from Strongly Agree to Strongly Disagree. The results indicate that all but two trainees believe they now hold constructivist views of mathematics teaching. Nevertheless, there were some inconsistencies in their stated perceptions; for example, one trainee believed strongly that 'a vital task for the teacher is motivating children to resolve their own mathematical problems', whilst at the same time also believing strongly that 'telling children how to find the answer is an efficient way of facilitating their mathematics learning'. These inconsistencies will be subject to further analysis as the research develops during the remainder of the academic year.

Inconsistencies apart, it is encouraging, from this tutor's perspective, to see that almost all the current cohort of PGCE Mathematics trainees profess to believe, prior to their second placement, that children should be active participants in the learning process and that it is the teacher's responsibility to motivate all pupils accordingly. There is scope for further analysis of the data collected, the results of which will be reported in subsequent papers.

Problems of Implementation in the Classroom

The trainees were also asked whether they had been able to implement these professed beliefs during their first placement. Many said they had, referring to the exceptionally positive support received from their mentors and other members of staff with whom they worked, who had encouraged them in every aspect of their teaching. There is certainly some disparity here with my own observations of their teaching during their first placement and this is an area which warrants further investigation during the second placement observations. It is possible that those trainees who had adopted constructivist convictions were unable to implement their beliefs in practice because they have not been internalised or integrated into their experience at this stage of their professional development. Five of the trainees experienced some frustration in their perceived attempts to adopt teaching strategies consonant with their professed beliefs. Their comments were:

- *I was limited occasionally by other teachers' views.*
- *Some teachers preferred me to adopt their style so as not to disrupt the pupils' routine.*
- *I was unable to put many of my beliefs into practice as I was placed with a fairly traditional teacher.*
- *The teachers were very concerned that their pupils should not get behind on their textbook work.*
- *Year 11 had strict rules on how and what they were to be taught because their mocks were approaching.*

Such comments raise issues about shared values of all participants in a teacher education programme. These issues will be addressed in forthcoming liaison meetings.

Lesson observation by the university tutor during second placement will provide further data on trainees' ability to translate their current professed beliefs into classroom practice. They may still face resistance from some of the teachers with whom they work, or they may be unable to translate those beliefs into practice, despite our efforts throughout the course. Additional data collection at the end of the trainees' second placement will allow the research to progress further and lead to new developments within the partnership for next year's PGCE Mathematics course.

Conclusion

It seems that on-going, supportive experience in a teaching environment which is consistent with recent reforms in Mathematics education can be beneficial, at least in terms of challenging and changing trainee teachers' widely-held initial beliefs that mathematics teaching is all about 'transmitting knowledge'. The ongoing study described here shows that most trainees have changed their initial professed beliefs and developed a strengthening conviction about the need to provide opportunities through which pupils may construct their own mathematical knowledge by participating actively in the learning process. The trainee teachers were not uniformly successful in translating these professed beliefs into action during their first placement. It remains to be seen how far they implement these current perceptions during their second placement.

In order to be sure that we can provide placements where all our trainees may have at least the opportunity to translate their developing beliefs and changing perceptions into actions, we may need to initiate some developmental work within the schools. This should increase some teachers' awareness of a constructivist approach to teaching and learning, which may mean that the teachers are more receptive to the notion of trainees 'trying out' strategies and approaches which these particular teachers do not utilise themselves. Such development could perhaps occur within mentor liaison groups based at the university, from which mentors could 'cascade' to the remainder of the department. Within the school, teacher development might also be facilitated by means of a member of staff teamteaching with a trainee who is committed to an active learning approach. In the 20% of secondary schools where Ofsted judged Key Stage 3 and 4 lesson content and activities to be broadly unsuitable for the purpose, they comment: '... there is much exposition by the teacher but very little opportunity for pupils to participate and respond ...' (Ofsted, 1995).

This research to date has raised issues for further study and highlighted areas which can be pursued in the quest for increasingly effective initial teacher education partnership courses.

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Why Secondary School Mathematics Departments Take Students

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This article is based on a number of informal discussions with heads of mathematics in secondary schools in the North East of England. They were asked for their reasons as to why they agreed to take Post Graduate Certificate of Education students into their department for the teaching practice component of the initial teacher training course. A wide range of reasons were given including financial and altruistic concerns. Despite a number of difficulties mentioned in allocating suitable classes to students most schools are happy to be able to contribute to the course.

Introduction and Background

Reading a recent issue of Mathematics Education Review I came across an article entitled *A Survey of Secondary Mathematics Initial Teacher Training* (Haylock, 1994a). It detailed the problem some institutions were finding then in securing sufficient school placements for mathematics student-teachers who were embarking on the oneyear Post Graduate Certificate of Education (PGCE) course, under the new partnership arrangements.

The course is generally thirty-six weeks long of which, in this institution, fifteen weeks are spent on block teaching practice in local schools. The first, or diagnostic practice, comprises of four weeks in the first term while the second, long, practice is split over the Easter vacation. For students to spend such long spells in schools obviously puts a heavy emphasis on the contribution made by the participating schools. The schools are referred to as partnership schools to highlight their important role within the PGCE course and receive a payment from this University's Department of Education to help offset any costs they may incur during their contribution to the training of their students. The realisation of the significance of their contribution set me thin kine as to why schools agree to take students at all. After all, schools are under considerable externally-enforced pressure without taking on additional roles and responsibilities.

I was initially tempted to investigate the reasons as to why schools agreed to take students in a formal manner with questionnaires and structured interviews when it occurred to me that, in considering their response, the accepting school may conclude that perhaps it was not beneficial to them to take students after all! Not wanting to lose any of our placements I decided that a more prudent approach would be to broach the subject in an informal conversational mode. The methodology was therefore similar to that used in a survey of why primary schools take student-teachers (Haylock, 1994b), although some of the findings are very different.

The University currently has students in 50 different schools across seven Education Authorities. The mathematics department is using 22 of these schools this academic year. They represent a wide range of institutions. The age range taken by the schools includes 11-18, 11-16, 13-18 and 8-14 years. The geographical location and social catchment is similarly wide with schools located in rural areas, in the inner city, by the coast and in prosperous suburbs.

Reasons Given

So why do so many schools agree to take students? Haylock (1994b) gives the reasons for primary schools taking students in four categories: benefits to the pupils, benefits to the staff, benefits to the school and commitment to the profession. In my survey of secondary schools I found the main reasons given to be: financial benefits, traditional involvement in initial training, an altruistic interest in helping newcomers to the profession, and the benefits for the department of being involved with the University.

Money was nearly always the first reason mentioned, often rather apologetically and with a wry smile. It is interesting to note that the majority of these schools took students under the 'old scheme'. Prior to the changes brought about by circular 9/92 (Department for Education, 1992: a Government initiative to

change the emphasis of initial teacher training) there was no financial incentive, as the placements were shorter and the role of the university was much greater in the monitoring of students during their teaching practice. It should also be noted that, during the first year or two of the implementation of the Circular, 58% of the respondents in the 1994 survey of secondary schools (Haylock, 1994a) stated that the reason for *not* taking a student was insufficient payment. However, as budget cuts start to be felt more and more in schools, the opportunity for a mathematics department to increase its capitation by up to 50% by taking a student is an understandable temptation.

The second most common response was that the department had 'always had a student', The rationale behind the decision had never been reviewed it was simply something that had always been done and had become an established part of the departmental year. Some institutions had got as far as discussing whether they could successfully accommodate two students but not whether to abandon their involvement.

Other responses included an element of altruism, wanting to help in the training of new staff, and a genuine concern and interest in the development of newcomers to the profession.

Departments also felt that by taking students they were maintaining important links with the University and would receive via their students access to new ideas and teaching material. I trust that they have not been too disappointed in that respect! The importance of having 'young blood' in a mathematics department was also mentioned (admittedly by a chronologically-challenged department!).

Cynics (or realists?) may also suggest that contact with the University may allow schools to 'get in first' if they have a vacancy that they wish to be filled with a newly qualified teacher, although this was not stated as positively as it was in Haylock's survey of primary schools taking students.

Few secondary school mathematics departments mentioned the reason which I thought may follow hard on the heels of the financial benefits and that is one of time. In my experience, it used to be the case that having a student to take one of your classes was something of a perk. It allowed the normal class teacher to benefit from some free time, for example, to complete tasks normally done out of school hours or to spend time developing new resource material. However this does not now seem to be the case. The increased length of the teaching practice means that teachers often feel there is the danger of losing control over their classes (both academically and behaviourally) and so need to monitor the student more closely and regularly than in the past.

Difficulties in Placements

During discussions on this topic however one concern was frequently mentioned that did not figure in the 1994 survey of secondary schools (Haylock, 1994a). This was the difficulty schools were having in finding sufficient suitable classes for students to teach. Some departments were concerned about the End-of-Key-Stage National Curriculum Tests (still referred to as SATs), the results of which may well form the basis of "league tables" of schools. This meant that they did not want to allocate Year 9 classes to students. There were similar concerns voiced about losing exam classes in Years 11 and 13.

Often there was the additional problem of modular A-level courses which mean that Year 12 pupils were also preparing for external examinations. This problem is particularly acute in this region where there is a large number of high schools (13-18 year olds).

Obviously there are strategies for overcoming these problems such as team-teaching or splitting the class into two distinct groups especially useful in a class where only some of the pupils are able to cope with level 7 work. We need to ensure that schools are aware of these alternative strategies and that they consider them, both when deciding whether to take a student and when allocating teaching groups.

Conclusion

While no quantitative data has been produced in this discussion it seems that schools are, on the whole, happy to take students on a regular basis without any great tangible benefit for themselves. It is an attitude for which we must be actively grateful.

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A Gifted Mathematician, Age Eleven

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This is an informal report of the author's encounters with an 11-year old, mathematically gifted pupil. Case studies of this kind contribute to our awareness of the special needs of such pupils and raise some challenging questions for a mathematics educator.

The Subject

For the purposes of this article I will call him Daniel. For the four years in which he has been a Key Stage Two pupil I have followed Daniel's mathematical career with interest. Although I have had the privilege of working with him on relatively few occasions, perhaps once or twice each year, I have frequently talked to each of his teachers about his progress and suggested suitable tasks to set him. Daniel is undoubtedly an able mathematician: all four of his junior school teachers willingly admit to him being considerably better at the subject than they are. Not surprisingly, he achieved Level Six in the Key Stage Two National Curriculum Tests; indeed had he been taught the appropriate material, he could probably have reached a higher level.

A Conversation About Mathematics

On the penultimate day of his primary school career, I asked Daniel if he would be prepared to stop watching the video in which he was engrossed to talk to me about the maths he had done at school. He happily did so. The discussion was spontaneous and therefore unstructured. Had I been intending to use the results as part of my research, I would undoubtedly have planned the questions more carefully. However, the conversation might then have been less interesting! The following is an account of our discussion.

I began by explaining to him that I was interested in primary school children who were good at maths and wanted to find out how teachers could help them to develop their mathematical ability. Initially Daniel simply responded to the questions I asked but eventually he chattered on uninterrupted.

"Which topics in maths do you enjoy most?"

"Geometry and anything I can do on my own. I like being independent"

"Do you ever get bored in maths lessons?"

"No. I always concentrate on what I am doing and think about it."

"Do you prefer the lessons in groups (ability sets) or class maths lessons (in which they sit in ability groups but remain in their usual classes and are taught by their usual teacher)?"

"Classroom lessons. We all do; and Miss X finds interesting things for me to do" (See later notes on discussion with Miss X)

"Why do you all prefer class maths?"

"Because we work with the people we know and we do more interesting things."

"Have you always been good at maths?" "Yes."

"Have you ever had a maths problem set to you at school which you have not been able to do?"

(After some thought). "There was this question I had about two years ago and it took me nearly half an hour. I remember what the question was about. I had to find the shortest time for a train journey between two places and we were given various bits of information about speeds, distances and times. Also the driver needed a tea break every two hours and this had to be taken at a station rather than between two stations. The trouble was there were lots of different ways of doing the journey and I

wasn't sure that I'd found all of them in order to work out the shortest time. In the end I was sure."

"Is that the only time you have really had to think about a maths problem?"

"Yes."

'~re you looking forward to maths lessons at secondary school?"

"Yes . but I'm not looking forward to meeting people or to playtimes. It's quite a nice thing being good at maths but people sometimes make fun of you - I don't find the social side very easy".

"You talk comfortably to me. "

"That's different. You're an adult. It's easy to talk to adults. "

To sum Daniel up as a typical gifted child would seem to devalue his uniqueness, but, nevertheless, from what I gathered during this discussion and from what I already knew about him, Daniel seems to exhibit a remarkable number of the indicators listed by Davie!. George (1992) and others to help teachers to identify able children. In particular Daniel prefers working alone to working in a group; he is articulate and revealed his mathematical giftedness at a very young age. He works hard, gains intrinsic pleasure from mathematics and shows persistence. He has supportive and encouraging parents and, in addition to his exceptional mathematical ability, he is also good at all academic subjects studied at Key Stage 2.

The Teacher's Perspective

I know his teacher well. At the time Miss X was a newly qualified teacher who I had taught when she did her PGCE. the previous year. She had found Daniel an interesting and rewarding pupil and had gone to great lengths to find appropriate tasks for him. His interest in Geometry had been recently stimulated by working with some Islamic patterns which Miss X had found in a publication from the Victoria and Albert Museum (Sharman, 1993). Miss X was aware of Daniel's fear of going to secondary school and was also worried that he might be bullied. She had spoken about him to the head of first year at his new secondary school but still foresaw that he would have difficulties. He is not a popular child and she also commented that he does not socialise easily.

Some Questions

Daniel's case has set me wondering ...

- Is Daniel well equipped to continue to set himself high mathematical standards during puberty or will he be content with being "above average"? How will he cope with not being able to do a problem? He has not yet experienced real mathematical failure or even much mathematical challenge despite the efforts of his excellent teachers. Has he developed the necessary persistence?
- Will he be adequately stretched at secondary school? Although he is more likely to meet a mathematics "specialist" he may easily be placed with a maths teacher who is not able to match Daniel's own mathematical ability.
- Will peer pressure to conform, combined with Daniel's identified difficulties in making relationships with other children, discourage him from demonstrating his mathematical ability to his teachers?
- Should I have played a more active part in Daniel's mathematics? I have more mathematical knowledge than any of the teachers in his school; I visit the school regularly and have an excellent relationship with all the teachers there who would have been happy to let me teach Daniel whenever I wanted to. I wish I had made more time to talk to him during his time at primary school. I guess we would both have benefited.
- More generally, how can we, as mathematics educators, give effective and active help to non-specialist mathematics teachers in primary schools in their efforts to satisfy the needs of their able mathematical pupils?

I hope to be able to talk to Daniel again when he is at secondary school. Since the reported meeting I have read " The Psychology of Mathematical Abilities in Schoolchildren" (Krutetskii, 1976). It is intriguing to note how Daniel demonstrates so many of the characteristics of mathematically capable pupils which Krutetskii identified: for example, his ability to recall the mathematical structure of a problem, and his striving for elegance and flexibility in solving a problem. Krutetskii provides a set of mathematical problems categorised according to the kind of strategies needed for their solution; I

should like to see how Daniel responds to these different types though I might not get through all 26 on Krutetskii's list! I think Daniel would enjoy trying out a variety of problems and would be interested in the categorisation.

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Empowering Teachers to Develop the Quality of their Teaching

Heather Scott (Northamptonshire Inspection and Advisory Service) and

Michelle Selinger

(School of Education, The Open University)

This article provides an account of the experiences of the two leaders of The Number Project, in which they worked with a group of teachers in exploring issues of classroom practice in the teaching of mathematics. The aims of the project are outlined and the extent to which these were achieved are considered in a dialogue between the two project leaders. The extracts below from correspondence between the two leaders establish a flavour of the development of the project.

Dear Heather,

So much for promises to write! Every evening I wanted to write to you about the last number project meeting and yet other more urgent things have prevented me. My thoughts are very distilled now, so the letter will be short but the significant things that stand out for me about the Number Project are the way we work with the teachers which results in the progress being made:

- we are all asking challenging questions of each other;
- there is a growing awareness about the different problems that exist for primary and secondary teachers;
- we all now believe that a mathematics investigation need not be seen as a one-off and that can lead to other activities;
- we saw the challenges that arise from motivating and sustaining activity;
- we realised that low attainers can also take ownership of the problems posed;
- we came to a realisation of the potential of this way of

working.

I was impressed too by how little we had to do as leaders, the ownership was theirs as we had fought for. Our decision to hold back as leaders seems to have been the right one. They seemed unthreatened by the questions that arose and tried to answer them honestly. They also looked to each other for support and ideas, wonder if the teachers have kept their diaries/notebooks going?

Perhaps we can develop this idea of letter writing where they pair with one other teacher and write to each other with a guarantee of less than a week before writing to each other say?

Best wishes, Ntichelle

25 February 1993

Dear Michelle,

Thanks for your letter. I agree with all the points you raised. I enjoyed the meeting tonight much better than the last time, because it was much smaller. One of the things that I am learning from a project of this nature is that the group must be small enough for sufficient reflection and in-depth conversation to take place without people feeling left out. I liked the way that people were able to bring back their work to the group. I noticed the discussion focused on descriptions of what the teacher and child did in the classroom. I was personally excited by the range of responses from the children. I was, at the same time, disappointed in the difficulty that the group had in talking about learning, on the part of the children, or on the part of ourselves as teachers. What I would really hope for is that one of the outcomes of the project is that people want to take responsibility for enabling 'learning' in the classroom and then to go on from that and really work to improve the quality of learning.

Any courses can only be a door opening to what may be possible ... maybe it can also give a little confidence to those people who want to step through. I know that people can only learn by doing it themselves. At the same time I know that learning doesn't take place in a vacuum. We can learn from

other people's experience. Similarly it may be that the more productive 'learning fields' are beyond our own vision, so we may need to be shown these previously 'hidden places'. The nature and the timing of our interventions as 'group leaders' are important, but they are also difficult given the limited contact we have with people on the course. I think the letters have really helped to keep people in touch in a more coherent way and one that also records our own learning process. I would really love to know how the teachers are feeling about the project, maybe we could ask them, "What have you learnt?" and, "What has helped you learn?". I know that we have all learnt that 'learning' takes a great deal of time. In our role, even in the sort of supportive group that we have there are still many constraints and interruptions. Therefore, we have to learn how to manage these in the most productive way possible. See you at the next meeting where we shall need to sustain our momentum.

Heather

2 March 1993

Aims of the Project

The group met at approximately three-week intervals over two terms. It involved eight teachers from Northamptonshire, two from a first school, three from a middle school, two from secondary schools and an advisory teacher. There was also an observer/evaluator, the deputy head from the middle school, who expressed an interest in observing the project as a way of developing school-based inset.

The number project was set up in January 1993 to explore some issues with which we had both expressed concern. We felt uncomfortable with the way in which classroom research projects had been set up in the past, which ostensibly claimed to be examining ways of developing teaching and learning in classroom, but which in reality used teacher participants in the project as guinea pigs with seemingly little concern for their professional development (see Fisher and Selinger, 1992). We were also concerned that teachers should look critically at their practice and seek, through their own impetus and classroom research, to develop and improve their teaching and thus improving children's learning. Our thinking in this area had been much influenced by Hans Freudenthal:

The future teacher should learn to observe and analyse learning processes, not only those of his pupils, but also his own, those of his fellows and his trainers. For the trainer this means that he leads and guides his students to the places where the learning processes take place, that he opens their eyes and minds to observation and analysis.

It is not so new but still rarely fulfilled requirement that mathematics is taught not as a created subject but as a subject to be created. For the same reason, armchair pedagogy in a standard package should yield to those pedagogues which are created by pupil, student and trainer in a common experience.

(Freudenthal 1980 p.72)

Opportunities for professional development have become much harder to access over recent years while at the same time there appears to be a growing demand. How could we support some of those teachers in a more effective way? We also believed that effective professional development came through sustained contact with other professionals interested in similar issues and we wanted to explore our beliefs and try to find ways to make this work in practice. Our aims for the project then were threefold:

- To explore ways of working with teachers which will empower them to develop the quality of their teaching.
- To investigate the nature of the teacher researcher partnership.
- To improve the quality of the teaching of number in school.

The project started by asking the teachers to explore through the use of metaphors, ways of describing their own learning of mathematics. From then on we allowed the teachers to develop the agenda and we offered support to the group in the form of asking questions and discussing research findings that would help them move forwards in their planning and teaching.

In the rest of this paper we shall explore the extent to which we believe we fulfilled our aims and the lessons we learnt for the way we might set up future projects. We have chosen to present our findings through a dialogue between ourselves.

AIM 1: To explore ways of working with teachers which will empower them to develop the quality of their teaching

M: From the outset of the project I wanted there to be at least two teachers from each school. As someone who had worked on changing my own practice in school several years ago, I recognised that support and interest from my colleagues, people to share ideas with, was for me the most important sustaining factor in my own professional development. The evidence from the project bore out my beliefs. The teachers in the first and middle schools did appear to develop and sustain their practice, but the two secondary school teachers, who were not partnered in the project by other colleagues found it much harder to change their practice and also to maintain their commitment to the number project. One dropped out due to a number of factors, another continued with the project but was unable to effect any lasting change. She became antagonistic too towards the development of the project and felt isolated as the only secondary teacher there. Perhaps she might have been less critical if she had a colleague who was also involved in the project?

H: I know when people learn they provide their own motivation, curiosity and purpose. Most importantly they pose their own questions which they are particularly keen to find answers to. When I'm involved in the in-service education and training of teachers (INSET) my role is to create supportive environments and opportunities which enable teachers to learn about teaching. Our philosophy and approach was to give the teachers responsibility for their own learning. Our main methodology was to bring a group of teachers together to discuss what they were learning about the teaching of number and improvements in the quality of teaching and learning in the classroom. I particularly learnt a lot about ways of working with teachers by trying to take a back seat by not taking a proactive lead. I knew I had my own beliefs about the teaching of number, but I also knew my role was to help the teachers be clearer about their beliefs and to test these against the real evidence that they observed in their classrooms.

M: By selecting the activities to work on and bringing their own ideas, the teachers were more involved in the project. The first session in which we asked them to decide how we should proceed was awkward and there were many silences. I felt they were waiting for us to tell them what to do and even though we had explained that we considered this to be their project too, I feel they still expected us to come up with suggestions. It was difficult not to come forward with ideas, especially when we had discussed what we thought they might do in such depth before we met any of the teachers for the first time. Eventually they came up with the notion of sharing teaching ideas to work on with children in the classroom, and after that there was little problem. The next session they were all eager to share their experiences and from then on the project seemed to generate its own momentum. The number spiders activity really showed them how much children had to offer and that differentiation by outcome was really possible. They were amazed at children's ability to spot patterns. Perhaps we should have been more rigorous at this point and challenged them more to look at how they would develop the next tasks from those they were describing. Many of the tasks were 'oneoffs' and we did not really encourage them to think seriously enough about continuity and progression. Although they

did question what they might do to build on these experiences, very few actually developed that question into action. Perhaps we should have been more challenging?

H: I found that discussing the 'learning' of pupils in the class, the teachers on the course and of ourselves was difficult. The meetings were very short. We spent a long time discussing the events that we had experienced in the time between the meetings. Teachers asked each other about the nature of what had happened in the classroom. I was looking for a type of evidence which didn't exist. On reflection, learning is a private process which evolves over a period of time. We collect information from a variety of sources and then we adjust our own views and actions according to the new framework that we understand. The meetings were being used as only one opportunity where teachers gathered information. In addition to the meetings they were also using their own classrooms as a source. My expectation that teachers would use the meeting as a vehicle for examining their learning processes was proved not to be right. In this respect we were successful in enabling teachers to choose their own methodology, they set the parameters for the meeting in order to meet their own needs.

Evidence that learning was taking place came from other sources.

For some teachers there were changes in classroom practice which I was able to observe by visiting the school in which they taught. The suggestion of writing letters and reflections was a key aspect in accessing the learning taking place. In addition the project evaluator also played a significant part in providing evidence of the progress which was made. This observed and written evidence was particularly valuable for me. It allowed me to make a connection between the different relationships we were building, the types of interventions that we were making, and the learning outcomes of the different teachers. I could clearly see that where relationships were built this contributed positively to the quality of learning taking place. Similarly, where interventions had been encouraging and supportive this added to the confidence and security which teachers felt with their own learning situation. It was also extremely important to be 'honest' within the group and to value all of the experiences of each person in the meeting.

AIM 2: To investigate the nature of the teacher/researcher partnership

M: I was pleased with the way we developed our roles. You were in a more difficult position than I was, as they saw you as an authority figure within the county. I am not sure how they perceived my role. I hoped I blew away the researcher/academic notion quickly. I think it helped there being two of us because they related to us differently and were able to focus their attention on one or the other when talking about their work. I think our philosophies about mathematics education were similar enough not to cause conflict or tension. How much were they trying to second guess what we wanted from the project and respond accordingly, the way children do with a teacher in the classroom? Did that matter? Our philosophy is based on many years of teaching experience and exposure to research evidence and to a firm belief that mathematics education can be improved. We have both been action-researchers in our own classrooms and have insights into and are aware of both the problems and rewards of changing and developing our practice, perhaps we did have some rights to influence the direction of their professional development. As co-ordinators I felt we achieved the aim of encouraging the teachers to contribute fully to the direction of the project and to feel they had a measure of control in the direction it took. It was shown by their commitment to the project in terms of their action in their classrooms, their enthusiasm and their attendance at meetings.

H: During the project I continued with personal research on improving the quality of my own teaching in the classroom. One of the most important aspects of the project was that we were all learners together. There was a wide variety of experience in the group, however, the view that each person brought to the group could only contribute positively to the learning of the group as a whole. My role as a County Inspector did inhibit the level of commitment which I could make to the meetings. I felt that this did detract from the project as a whole, not so much that I was not at the meetings to make contributions, but that I was not able to learn as much as I could from the progress which the project was making. The way in which the whole group cooperated and communicated with each other was extremely helpful. I genuinely felt accepted as a group member.

M: Encouraging them to write about their involvement in the project was a success in many ways

although at the time I felt as though I was asking them to cut off their right hands! I also feel it was important to take them away from school and give them time to write together. For some this was the only writing they did although some subsequently refined their writing and I have sent it to various sources for possible publication. They did tell me afterwards that they were pleased they had been coerced into writing as it had made them realise how they had moved on in their thinking about teaching number. Taking along journals to show them examples of other teachers writings also encouraged them and provided them with a source of ideas which many wanted to follow up. It gave them an added interest in reading about mathematics teaching.

H: On reflection there were two teachers with whom I would have liked to have spent more time with in their schools. With one, I felt I had little time to develop a worthwhile relationship and at the same time had made inappropriate interventions which detracted from the quality of her learning opportunity in the group. With the other teacher I felt that some very interesting and productive initial work had taken place but that there was little opportunity to observe the final outcomes as she was unable to continue with the meetings.

AIM 3 To improve the quality of the teaching of number M: It is difficult to judge whether this aim has been achieved. Changing your practice takes time and measuring success can be very subjective. Children rarely stay with the same teacher for more than two years so improvements in the quality of teaching by one teacher can if not completely undone by another the following year, certainly held back. The teachers I have visited since the project ended however appeared to continue to develop their idea about teaching number and they are all keen to listen to the children and to offer them tasks that demonstrate their understanding. One particular teacher has also developed her ways of teaching number into other areas of mathematics and also in other areas of the curriculum. She allows the children to explore mathematics for themselves, ask challenging questions and encourages them to develop their own lines of enquiry while requiring them to be precise about what they are doing.

H: Yes, we did make progress in this area. The work with children opened all our eyes and there were surprises around every corner in this regard. Most importantly children surprised us with what they were capable of. I think that the letter from Pat, one of the secondary project members, summed this up for me with the paragraph about Sam. He was thought to be only capable of answering questions that used only one-digit and two-digit numbers and did not appear to have a good understanding of place value:

Sam is one of those towards the lower end of the ability range (perhaps!). He has been materials to develop his ability to use the four operations with hundreds, tens and units. His junior school record indicated that he was competent with using the four operations with 2-digit numbers and he had been introduced to hundreds. His work showed no evidence of 'carrying figures' or 'borrowing', but he was getting most of the answers right. (Had he done his working out on scrap paper? Had he used a calculator?) I asked him to show me how he had done this sum:

$$\begin{array}{r} 400 \\ -136 \\ \hline 264 \end{array}$$

His explanation went something along these lines: "400 take 100 is 300 but I need to take 36 so its 200." (He writes down 2 in the hundred column.) "100 take 30 is 70 but I need to take 6 so its 60." (He writes down 6 in the tens column. He then writes down 4 in the units column.) He could also do sums like 438 - 175 in the same way using the correct place value names for the numbers. I was quite excited by the fact that Sam was using place value to explain his process. I also felt that I could actually say that Sam understood the process of subtraction.

M: Improving the quality of teaching number requires more than 'trying out' new ideas with pupils. It means finding out what children already know and can do and building on that. I think. this process was started with number spiders, but as I said earlier, this was not developed. Perhaps we ought to have spent more time thinking about diagnostic assessment before exploring ways of developing children's learning. It was difficult to do this without directing the development of the project but I think with more careful questioning and challenging of the teachers statements we could have started to move in this direction. It also begs the question about whether we ought to have allowed the teachers to dictate what we did next? Perhaps we were too laissez faire?

Time is an important factor. I think, changes take place slowly. Some time after the project finished I visited Dawn in her classroom. The class were working on an algebra task that involved constructing overlapping triangles using matches. Many children had misinterpreted Dawn's instructions but she did not redirect them. The ensuing discussion was richer and more rewarding. She told me afterwards that it was her involvement with the Number Project that had made her rethink the tasks she offered and the way they were interpreted gave ownership to the children and encouraged motivation and subsequent learning.

Conclusions

Our reflections on the project highlight many areas which could provide a fruitful basis for further work and research. Everyone brings their own assumptions and expectations about how courses will proceed and which types of methodology will be valuable. Learning will take place in an environment where the agendas are clear and the methodologies have been discussed and agreed. Situations need to be flexible to cater for a variety of needs. One of the key roles of a group leader is to enable the group to work open and honestly together towards ensuring that all needs are met without compromising the needs of any individual.

Becoming a reflective practitioner takes time, and sometimes it is uncomfortable too. Realising that you could do things another way and potentially better can be very threatening. It takes time in your own mind to resolve the fact that you are growing as a teacher rather than having necessarily doing things badly before hand.

Evidence of the learning which happens can be found in a variety of places. This evidence is not always obvious, and is sometimes surprising. It is important to expect progress, it is also important to expect that progress will take time. Researchers need to be flexible in picking up evidence from a variety of sources. In this respect the partnership between us was extremely useful. In addition, the information which the objective evaluator contributed added a great deal to the learning process. People who find themselves in the role of a teacher need sound evidence in order to make appropriate interventions in the learning process. When interventions can only happen in a very small time slot over a period of weeks this evidence is even more crucial in enabling successful learning.

This project has highlighted the importance of varying the nature of INSET. Teachers might:

- come away from their classrooms to discuss their teaching, share ideas and write about their experiences;
- have the opportunity, within their classrooms, to work with other colleagues either from within their school or from other schools within the INSET group;
- be supported by advisory teachers;
- visit the classrooms of their INSET colleagues.

It is important to recognise that we are all 'learners' together. When this atmosphere and ethos prevails it contributes positively to the learning process. Therefore this confirmed my initial view that where a 'leader' gives responsibility for the learning to the group this is a positive feature which does enable learning to happen. At the same time it was important for the 'leaders' to be involved in the same research that the teachers were undertaking, since only then can genuine dialogue about all experiences take place on an equal level.

The leader's role of 'making appropriate interventions' is also crucial in enabling the learning process. At times the group did feel that it lacked direction. On future courses I would concentrate on reflecting upon the way in which we focused the group on it's own learning process. Maybe at times we were not able to enable the individuals in the group to be fully aware of what was going on. In the future I would remind us that part of the responsibility which has been given over, is a responsibility to say 'help' when it is needed.

The relationship between teacher, researcher and inspector was problematic to begin with. Heather had to work hard at reassuring through her actions that she was not 'wearing her inspector's hat'.

Michelle was more readily accepted although the teachers seemed unsure about what her 'hidden agenda' might be. We both felt uncomfortable about being regarded as 'experts' or about making the teachers feel as though they were guinea pigs. It was only by staying silent and holding back that the teachers were able to move into the centre stage. This was only then could we support and encourage them and ideas were accepted and shared in the tenor in which the whole project was intended to be set.

We should never underestimate what children are capable of and able to do in mathematics. As teachers we need to learn this capability and ability more effectively in the classroom. We need to build upon pupils achievements to help them make appropriate progress.

A special thank you goes to all those teachers involved in the project. This was a special event for us and it would not have been possible without the commitment to learning about teaching by all of us who met, and all of the teachers who continue to work towards improving standards in their schools.

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Teachers' Notions of Mathematical Ability in their Pupils

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This paper is an analysis of the comments about mathematical ability made by a sample of 25 teachers when interviewed about factors which enable children to achieve well in mathematics. The teachers interviewed use a wide range of working notions, both behavioural and cognitive. A few notions are specific to mathematics, but these are infrequent in teachers' responses. A comparison is made between teachers' utterances about achievement and ability in mathematics and Krutetskii's definitions of mathematical ability: some overlap is identified, but this is limited.

Introduction

Assumptions about the existence of such a thing as mathematical ability run through much that is said or written about the teaching, learning and assessment of mathematics. Teachers are frequently heard referring to pupils as low, middle or high ability, less able or more able. I have transcripts of twenty-five interviews I have had with primary, middle and secondary school teachers about their assessment practices. Many of the transcripts contain the word *ability* in some form or other. I have analysed these references, both direct and oblique, in order to obtain a broader picture of what these teachers regard as factors contributing towards a child's success, or otherwise, in mathematics. I picked out any mention of ability, particularly if the concept had been elaborated, any general comments about the attributes, skills, attitudes and characteristics of pupils who were particularly good at mathematics, and also any such characteristics which were listed by teachers as being desirable or essential to the successful learning of mathematics. I was concerned to find out if the teachers had a deeper sense of ability in mathematics than that described by Askew and others in their evaluation of the first years of the National Curriculum.

Across all three key stages differences in pupils' abilities in Mal were more often described in terms of personal qualities particularly confidence, than in terms of mathematical abilities ... (Askew, 1993)

This statement somewhat begs the question of what mathematical ability is. My personal view is that *Using and Applying Mathematics* in the National Curriculum is a good place to start, but that we need to look elsewhere for a fuller definition. The work of Krutetskii (1976) provides a thorough and highly-regarded analysis of mathematical abilities in schoolchildren (Bishop, 1976). This was developed by observing the problem-solving approaches adopted by mathematically-capable pupils and comparing these with those of average and mathematically-incapable students. Krutetskii's analysis is based on the assumption that ability is independent of genetic or class considerations. The components of mathematical ability which he identified in schoolchildren were the abilities to:

- grasp formal structure;
- think logically in spatial, numerical and symbolic relationships;
- generalise rapidly and broadly;
- abbreviate and curtail mental processes;
- be flexible with mental processes;
- appreciate clarity, simplicity and rationality;
- switch from direct to reverse trains of thought;
- memorise mathematical objects, schemes, principles and relationships

In addition there is a general synthetic component called a mathematical cast of mind. Krutetskii found that mathematical ability can involve speed of thinking, computational ability, memory for facts and formulae, visualisation and spatial ability but these are not obligatory. It was possible to be a good mathematician but weak at computation, for instance.

I shall use *mathematical ability* to refer to an amalgam of factors which the teachers in my sample associated with a pupil's past and future achievements in mathematics. I identified several categories of responses. Some were references to factors which were present at the start of the teacher's contact with the pupil. Others were comments about behavioural, emotional and intellectual traits. These are described below. Passages in italics are quotations from the transcripts chosen to exemplify a category.

Starting State

In discussing pupils' achievements in mathematics, the teachers in my sample recognised the significance of what a pupil brings with them at the start of their teacher-student relationship:

- prior knowledge and experience
- retained knowledge
- mental images
- an attitude to the subject
- preferred ways of working
- awareness of personal achievement or failure to achieve

However teachers recognised that their initial judgements of a pupil are likely to be affected by:

- the view of the previous teacher as expressed in reports or through teacher assessment
- the view that the new teacher has of the previous teacher's work or style

Behaviour While Learning

From this starting state the first things the pupil's new teacher is likely to notice and comment on are aspects of pupil behaviour. Some of these are easy to observe and seductively easy to interpret superficially. Teachers here are in the familiar territory of interpreting non-verbal signals between human beings. Teachers notice which pupils pay attention and which, apparently, do not; those who appear keen, those who appear bored; those who talk a lot, those who stay silent. These observations are readily interpreted as indications of potential for achievement in mathematics. One teacher thought an indicator of ability in mathematics was that pupils should *desire to know*, should be *curious* and *want to understand*. Another felt that the will to push forward, to ask for help or ideas in order to progress, not just when stuck, was significant.

She spends too much time on the minutiae rather than pushing herself on ... it is important to have some idea of how to get themselves from one process to the next without just thinking it happens automatically.

There seems to be a progression in this category through: (a) willingness to provide the external appearance of expected learning behaviour; (b) willingness to do the work offered; (c) wanting to finish it; (d) having some internal goals and drive so that the teacher is not the only propellant. All of these are learned behaviours in the sense that they depend on the perceived effects of previous behaviour. They are therefore strongly related to the starting state of the pupil-teacher relationship, and thus to the past. If the teacher wishes the behaviour to change it will therefore not necessarily be easy.

Emotional Factors

The next set of factors referred to by teachers relate to the emotional state of the **pupil** in various mathematical learning situations.

Reactions to being right or wrong

"Her character is very precise, she gets upset if she gets things wrong. "

"He has pride in being right. "

Reactions to being stuck

"She'll ask very quietly and politely, how do you think I could do it?" "N is very bright. If there is something he did not recall he would take the trouble to find it out if he needed it for a particular problem."

Reactions to different teachers' styles

"There are still some who will just go for answer and sit there waiting. I try to encourage them to talk and to push it further. "

"One style might suit one pupil but another may suit another. I think it is very important that they have a change of teacher. "

Reactions to different types of activity, especially open versus closed activities

"She is not interested in puzzles, she is more interested in saying, I can do that, or, I did that before you did. Mastery is her motivation, not curiosity. "

"If they have to do some work where they have to work things out themselves they own it more and they are trying to create something instead of doing something that already exists. I think there is a hurdle that is removed."

Sense of self-worth

"One little boy wrote, I am a bit silly and a bit slow. Well that would need tackling through his work and not through the label he had been given. To take that away would be a starting point for his working."

Confidence

"I think mathematics is a bit of a confidence trick. I believe you can do almost anything you can believe you can do, most children can too. " "He's lacking in confidence about everything because there are some mathematical things that are missing."

Liking of challenge

"When I set work for the brighter pupils they generally get on and finish it. They like more of a challenge"

All these factors are heavily influenced by what has gone before, and the current teacher may never know why the emotions are what they are. Nevertheless teachers seem to be convinced that they have an effect on mathematical learning, This was particularly true of *confidence*, which was mentioned by nearly all interviewees from primary and middle schools in the sample. Interestingly, compared to the number of times that confidence was mentioned there were very few occasions on which teachers talked directly of being able to alter or affect confidence, as in this example:

She is under-confident. She is in that group to give her encouragement.

There were even occasions when the word *confidence* was used in a way that almost suggested it as a definition of ability.

The lower ability pupils do not have the concepts or confidence (to work investigatively).

Intellectual Factors

Few of the aspects I have mentioned so far were used explicitly to describe differences in ability, but all were given as factors which influence progress and learning. Those which were related most closely to learning were: reactions to being stuck, curiosity and desire and reaction to challenge. I was encouraged to find that, on analysis, there were several utterances about pupils' ability in mathematics which were based on observations and interpretations of their cognitive or intellectual activities. I have categorised

these below and, within each category, isolated some factors which are specific to mathematics.

Communication

Reading ability is linked to mathematical achievement where teaching is usually through written material. Some teachers linked the two with no explanation, but others did talk about the link, one teacher, for example, referring to ... *the ability to overcome hurdles of reading textbooks*. Children's ability to communicate their own ideas verbally was considered very important, because otherwise the teacher has no window into their minds. One teacher said that... *linguistic ability is not far from mathematical ability*, and others talked of *encouraging discussion*. One teacher recognised that some pupils who are very good at mathematics may produce high-level but very terse arguments which need careful interpretation by the teacher. He felt a teacher might have difficulty in grasping the sense of what was going on. This suggests that teachers may recognise mathematical ability by how well they themselves can follow a pupil's arguments, or how far the pupil's articulated understanding coincides with theirs. No teacher talked of the standard of written work as an indicator of ability, although one teacher thought that the ability to write creatively and the ability to spell and punctuate perfectly were analogous to creativity and precision in mathematics. From these aspects of communication only the possibility of using terse or shorthand arguments that was identified by some teachers seems specifically mathematical.

Attitude to mathematics

Some teachers talked of pupils who were interested in mathematics, who responded broadly and deeply to mathematical stimuli, who responded positively to teacher attempts to extend their thinking, who could think on their own, who looked for patterns and other recognisable structures and who understood the concept of *trying*. It was noted by one teacher that his low-ability pupils had responded much better when offered a mathematical situation to explore than when offered someone else's thinking to follow in a book. These suggestions were given in the context of mathematics, but most could be applied to any subject. Interest in pattern and structure, however, are specific to mathematics.

Planning and Approach

Organisation of work was often the first thing mentioned about individual pupils, along with being methodical. The ability to collate relevant information and decide what mathematics to apply in open situations was mentioned. If the latter point means expressing a situation in mathematical terms so that mathematics can be applied to it, then this could justifiably be recognised as something specific to mathematics, But I suspect that in a classroom situation pupils usually know that it is mathematics they are supposed to be doing, and very often are given clear indications as to what mathematics they are supposed to be applying. Some teachers say that being methodical is a sign of mathematical ability, but there would be a case for regarding this as an aspect of a more general trait which is relevant to all subjects.

Implementation

Once a piece of work has been started, teachers consider that mathematical ability is shown by the pupil being able to move from one process to the next themselves, to apply mathematics correctly, to get right answers, to approximate appropriately, to develop a range of strategies and processes, to work creatively, to think and to take jumps which are longer than step-by-step approaches. Most of these aspects of doing mathematics are specific to mathematics, but they provide a rather superficial view of the subject. Of particular interest is the requirement that pupils should be able and willing to "think". This implies that there are times when they do not think, and that a general instruction to think will make a difference to achievement!

Understanding

An able mathematician might be someone who is interested in finding out why things are the way they are, but skilled performance of skills and techniques can also be described as mathematical ability. Several teachers drew attention to this difference. One Year 6 teacher suggested that GCSE may be a test

of the latter rather than the former. This possibility that a pupil might succeed through successful performance of routines rather than through genuine understanding of principles may be a feature particularly associated with mathematics as a school subject. An aspect of understanding mentioned by teachers was that pupils should make links with other knowledge and concepts and should be able to make both specific and general statements about the situations with which they were working. This is, of course, common to understanding in all subjects, although there are some ways of formulating generalisations which would appear to be specific to mathematics.

Other aspects

Memory for mathematical facts and formulas was considered to be a useful contribution to mathematical achievement. Teachers also mentioned quantity of work or pace of work as a good indicator of ability. Some said that creating the pupil's own mathematical ideas is important. Other teacher utterances about achievement were about matching teacher intentions to pupils' actions. Comments were made about whether the pupil could work with what was provided and the importance of ensuring that the level of challenge is right.

Descriptions of Specifically-Mathematical Ability

Cognitive activities mentioned by teachers which were specifically mathematical were:

- using terse or shorthand arguments
- showing interest in pattern and structure
- seeing and expressing situations mathematically
- being able to string processes together
- applying mathematics correctly and appropriately
- developing mathematical strategies
- making multi-stage jumps
- having useful images of abstract concepts, eg number
- memorising mathematical facts and formulas
- having mathematical ideas
- generalising mathematically

Some of these activities reflect components in Krutetskii's framework. They span his descriptions of giftedness and his non-obligatory aspects of mathematical ability. I was encouraged that teachers were able to say more about ability than had emerged elsewhere (Askew, op cit). However, I was discouraged by the infrequency of reference to thinking processes specific to mathematics. Whereas behavioural and emotional factors were mentioned by nearly every teacher, and there was much agreement across the sample, the general cognitive activities listed above were usually only mentioned by one, or at most two, teachers in the sample. Most teachers who offered anything only offered one or two ideas about useful factors. The isolation of ideas which I regard to be specifically mathematical represents an even smaller subset of the sample. Only seven teachers contributed to the final list, and the majority of those were secondary teachers. (I wondered what the outcome would have been before the introduction of the language and concepts in *Using and Applying Mathematics*, even though these are rarely used in the transcripts in spite of the fact that they have been available to all teachers for some time.) Teachers' working views of mathematics and mathematical ability therefore differ widely from Krutetskii's, being predominantly about behaviour, emotion and general cognitive activities. Whereas Krutetskii's framework does seem to capture a flavour of something special to the subject, the teacher utterances recorded in this study could usually be applied to any subject with few changes of words.

Conclusion

In spite of the fact that the focus of the research was not directly about notions of ability, teachers' comments contained a wide range of information about possible factors contributing to achievement in mathematics. Their working notions include emotional and behavioural as well as intellectual factors. This is not surprising. It is worrying, however, that the intellectual factors mentioned do not, in general, address the essential cognitive activities involved in doing mathematics, as described by Krutetskii. Since the main question asked was about recognising what a child could do in mathematics I could have

expected more comments of this kind. Their absence could indicate lack of awareness, lack of personal experience or inability to find appropriate language.

I gave the above findings to students and asked them what was teachable, what the teacher could affect and what the teacher could not affect. They agreed that nearly everything on the list, including difficult aspects like interest, willingness, generalisation, memory, image-making and curiosity, could be affected by the teacher. I hypothesise that other components, like those identified by Krutetskii, might be teachable too, but to be taught they must first be known about, recognised, and articulated.

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Numbers with LOGO

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In this article six difficulties in programming with LOGO are identified and addressed.

The Place of LOGO within the Mathematics Curriculum

There are two reasons for writing this article. The first, and more important one, is to do with the place of LOGO within the mathematics curriculum. LOGO has been available for more than ten years, and for most of the last ten years intending primary teachers, and especially those with some specialism in mathematics, have been initiated into turtle movements. Some LOGO has been used in school, and yet, in most schools it has a marginal place, and appears weakly related to the rest of mathematics. It could disappear from many schools without trauma. In early LOGO trials it was commonly said by both teachers and pupils that LOGO was fun, but no, it was not mathematics. There have been claims that turtle geometry reinforced children's notions of angle, and that the writing of procedures with variables provided a transition into algebra. But these effects have not brought teachers cheering to their feet, and some of the strongest advocates of LOGO have positively revelled in its independence. My experience is that, during initial teacher training, programming in LOGO evokes the generation of problems by students more readily than any other mathematical context, and it is worth exploring whether this problem-generating facility can be extended more widely through mathematics by opening appropriate programming windows. Current work on how to explore symmetry with LOGO has been reported in *Micromath* (Burn, 1995) and *Mathematics in School* (Burn, 1996).

The procedures I discuss below (all in RM Nimbus Logo) will bore the LOGO-buff. But most of us in initial training are not LOGO-buffs. Perhaps the learning experiences of someone new to the language are worth sharing. They may help others to open their own LOGO windows!

More than Turtle Geometry

The second reason for writing this article is to give a little flesh to the claim that turtle geometry is not the whole of LOGO. The published materials which capitalise on this are not as user-friendly as LOGO itself (Cuoco, 1990; Fletcher, Milner & Watson, 1990), and advice on the non-geometrical use of LOGO is not easy to come by. Difficulties I have had in writing procedures in LOGO have sometimes been with me for one, two or three years. The solutions to these problems have come to me in unexpected ways through chance encounters at conferences, through adapting ideas in *Micromath* or *Mathematics in School*, through spontaneous illumination or by accidentally pressing the delete button!

Six LOGO Procedures with Numbers

I offer six LOGO procedures which are about numbers, to indicate to the curious how accessible is this area, and to describe some difficulties in LOGO programming and some solutions. I learnt the first procedure through an NCET conference:

```
FIB 'A 'B  
PRINT :A  
FIB :B (:A + :B)
```

FIB 1 1 prints out the Fibonacci sequence. Of course a STOP condition may be inserted. The first two terms may be chosen arbitrarily, or the print-outs made in a modular arithmetic. There is plenty of arithmetic to investigate. But the real insight which this procedure gave me was how two variables might interact. Many recursive procedures treat the variables independently, and I had not recognised

how creative their purposeful interchange might be. Of course the format here gives a way of writing out the terms of any two-dimensional recurrence relation, but the way in which the variables are used in the recursion suggests how spirolaterals, and suchlike, can be drawn using a recursive procedure with a single drawing command.

The second procedure I offer was designed to implement the process in Euclid, Book VII, Proposition 1, for finding the highest common factor of two positive integers. If the two numbers are A and B, I knew that there were three circumstances, $A = B$, $A > B$ and $B > A$, to which the computer must respond, so I wrote the following procedure:

```
HCF 'A 'B
IF :A = :B [RESULT :A]
IF :A > :B [HCF :A - :B :B]
IF :B > :A [HCF :A :B - :A]
```

But this only works sometimes. I needed to digest the information which appears in every manual, that RESULT (or in Logotron Logo OP) or STOP only stops the procedure which the computer is actually running. If I have just typed in the command RCF 12 5 (say) it is understandable, though wrong, to suppose that that was the procedure which would be stopped by RESULT. When this procedure is rewritten, using a thoroughgoing tail recursion, with

```
HCF 'A 'B
IF :A = :B [RESULT :A]
IF :A > :B [HCF :A - :B :B][HCF :A :B - :A]
```

the procedure is no longer problematic, even though the falsity of 'greater than' is encompassed by 'less than or equal to'. Some reflective arithmetic (as in Euclid) is needed to justify the procedure.

Another problem about computing with numbers is that PRINT 2 makes the number 2 appear on the screen but then loses it from memory. If a single number is to be generated and used the command RESULT (or in Logotron Logo OP) will retain but not (usually) print the number. When I wished to get a list of numbers to keep and use, I had to use a list as a variable, an unaccustomed experience, as most of the lists I had met were sentences or phrases. In this third procedure, 'COPRIMES is a list which starts empty and is filled gradually.

```
FINDCOPRIMES 'D 'N 'COPRIMES
IF :D = :N [RESULT :COPRIMES]
IF (HCF :D :N) = 1 [FINDCOPRIMES :D + 1 :N SE :COPRIMES :D]
[FINDCOPRIMES :D + 1 :N :COPRIMES]
```

The command required actually to produce the coprimes to N is FINDCOPRIMES 1 N [].

The family of commands which may be used on lists may then be brought into play. For example, if in the FINDCOPRIMES procedure the word COUNT is inserted between RESULT and :COPRIMES, Euler's $\phi(N)$ is the result.

Despite the incredible facility with numbers which computers provide, there may be difficulties with very large or very small numbers. Checking Fermat's (little) theorem is something that often involves huge numbers. Here is a fourth procedure which will deal accurately with four-digit numbers to a four-digit index. It calculates X to the power POW modulo MOD.

```
FERMAT X 'POW 'MOD
IF :POW = 1 [RESULT REM :X :MOD]
RESULT REM (:X * (FERMAT :X :POW - 1 :MOD)) :MOD
```

If a series converges, a computer print-out of the sequence of partial sums will usually provide a helpful indication. But if the terms of a divergent series form a null sequence, then a computer print-out of the sequence of partial sums gives one's intuition little to work on, and eventually the terms are, as far as the computer is concerned, indistinguishable from zero. The classic case is the harmonic series $1 + 1/2 + 1/3 + 1/4 + \dots$. Although it is divergent, it is not easy to produce good evidence for this on the computer. This fifth procedure is based on an idea of my colleague David Hobbs. It prints out the number of terms

of the harmonic series which are needed to reach a declared target.

```
HARMONIC 'TARGET 'SUM 'TERMS
IF :SUM < 1/:TERMS [SAY SE :TERMS SE
[terms of the harmonic series are needed to reach] :TARGET STOP]
HARMONIC :TARGET :SUM - 1/:TERMS :TERMS + 1
```

The command HARMONIC N N 1 tells you how many terms are needed to reach N. Commands which reveal an indication of a recognisable pattern are: HARMONIC 2 2 1, HARMONIC 3 3 1, HARMONIC 4 4 1 ... An arithmetical progression in the targets reveals a geometric increase in the terms needed.

The sixth and final procedure deals with the embarrassment of discovering that there is no command which gives a print out from the textscreen in RM Nimbus Logo. Tabled displays of numbers are part of the basic raw material of number theory. The patterns from which arithmetic modulo n derives emerge from properties of the integers displayed in n columns. This can be achieved on the turtle screen with the command ROWCOLUMN 1 1 N with the following procedure:

```
ROWCOLUMN 'R 'C 'N
SETX -15*:N + (:C - 1)*30
SETY 90 - 15*:R
IF YCOR < -90 [STOP]
IF :C = :N + 1 [ROWCOLUMN :R + 1 1 :N]
[LABEL (:R - 1)*:N + :C - 1 ROWCOLUMN :R :C + 1 :N]
```

(In Logotron Logo, the command TITLE is equivalent to LABEL in Nimbus Logo.)

These six procedures have been exhibited to illustrate the surmounting of various programming difficulties. None is definitive, but I will admit that HCF is my favourite.

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Constructing a Liberatory Discourse for Mathematics Classrooms

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This paper is based on the author's conviction that the discourse which informs much current writing by newspaper journalists and some university-mathematicians contributes neither to improving mathematical attainment in schools nor to creating an emancipatory mathematics pedagogy. After a brief discussion setting this discourse aside, three characteristics of an alternative discourse of mathematics education are proposed. These are: that the learners make the mathematics; that mathematics involves thinking about problems; and that difference and individuality should be respected. There is evidence linking the application of these principles to the raising of achievement in mathematics and some justification for the assertion that each also contributes to a liberatory pedagogy.

Challenging the Hegemonic Discourse of Schooling

In this paper I challenge the convictions about schooling of those with power and influence in this country, what can be referred to as the hegemonic discourse of schooling, particularly with reference to the teaching and learning of mathematics. I question the opinions about teaching mathematics that are perpetrated as 'common sense' by the media and that are now reinforced by some parts of the academic community concerned with mathematics. I challenge the view that decisions about the teaching and learning of mathematics can be taken outside of a value-based framework for education. Rather I attempt to make explicit the democratic principles that can render problematic those views about teaching mathematics which are currently taken for granted by those whose voices are heard most loudly in the public domain.

It is a commonplace that things are not always what they seem nor are they always how they are made to seem by those with power to influence our seeing. We are surrounded by the way of seeing, speaking about and thinking about the world that is congenial for those who have the power in our society (see Restivo, 1983, chapter 7, for a discussion of the struggle for an alternative epistemic strategy).

For me, a central function of mathematics education research is to provide the community with an opportunity to affirm an alternative way of seeing, speaking about and thinking about the world. This includes affirming a commitment to acknowledging values rather than hiding them. It includes striving for a more adequate representation of the world which respects evidence whilst recognising that all knowledge reflects the position (or positioning) of the knower. Acknowledging the influence of the knower on the known is likely to give us better representations of the world than pursuing a no-longer coherent 'objectivity' (see, for example, Harding, 1987; Salmon 1992).

What are the purposes of schooling? Personally, I accept that one of the functions of schooling is to reproduce the existing values of the State, thus to maintain the status quo. It is clear that, in part, 'education is the dominant ideological state-apparatus because all children ... are to partake of it' (David, 1978, p173). But that is not the whole story

"Schools ... socialise future generations of young people into the appropriate niches they are destined to fill as adults. But they do not do so unhindered - there is no direct correspondence between the social relations of production and the social relations of education. It is precisely the failure of this correspondence that has brought about the intense pressure to bring about change into schools, to narrow the space between the needs of the system and the needs of the pupils. It is within this space that teachers and pupils may find room to manoeuvre."(Noss, 1990, p22)

We operate within constraints and at times those are more severe, more constraining, than at others.

Part of the constraint is the extent to which the prevailing discourse shapes our thinking despite our intentions to the contrary (Kenway *et al*, 1994, p197). To the extent that we are so constrained, to that extent we are able neither to exploit those spaces that are available for an alternative position, nor to help our students to understand the structures within which they are required to operate. There *are* opportunities in education to do some of the things we want to do some of the time. At the very least,

when we find ourselves constrained by external authority we can articulate and discuss those constraints with our students.

Some independence of judgement and practice is left to us even under current conditions. Indeed, as Giroux (1983, p87f, discussing Bourdieu) has pointed out, it is the perceived relative autonomy of schools that lends them credibility as maintainers of the *status quo* if they were seen wholly as state-apparatus they would lose credibility - and this relative autonomy also allows us to insert other perspectives and gives us a space to explore a different vision of how things might be. It allows us, in fact, to be *utopian* in the sense used by Freire (1985):

"To be utopian is not to be merely idealistic or impractical but rather to engage in denunciation and annunciation." (quoted in Weiler, 1991, p452)

By denunciation, Freire refers to the naming and analysis of existing structures of oppression. By annunciation, he means the creation of new forms of relationships and new ways of being in the world through a process of struggle against that oppression (Weiler 1991, p452).

Denunciation

We do not engage in nearly enough denouncing. No doubt we are all ready to denounce this sort of remark: "The pursuit of egalitarianism [in education] is over." (Baker 1987) We see it clearly for what it is: a statement by someone who has power in society designed to enable him to hang on to that power. Harder to resist are those that come from outside of the overtly political arena. Within the last twelve months (I write in December 1995) we have had newspaper headlines such as "School maths is in crisis" (Guardian, 28 December 1994) and "Maths disaster for schools" (Guardian, 1 November 1995), which have been based more on a media-supported discourse of failing schools, failing teachers and failing children than hard analysis of evidence.

But what has perhaps been more alarming (assuming that we would expect better from academics than from journalists) has been the contribution of some eminent university mathematicians. I have three fundamental criticisms here: first, the relationship of argument to evidence; secondly, the lack of knowledge about disciplines other than their own; and thirdly, the internal quality of the arguments presented.

As an example of the first, when presented with some evidence that mathematics at Advanced-Level might be harder now than in the fifties, Professor Saunders of the Department of Mathematics at King's College, London (quoted in the Education Guardian, January 1995), said, "Everybody's (*sic*) perceptions might be wrong but it would be surprising."

As an example of not understanding other disciplines, in this case theories of learning, we read;

"A common concern is that there is far too much emphasis on self-discovery rather than the presentation of material as a body of knowledge. Such knowledge is the culmination of the work of very smart people over a very long period of time. It is laughable that pupils can achieve mastery of such work through self-discovery." (Professor Erington, head of the Department of Applied Mathematics and Theoretical Physics at Cambridge University, quoted in Times Higher Education Supplement, February 24, 1995)

This is like suggesting, following Seymour Papert (1972, p236), that the purpose of getting students to write poetry is to enable them to discover a line such as 'Mary had a little lamb', rather than to be involved in a personal and creative act.

As an example of the poor quality of argument presented, we might consider *Tackling the Mathematics Problem* (London Mathematical Society *et al*, 1995), where evidence is offered about current levels of attainment. It is suggested (in two places) that the evidence should be studied bearing in mind the question: "Are these standards appropriate?" rather than "Were students better in the

past?" (p10, 31). An informal reading of the presented evidence suggests a not-unfavourable comparison (at age 15 years) with, say, algebra results obtained fifteen years previously (Hart 1981); but the suggested 'possible causes' of the current ('unsatisfactory') state of affairs relate to changes in the recent or even very-recent past. It is very hard to read these responses without being led to question why they are awarded high media-status, and without recognising them, whilst acknowledging some of the legitimate concerns from which they spring, as being part of the maintenance of the current hegemony.

First Annunciation

In my annunciation of an alternative view of mathematics teaching and learning to the hegemonic discourse, there are three characteristics on which I particularly want to focus:

- the learners make the mathematics;
- mathematics involves thinking about problems;
- difference and individuality are respected.

I shall rehearse some evidence that the application of each of these principles contributes to raising achievement, and I shall also argue that each contributes to creating a liberatory discourse in mathematics classrooms.

The learners make the mathematics

Involved here are ideas about the authorship of the mathematics and control of one's own learning, about the mathematics being personal in the sense of personally directed and personally chosen, about being responsible for one's own learning, about learning that is involving and participatory. If learners make the mathematics then the model of learning cannot be that of transmission. We are not going to be in the business of the *delivery of knowns* but rather the *investigation of unknowns* (Burton, 1992, p2). Meaning is negotiated, with the mathematics being co-constructed by a community of validators (Cobb *et al*, 1992, p594), a community which includes the learners and the teacher. The learners have the opportunity to produce as well as criticise classroom meanings. Justifying and explaining are fundamental activities, becoming deep-seated habits of mind which form the building blocks of the negotiated meaning. Discussion is therefore central.

There is evidence about what happens when the model of learning is not that the learners make the mathematics but that the learners' task is to acquire proficiency in the teacher's mathematics. Alan Schoenfeld (1988), reporting his research under the title, "When good teaching leads to bad results", writes:

"On the one hand, almost everything that took place in the classroom went as intended - both in terms of the curriculum and in terms of the quality of the instruction. The class was well managed and well taught, and the students did well on standard performance measures. Seen from this perspective, the class was quite successful. Yet from another perspective, the class was an important and illustrative failure. There were significant ways in which, from the mathematician's point of view, having taken the course may have done the students as much harm as good. *Despite gaining proficiency at certain kinds of procedures*, the students gained at best a fragmented sense of the subject matter and understood few if any of the connections that tie together the procedures that they had studied. More importantly, the students developed perspectives regarding the nature of mathematics that were not only inaccurate, but were likely to impede their acquisition and use of other mathematical knowledge. (Schoenfeld, 1988, p145, my emphasis)

Schoenfeld provides compelling evidence that gaining proficiency in procedures which can get in the way of learning. Not only this. Teaching which focuses on the acquisition of the correct procedures to be followed teaches some other things as well. Schoenfeld found that the students had also learnt the following:

- The processes of mathematics ... have little or nothing to do with discovery or invention.
- Students who understand the subject matter can solve assigned problems in five minutes or less.
- Only geniuses are capable of discovering, creating, or really understanding mathematics.
- One succeeds in school by performing the tasks, to the letter, as described by the teacher.

There are some clear implications for how students experiencing this sort of teaching will construct themselves as (not being) authors of mathematics. For example, thinking about your work is not a worthwhile strategy. It is better passively just to accept what is handed down from above without the expectation that you can make sense of it yourself, let alone offer a challenge. You see yourself as a passive consumer of others' mathematics. Learning and thinking are incidental to 'getting the work done'. Such schooling will not support the development of learners who themselves make the mathematics.

Mathematics involves thinking about problems

A second characteristic of an alternative view of mathematics is that it involves problem-posing and problem-solving. In a problem-based approach, the work is not broken down into pre-digested little bits, each isolated from each other, because the learners must make meaning for themselves. They are part of a classroom tradition in which it is acceptable to take risks, where questioning, decision-making and negotiation are the norm, and where techniques are learnt in the context of solving a problem.

Alan Bell (1994) reported some interesting research in which two parallel classes of 10-11 year olds in the same school with the same teacher were taught fractions using two very different methods. One involved carefully and gradually graded exercises including a large number of examples worked through individually; the other involved the students working in groups at fairly hard challenges involving the production mostly of their own examples. The groups performed comparably at the beginning and showed similar improvement in performance at the end of the nine lessons. However, when they were tested again after the summer holiday break the attainment of the graded exercise group had fallen off to a lower level than before the work began, whereas the learning of the other group was well retained.

There are of course many differences between the two approaches adopted in this experiment, but I would suggest that what the researchers called the 'conflict and investigation' approach accounted for the difference. The students had to grapple with worthwhile problems, they had to take risks in their own thinking, they had to negotiate meaning and justify and explain what they were doing. They had to pose their own (sub) problems, to make decisions of a variety of kinds. The first class went from being highly motivated to being bored and lethargic, whereas the interest and involvement of the 'conflict and investigation' class increased. (Incidentally, we should note that the teacher, unsurprisingly, found the problemsolving class noisier and more stressful than the graded-examples class.)

Classroom activities that support an alternative point of view about what constitutes 'good teaching' might include: the student being offered several methods not just one; there not being a standard procedure to follow; there being the opportunity to test out theories and to self-check. There will need to be aspects of the activity which encourage the students to be their own authority; which require them to generate some of their own questions so that the task cannot be just to reproduce what is in the answer book. Their own questions may be more or less interesting, more or less appealing, more or less difficult: but they can be encouraged to reflect on those qualities.

Difference and individuality are respected

The third characteristic of the alternative to the hegemonic discourse is that individuals should be respected. The difference of each is respected. The authority of each is respected. There is evidence that teacher-behaviour which offers students respect helps the student to develop and enhance their self-image and their own expectations, which in turn enhance the students' academic performance (see, for example, Charlton and Hunt, 1993). Self-esteem, as well as being a moral end, is also related to what students are able to achieve. Self concept correlates positively with achievement in mathematics and a key attribute of teachers who are able to enhance it is that they offer learners respect (Rogers, 1983, p197-224).

What I argue for here is currently unfashionable. It relates to the issue of how we group students. It is difficult to see how most students' self-concept can conceivably be enhanced when learners are segregated, by others, into perceived ability-groupings. Challenging our 'feudal' notion of ability (Tahta, 1994, p25) is essential if we are to enable schooling to offer students equal regard. Rejecting such a notion and the organisational structures that go with it does not guarantee anything in terms of the

relationships in school classrooms but it does offer the opportunity to offer all the students respect. This issue, of course, impinges on issues of gender, 'race' and, especially, class. Students not belonging to the hegemonic group have all been under-represented in 'top' groups. If you allow students to be grouped according to someone else's estimation of their capabilities this seems to me to be inherently disrespectful and undermining of those who are not 'class one'. If, on the contrary, we work to challenge the discourse of 'ability', we make for ourselves the opportunity to respect difference and the individual. The opportunity is there to exploit or deny.

Second Annunciation

As has been argued above, evidence suggests that classrooms where the learners make the mathematics, where mathematics involves thinking about problems and where difference and individuality are respected make for more effective learning. In addition, however, a classroom with such characteristics is potentially *emancipatory*. Emancipatory education involves:

"... a process of empowering people with the understanding and competences which increases effective participation in our society, and enables people to define and realise their identity, think critically about the world, and to change it." (Hill, 1991, p20)

It involves a curriculum and a model of learning which rejects the transmission-delivery mode ...

"the 'banking' concept of education, in which the scope of action allowed to the students extends only as far as receiving, filing and storing deposits [... where] knowledge is a gift bestowed by those who consider themselves knowledgeable upon those whom they consider know nothing." (Freire 1972, p46)

Because we live in an unequal world, unequal in terms of power, opportunities, access to resources and so on, a curriculum based on transmitting received knowledge is oppressive. It gives no opportunity for challenging the (unequal) *status quo*. Nor does it provide an opportunity for hope.

"The transmission model of teaching, in a traditional formal classroom ... is the opposite of what we need to produce learners who can think critically, synthesise and transform, experiment and create. We need a flexible curriculum, active co-operative forms of learning, opportunities for pupils to talk through the knowledge which they are incorporating." (Gipps, 1993, p40)

I am advocating mathematics teaching which rejects a content-driven, hierarchically-organised agenda and test-dominated assessment (Burton, 1993, p10), that is antithetical to the development of self esteem. In order to teach the acceptance of inequality, schools must be sites of differentiation, of fragmentation or of dominance. Recent changes in schooling need to be seen in this light.

"The National Curriculum is not about the enhancement of curricular content or the improvement of assessment procedures, and not about disagreements over the kinds of strategies which will improve children's (mathematical) learning: it is about a centrally imposed and nationally validated system of grading children, schools and teachers." (Noss 1990, p28)

The basis of the National Curriculum is differentiation, selection, streaming. Differentiating between people according to some characteristic and then grading them allows that characteristic to be used as a justification for inequality. This process of differentiation, selecting, grading and streaming is necessary so that those who are at the bottom are firmly aware that that is where they should be - otherwise why should they accept it? This grading and valuing is something very deeply embedded in our culture of schooling.

Almost always when we as teachers talk about a 'good' student what we actually mean is 'a high attainer'. What are we doing to those who are not described as 'good' when we use language like this? We must recognise through such use of language the extent to which our thought patterns are framed by the hegemonic discourse. We need to be able to "jump outside" the frames and systems that authorities provide (Belenky *et al*, 1986, p134) and to create and recreate our own.

We can so easily ourselves be silenced, to lose our voice even to ourselves. It happens to us all from time to time. It is a position of muteness and terror (Perry, 1988, p149). It cuts us off from both internal

and external sources of intelligence and stops us from being able to see ourselves as developing, acting, planning, choosing. It can be evoked when one is challenging or resisting those who are dominant in society and speak most loudly. It can be generated by our friends as well as our enemies, especially when we are trying to understand 'folks different from myself' (Pratt, 1984, p18). In particular it is evoked by the presence of an alien discourse that defines and positions us in ways we want to resist.

Remaining positioned by the prevailing discourse does not, of course, preclude, for example, 'doing investigations' where this is seen as a new area of rule-bound work (but with the rules harder to discover), a new area to be 'mastered' (*sic*), a new set of (given) strategies to be acquired (Love, 1988). Our own education and enculturation encourages this reliance on and deference to external authority. A telling instance was provided by a group of the PGCE students with whom I work. They were asked to report their perceptions of the personal skills and qualities possessed and/or desired by them as relevant to the job of teaching. The only aspects of their ability to work with others that they were satisfied with when they began the course were 'co-operation with a boss, taking orders' and 'tact, diplomacy, politeness' (Payne, 1991).

Further, the dictates of external authority, when they are part of what I called the hegemonic discourse, seem just common sense.

"Some of the representations of dominant groups are likely to be labelled as self-evident, and put to use to enforce conformity, put a subject beyond dispute, and deal with ambiguous and anomalous events. These representations will be prime targets for those who want to criticise, change or demolish the reigning social order." (Restivo, 1992, p125)

Thus how we know things and how our students know things are of central importance. We and they have to accept the unguaranteed nature of knowledge (Restivo, 1983, p140f). We have to maintain our vision of meaning, coherence and value while being conscious of the fact that our vision is partial, limited and contradicted by the vision of others. We need to hold knowledge tentatively but with commitment, with what has been called a kind of provisional ultimacy (Fowler, 1978, quoted in Perry, 1981, p96). What is important here as far as mathematics teaching is concerned is the habits of mind which our teaching helps inculcate. We want to inculcate those habits of mind in our students which encourage them not to accept received wisdom (including ours), to be intellectually critical, to see final authority as lying with themselves rather than with external sources. We want a shift to an internal locus of control so that they come to believe that they can do things, that they have power over themselves and power to effect change over the things they believe to be right (Perry, 1981, p94). Such authority is fostered by a curriculum in which is embedded the view that the learners make the mathematics.

Similarly, an approach to *mathematics through problem thinking* links with a rejection of an external authority model of knowledge.

"Once knowers assume the general relativity of knowledge, that their frame of reference matters and that they can construct and reconstruct frames of reference, they feel responsible for examining, questioning, and developing the systems that they will use for constructing knowledge. Question posing and problem posing become prominent methods of enquiry." (Belenky *et al*, 1986, p138f)

Problem-posing and re-posing allows the linguistic assumptions hidden in their original formulation to be questioned (Brown, 1986).

"[A problem-posing pedagogy] represents a powerful emancipatory teaching approach, and when successfully implemented, empowers learners epistemologically ... it encourages active knowing and the creation of knowledge by the learners, and it legitimates that knowledge as mathematics." (Ernest 1991, p291)

If we reject a curriculum in which the teacher is the sole validator of what counts as legitimate mathematical activity, this implies respect for multiple ways of experiencing mathematics and its interactions with the world of each learner. We want to encourage the critical capacity to challenge, to find one's own voice, to take risks, to allow for the possibility of hope (Giroux, 1992, p73-80). Each of these encourages the capacity to see beyond the world as it is, an essential prerequisite for devising a plan of how it might be.

Central to it all is talking:

"[We can] make a distinction between 'really talking' and what [we] consider to be didactic talk in which the speaker's intention is to hold forth rather than to share ideas. In didactic talk, each participant may report experience, but there is no attempt among participants to join together to arrive at some new understanding. 'Really talking' requires careful listening; it implies a mutually shared agreement that together you are creating the optimum setting so that half-baked or emergent ideas can grow. 'Real talk' reaches deep into the experience of each participant; it also draws on the analytical abilities of each." (Belenky *et al*, 1986, p144)

Thus 'real talk' involves listening and that can only happen effectively in a context in which difference and the individual are respected.

An alternative view of mathematics, then, to that presented by the hegemonic discourse will include an understanding that at least some of the characteristics of critical mathematics education are also general characteristics of effective learning. In addition, inasmuch as the 'real talk' described above occurs within the mathematics education community, it will also support us if we decide we are not prepared simply to adjust ourselves to reality but also to try to make reality adjust to us. Without a commitment to ideas beyond the everyday, taken-for-granted such a perspective seems impossible. There are formidable problems in trying to do so in the face of the paradoxes, ignorance, contradictions and compromises that we all experience and which obscure, harass and destroy emancipatory motives, aims and ways of life (Restivo, 1983, p129); but there are pay-offs too.

"The relationship between a person and an idea seems doomed to be one-sided, since an idea cannot reciprocate the care lavished upon it by a thinker. But ... when we understand, we feel that this object-other has responded to us ... we hear it speak to us. The joy attendant upon intimacy with an idea is not so different from the joy we feel in close relationships with friends." (Belenky *et al*, 1986, p102)

It is my conviction and contention that freeing ourselves, in some small part, from the prevailing discourse of schooling will support that process of understanding.

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