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Going Beyond the National Curriculum in Preparation for Teaching

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An important function of teacher training is to encourage teaching approaches which stimulate the intellect of learners. I argue that this requires a good appreciation of key ideas in school mathematics. With reference to ratio in the national curriculum, I illustrate two strands which contribute to such an appreciation. One strand is the realisation of how engagement with an idea can develop over a period of years in a child's education. In general, the progression will be from intuitive to more formalised, self-aware understanding. The second strand is a recognition of those concepts which are powerful within mathematics itself and of the various contexts in which they recur.

Introduction

Both a national curriculum and a school scheme of work tend to be catalogues of material to be taught. Whilst a well structured and ordered catalogue has many virtues, it is potentially dull and lacking in appeal. Who fancies *learning* a catalogue?

Reminiscing one day, a friend told me about learning history thirty years ago. "We had two teachers," she said. "One told us we had this topic and then this topic to do and we would need to know such and such for the exam; with her, learning was a chore. The other came with enthusiasm: we are going to look into this and this question, you'll need to find out, and so on. With her we really worked, but it never felt like work,"

The story could equally be told of mathematics teaching today.

To engage the genuine powers of students, learning needs to be an intellectual quest.

To achieve a challenging, rather than a piecemeal approach to teaching, it is certainly advantageous to be enthused by the material, but it also requires a range of insights. I begin by examining a schema which can be helpful in developing insights about children's learning.

Genealogy of ideas

A *zone of next development* (Vygotsky, 1962) is one of the notions from educational theory which resonates with the experience of many teachers.

On the one hand there is the familiar territory of what the learner already knows and has thoroughly mastered. This consists of concepts, language, facts, habits of thought and problem-solving competences which are securely embedded in the leamer's mental framework.

On the other hand, there are concepts, algorithms, proofs and so on which are so far ahead or apart from a person's current mental framework that s/he has no real hope of making sense of them, even if they are carefully expounded by a teacher. 'Out of one's depth' describes the feeling of trying to operate in this territory. When such material is offered in the classroom, a learner has little option but to adopt a rote-learning mode. There are likely to be feelings of boredom, anxiety, and very limited success.

Now between these two extremes is a mental tract, the zone of next development or zone of potential development, in which progress with understanding can occur. This fertile territory is the natural place for educational effort. With appropriate guidance, learners can explore and consolidate parts of this zone and push forward its frontiers.

The boundaries of these regions are never precisely defined; skills can become rusty, ideas slip under stress, or moments of inspiration may occur. It is also important to bear in mind that when progress is made, it normally needs to be consolidated. Over a period of time, during which new knowledge and understandings are used, they become familiar and comfortable tools. Part of this digestive process involves establishing links within the learner's own mental framework and thus coming to realise the significance of the new material.

When adequately absorbed in this way, learning becomes available to be applied in the exploration of yet more ground. Formalisation also makes sense at this stage. Definitions and formal structure add precision and strength, whereas given too early they can stultify. I quote Mary Boole, who lectured in the 1860s at the first college of higher education for women (Tahta, 1972). She argues the folly of using Euclid's books of theorems as an introduction to geometry. We seem to have made some educational progress since then, but how far?

"Euclid wrote, not a geometry for beginners, but a book about the logical concatenation of geometrical facts for men already geometers ... men for whom the words triangle, circle, parallelogram were already charged with associations; and he gave definitions intended for the purpose, not of telling something fresh, but of clearing up and settling conceptions which were hazy from long familiarity ... If children of twelve are to learn what Euclid wrote for advanced men, children of three should be acquiring the subconscious physical experiences ... "

There is a real sense in which all learning is idiosyncratic; no one can prescribe the actual route a learner will take. Yet it clearly makes sense to construct a curriculum which follows any natural logic within the subject itself and helps teachers to bring out links where important links exist. A curriculum should also allow time for this gradual assimilation and maturation of ideas.

Mter the initial traumas of implementation, one major benefit of a national curriculum is surely its potential to assist communication between different teachers of the same children. Children, whose own development is a continuous strand, can only benefit if it becomes easier for their experience and intuitive reasoning, gained with one teacher, to be appropriately formalised and extended by another.

Teacher Training

In order to engage pupils in an intellectual quest, it is important for teachers to have an internalised consciousness of the key ideas in school mathematics. This is helpful both for planning a programme of work and for dealing with the multitude of unplanned interactions taking place in the classroom.

Along with an appreciation of what is essential to the subject, there needs to be a feeling for the way in which these ideas can be sown and then grow to maturity in the mind. Without a sense of what might be significant insights for a child, how can a teacher know when and how to prompt, when to hold back and wait for a thought to emerge, where to place an emphasis and so on? S/he also needs to be able to recognise children's perceptiveness, even when expressed in unconventional ways in what they write or say. One can go on developing such awareness throughout a lifetime, but it does seem an appropriate task with which to engage during initial training.

If we were all to make a back-of-the-envelope list of the key ideas in school mathematics, I wonder how much agreement there would be? Lookin~ in the new national curriculum (DFE, 1995), some of the items on my list, such as the notion of a function, have a high profile, and others are more latent than explicit.

The argument so far suggests that there is more to 'progression' and 'links' in the subject than could feasibly be written into a curriculum, but here is a danger. Because national and school schemes are so thoroughly documented, there may be a temptation to regard them as the whole story. Nevertheless, identifying key ideas may well arise from a study of curricula. Attempting to trace the potential evolution of ideas throughout the scheme will also provide food for thought. Whatever the starting point, some such reflective activity is a valuable preparation for 'delivering' in a meaningful and intellectually stimulating way.

The Quest for Ratio in Practice

I have two distinct reasons for attaching importance to the idea of ratio. The first is its prevalence in everyday life, as a fairly random collection of examples will show:

- For a plain cake you need the same weight of sugar, fat and eggs, but twice as much flour.
- It is ten times as far to the supermarket as to the corner shop.
- I am building a 15:1 model.
- What does this pie chart mean?
- The hedge needs cutting back to two-thirds of its height.
- The odds have shortened to 5:4. • They are offering a 20% discount.

The last example may look like percentage, but the idea is the same, 20 out of every 100 is a ratio. The National Curriculum does list these terms together, but the identity of the basic idea is not made explicit. Beginning teachers need to perceive that 'doing ratio' should have much in common with 'doing percentages'.

The essence of ratio is simply a comparison by means of 'times'. We make comparisons in many different ways, bigger/smaller, > or <, or in more precise vein, A is 47cm taller than B. In the latter it is the difference between A and B which is reported. To say that their heights are in the ratio 152:105, or that A is 1.45 times as tall as B, would sound rather odd, but 'A is about one-and-a-half times as tall as B' is natural enough and brings us back to a ratio comparison.

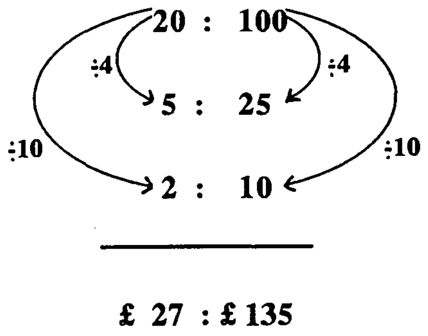
The distinction between difference and ratio is an important recurring theme in mathematics. For instance, a sequence of equal steps (differences) gives an aritbmetic progression or linear function. A sequence with equal ratio between terms gives a geometric progression or exponential function.

Proportionality (of the direct type) is another big idea, which involves application of the same ratio in two or more instances. For example, 'What is 20% of £135?' This requires the construction of a ratio (? : £135) which is equivalent to 20:100.

There is plenty of research (e.g. Hart, 1981) which shows that children find proportionality very difficult, yet doubling and halving are easy and intuitive, and we are surrounded by uses of scale: plans, maps, models, photographic prints and enlargements etc. Between the familiar and the baffiing, there should be a 'zone of potential development'.

One clue to this comes from an unlikely quarter, namely, V.A T.

The improbable figure of 17.5% is presumably chosen because of the ease, even mentally, of finding 10%, then 5% and 2.5% by successive halving. Adding the three results together gives the required 17.5%. Children have been observed (e.g. DalrympleAlford, 1979) to invent and apply this sort of strategy. It works in the case above (and many others, given the sort of ingenuity children display when they feel they are in control):



The National Curriculum is not the place to spell out detailed teaching approaches - that is for the professional expertise of teachers - but in the case of ratio, it has offered scant guidance on the kind of progression over the years that might be useful (Kiichemann, 1990). Currently (DFE, 1995) the following are offered in the programmes of study:

Key Stages 3 & 4: *General Rubric*

Ratio & proportion should be linked to probability, geometry and to solving numerical problems.

Key Stages 3 & 4: *Number*

Calculate with .... fractions, percentages and ratio

.... solving problems, including those that involve ratio

Key Stages 3 & 4: *Shape, Space & Measures*

Develop an understanding of scale ....

Although the use of fractions and percentages to estimate, describe and compare proportions of a whole is in the Key Stage 2 Programme of Study, it is disappointing that this general statement in the draft orders (SCAA, 1994) is now lost:

Key Stage 2: *Space, shape & measures:*

.... begin to develop an appreciation of relative size and scale.

Nothing of this is mentioned at Key Stage 1, where simple comparison by 'times' would surely be appropriate, eg cut a piece of tape that is twice as long, and another three times as long. First School pupils might also proceed from one-one matching to mixing potions in proportion eg, one of lemon and two of orange, one of lemon to two of orange, and so on.

The Quest for Ratio in Mathematics

The second reason for attaching importance to ratio is its relevance at some of the higher levels of school mathematics. Not all children will reach this stage, but fortunately there is not a conflict of interest, since long-term effort put into developing familiarity with the concept is beneficial to everyone, as demonstrated above.

At level 8, pupils are required to use trigonometric ratios. One

way to generate some intellectual excitement is to pose a question:

You know something about angles of triangles (they total 180^0), and you know how the sides of a special triangle are related (Pythagoras' theorem), but what connection might there be between sides and angles?

Initial exploration may suggest none. Even if the search is narrowed to right-angled triangles, you can have 40^0, 60^0, 90^0 triangles of any size. Someone may observe that, though varied in size, they are mathematically SIMILAR. So all 40^0, 60^0, 90^0 triangles have the same ratios between their sides, and those are different from the ratios which exist in, say, all 41^0, 59^0, 90^0 triangles.

Enlargement and similarity are explicitly mentioned rather late in the National Curriculum, whereas early work in geometry is strongly focused on symmetry and thus on congruence. But at Key Stage 2, for instance, when making 3D shapes it would be fun to make a family like Russian dolls, each so many times bigger than the baby.

Likewise in LOGO, having developed a procedure to draw a shape, to modify it to draw the same shape but smaller/bigger. These are the sort of early experiences from which an appreciation of mathematical similarity can be expected to mature.

Finally, for exceptional performance (in Number), pupils are required to understand and use rational and irrational numbers. It remains to be seen how this will be interpreted but in the 1994 draft (SCAA, 1994) this was elaborated to, 'distinguish between rational and irrational numbers and appreciate that irrational numbers complete the real number system.' Since we habitually work with rational approximations to quantities such as 1t and .J2, ordinary computational experience may leave the confusing impression that these, indeed all numbers, are rational! Irrationality is an essentially theoretical concept and the pedagogic question is how to pave the way for it in an intelligent manner.

The Hungarian National Curriculum (Howson, 1991) suggests one way of approaching the idea:

Grades 6-8 *(age* 12-14) *Topics include terminating and recurring decimals as rational numbers.*

Gymnasium, second grade (age 15-16) *Extension of number concept, irrationals as a periodic decimals.*

It is not difficult to show that every ratio has a decimal expansion which is terminating or recurring. Then it follows that a nonrecurring, non-terminating decimal must be non-rational. Constructing examples of such decimals can help to give a feel for the existence of irrationals.

For establishing that familiar quantities come into the irrational category, there are standard proofs, showing for instance that $\sqrt{2}$ cannot equal p/q for any integers p and q. Many students find this baffling. The choice of teaching approach will depend on judgment about what is, or could be, within their zone of potential development. It may be that formal proof would be accessible if the logical form, 'proof by contradiction', became familiar first in an easier context.

On the other hand, proofs may be inappropriate. Work with successive approximations may serve sufficiently to raise the relevant awareness in pupils' minds. For example, we could consider: historical efforts to specify 7t; Fibonacci ratios approaching the golden section; or a problem about the size of A4 paper .... normally given as 300 x 210 mm., when cut in half, each sheet of A5 is supposed to have exactly the same proportions as the A4, but 210/150 is not equal to 300/210, so what ought the dimensions of A4 to be? Raising genuine doubts about our ability to specify these quantities with ultimate precision is one possibility for bringing the concept of irrationality within grasp.

Conclusion

It has not been my main purpose to suggest modifications to the National Curriculum, but rather to consider ways of looking within and beyond curricula in order to make appropriate pedagogic decisions. There are both learning and mathematical issues which impinge on such decisions. I have tried to illustrate the sort of mathematics-specific debate about children's learning which could contribute towards the training of stimulating teachers.

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Mathematics Education: Yet Another Change

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Ten years ago the concept of a National Curriculum for Mathematics was virtually unheard of within the teaching community and yet, in the Autumn we are required to implement the third version since its inception. In the light of the dismal history of "new mathematics", this article examines its chances of success and offers some cautionary comments regarding its management.

Introduction

Yet another change in mathematics education is upon us: the Schools Curriculum and Assessment Authority (SCAA) have produced their proposals for the third version of the Mathematics National Curriculum and, no doubt, by the time you read this these proposals (with a few minor amendments) will have become law. We have been assured that we will be left alone for five years after that but just what are the chances that Mark 3 - as opposed to Marks 1 and 2 - will succeed both in terms of raising standards and improving attitudes?

Casting our minds back to the exhilarating days of "new mathematics", some may remember the zeal with which this revolutionary approach was introduced. Perhaps fewer will recall its decline into obscurity: within less than twenty years of its inception Hayden (1983) wrote,

"Today, mathematics educators are likely to look back on the 'new math' era as on a wild, and perhaps misspent youth; filled with energy and enthusiasm, idealism and naivete, but faintly embarrassing to look back on today ... Never again shall we attempt to radically alter the school mathematics curriculum left to us by the last century." (p. 1)

Hayden (1983) argues that the reasons for the downfall of new mathematics are multi-faceted and complex. One of the basic problems, however, was a failure to understand teachers and teaching: can the same be said of Mark 3 of the National Curriculum? Are there parallels to be drawn? To consider this, three issues should be addressed: firstly, the extent and efficacy of *piloting;* secondly, technicalities associated with *application and content,* and, finally, *ownership* of the revised proposals.

Piloting

Hayden (1983) points out that the piloting for new mathematics was minimal and yet it swept through Britain and America in almost a matter of months. The same could be said of the initial version of the Mathematics National Curriculum. There was some limited, short term piloting but, in effect, the new approach was trialled on an entire generation of school children and their teachers.

While one might wish to question the morality of such an approach one could also argue that such an extensive trial, together with the resulting modifications, ensures that much of the new curriculum will have been thoroughly piloted. To a certain extent this may be true but it overlooks the fact that, over a five year period, two versions of the National Curriculum were "piloted" and that, during this time, a great deal of adjustment was taking place in schools. Possible implications are many but three are of particular relevance here:-

(i) Was there sufficient teacher time and attention for the material to have been thoroughly trialled?

(ii) Was there sufficient piloting to assess progression?

(iii) Was there any improvement in the children's mathematical performance?

Taking the first of these: it has been well documented that change often results in "deskilling" and a subsequent - albeit usually temporary - reduction in performance levels (MacDonald, 1973). This, coupled with primary teachers' attention being diverted to an expanding science curriculum, suggests that teachers - and correspondingly their pupils - may not have been performing as they might have been had the situation been more stable. Indeed a SCAA report (1993) suggests that teachers at all key stages, " ... needed time to digest and evaluate the complexities of the mathematics National Curriculum" (p.ll) and" ... were experiencing difficulties due to the rapid pace of change." (p.ll).

Secondly an insufficient time elapsed for a real assessment of a stable programme over a significant period of a child's schooling.

Brown (1994), for example, questions the progression between levels suggesting that longer-term evaluations were required.

And finally, were there any significant data to show changes in children's mathematical performance? Brown (1994) argues that the answer is far from clear, there being no reliable evidence available to make pre- and post-National Curriculum comparisons.

To conclude, while superficially it would appear that, unlike new mathematics, the National Curriculum has undergone extensive albeit implicit - piloting, on closer examination, we are left questioning its reliability. Are the new proposals based on amendments to an unstable and evolving mathematics curriculum?

Technicalities

Ormell (1981) suggests that teachers were suspicious of the unfamiliar jargon and techniques which were an integral part of new mathematics. To some extent this may have been true with the National Curriculum but, I suspect, few have considered this to be a significant factor in hindering its apparent implementation. Firstly, because the documents included frequent examples; secondly, because, more than ever before, teachers were encouraged to work together and discuss issues of planning and concern and, thirdly, because there was an abundance of commercial schemes for teachers to call upon.

This, however, does not leave as much room for complacency as one might hope for. If the Draft Proposals (SCAA, 1994) are anything to go by, there are to be fewer examples in Mark 3 of the National Curriculum documents (Brown, 1994). Moreover, while it is recognised that staff room discussions may be illuminating, they may also be forums in which misunderstandings are shared and ignorance is refined and perpetuated' under a cloak of apparent security and confidence. And, finally, it has been suggested that some aspects of the curriculum (particularly Mal and Ma5) were not well covered by commercial schemes (SCAA, 1993). Whether this will hold true for Mark 3 time will tell.

In brief, unless we take care, perhaps one of my accounts of the demise of new mathematics (Cockburn, 1986) might equally apply in the future,

" ... Stephens and Romberg (1985) argued that, in lacking a real appreciation of the philosophy behind new maths, many teachers were unaware of its rationale and the values associated with various activities and actions ... One thing is certain: the innovative techniques of new maths were never actually implemented by the majority of teachers (page, 1983). They may have altered their content but their methods remained the same (Sarason, 1971)." (p. 9)

To monitor and guard against this I suggest that we must not content ourselves with scrutinising teacher forecasts and other documentation but we must also observe and evaluate teachers' practice in the classroom: will they practice what they (are supposed to) preach?

Whose agenda?

New mathematics seemed to arise because, " ... a large number of forces happened to be pushing in the direction of mathematics curriculum reform at the same time." (Hayden, 1983, p.1) In America these were partly in reaction to the poor mathematical standards of military recruits during the Second World War. New technology, advances in mathematical thinking and a worried government were also influential factors. It transpired, however, that the goals of these various groups were not always compatible with those of the pioneer reformers (Hayden, 1983).

Certainly various interested parties have proposed different priorities and desirable outcomes for mathematics education in this country. The government seem particularly keen on children acquiring the "basics" with pupils readily being able to recite their tables, recall their number bonds etc. etc. Many mathematicians, on the other hand, tend to more concerned with children developing a fundamental understanding of mathematical processes which will enable them to apply their knowledge in a wide variety of ways (Cockburn, 1994; Haylock and Cockburn, 1989). Teachers, I suspect, fall between the two camps, but frequently - sometimes unwittingly - adopt a skills-based, rather than concepts-based, approach: the former being easier to implement than the latter (Davis and McKnight, 1976; Desforges and Cockburn, 1987).

The crucial point in this instance is that, unlike new mathematics, everyone has had an opportunity to put forward their viewpoint. Thus classroom practitioners have been able to make contributions to the documentation as it has evolved increasing the likelihood of their feeling some ownership and decreasing the chances of their being, "mere executors (if not executioners) of someone else's decisions" (Kamii, 1985, p. xiv). This notwithstanding, it is interesting to note that a national study (SCAA, 1993) evaluating Mark 2 concluded:

"Any costs, human or financial, resulting from the imperfections of the current Order were felt to be significantly less than the costs of yet another change." (p.14)

In other words, while the teachers had an opportunity to make their voices heard it was perhaps not to the extent they might have liked: whether this will affect the success of Mark 3 remains to be seen.

Some Recommendations

Whether we like it or not a new version of the National Curriculum is upon us for - if we are to believe the politicians - at least five years. Certainly some will subvert it; others will misinterpret it but perhaps if we are serious about developing mathematics education in this country we should run with it. To do this effectively we need to ensure that, unlike new mathematics, the National Curriculum is fully implemented in the manner intended. As discussed above the implications of this are extensive and involve a thorough appreciation and examination of the various players and processes.

During the induction period we need to be aware of the upheavals, stresses and strains which almost invariably accompany change (especially after such an unremitting succession of new proposals). When relative calm and a more settled situation reassert themselves intensive and extensive monitoring need to take place in a low key, but thorough, fashion. One of the main reasons for this are that we need to determine how effective the new version of the National Curriculum really is in enhancing children's mathematical education and preparing them for adult life in the 21st century. This, as implied above, must include longitudinal as well as more short term evaluations.

A second reason for in-depth monitoring is to ascertain exactly how the proposals are implemented and, if applicable, for possible causes of limited implementation. By this I do not mean lack of commitment or understanding although these, of course, are important and need to be addressed (particularly when considering the interface between teachers and schemes.) Rather I am referring to weaknesses in the proposals which may make them impossible to fully implement as intended. These may be the result of factors such as poor continuity as discussed by Brown (1994). They may also, however, be in response to classroom processes which may militate against effective implementation. Researchers (see, for example, Davis and McKnight, 1976; Doyle, 1986; Desforges and Cockurn, 1987) have been aware of these for some considerable time but the ramifications for schooling are not entirely understood.

For the first time since its introduction we have been promised some time to work with the latest version of the National Curriculum. Five years might seem a reasonable period in which to iron out some of the problems of innovation discussed above and, indeed, it should be sufficient to give an indication of the success of the venture. Nevertheless policy makers need to be aware that effective change takes considerable time and American research (Shreshly and Bernd, 1992) suggests that major school reforms (never mind National ones) can take up to ten years. Moreover let us all hope that there are no suggestions of change within the five year period for, if the past year is anything to go by, we all know how unsettling - and even demoralising for some people - the thought of yet another curriculum upheaval can be.

In conclusion, it is too early to say whether Mark 3 of the Mathematics National Curriculum will prove more successful than new mathematics. Judging from the above its success will require considerable application, understanding and effort. This will only be possible if the politicians and policy makers leave us alone for a while and allow the professionals to tackle the task in hand.

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BA(Ed) Students' Initial Experiences of Mathematics Education

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Stimulated by the article by Rod Bramald and Alison Wood in Mathematics Education Review Number 4" *May* 1994, *we offer, in a similar spirit of professional sharing, a description of the mathematics professional element in the first year of our BA(Ed) programme together with the rationale for this part of the course. The thought and discussion about similarities in aims prompted by their article provided the stimulus for our writing. Below is an account, to which responses are invited, of how we approach an introduction to mathematics education with our students.*

Introduction

Reading the article written by Bramald and Wood (1984) reminded us of a question asked at an interview for a teacher training post:

"What would it be vital to include in a mathematics education course for students before they go out to school for their first block school experience?" This question has re-surfaced many times and, although at the time it was hypothetical, it has since required close consideration in reality. In 1993, during the writing of our new degree, the Primary Mathematics Team reflected upon this issue in order to draw up a new programme. The team felt that the impact of the first mathematics education experience upon the students' attitudes is of great importance and that, whilst the mathematical content is also important, it is possible to accommodate this in the context of addressing the over-arching aims related to student confidence and enjoyment.

We noted similarities with the thinking expressed by Bramald and Wood relating to the vital importance of first impressions and the need to examine the students' perceptions of, and attitudes to, mathematics.

Aims, Intentions and Structure

The aims of the sessions described in Bramald and Wood's article which relate to challenging students to question their assumptions about mathematics and to reflect upon their experiences of learning mathematics resonate with some of our own thinking. They closely correspond with our aims for the sessions which form the mathematics professional element of our first year BA(Ed) programme. This applies not only to the specific intentions of our first session with the students, but permeates all six weeks of which the block consists. The block is comprised of six weekly three-hour sessions, with time in school between each session.

The course that our first year students follow has been developed in order to ensure that the experiences that students have enable them to examine their perceptions of mathematics and to experience mathematics in a variety of contexts in which they can feel confident and enjoy doing the mathematics. Enjoyment is regarded as a key factor in affecting the students' future approach to teaching mathematics. The block aims to enable students to:

- examine their perceptions of mathematics
- articulate these perceptions

- relate them to mathematics teaching and learning

This is approached through providing the students with experience of mathematics in a variety of contexts.

Other aims relate to:

- gaining pleasure from, and confidence in, doing mathematics
- consideration of styles of teaching mathematics appropriate in different contexts
- examination of different teaching strategies
- enabling students to develop a familiarity with the National Curriculum
- an introduction to the development of number • developing students' independent learning

The titles of the six sessions broadly indicate the content of each:

1. An introduction to the course: what is mathematics?
2. An introduction to counting and number: making a number game.
3. Mathematics in the environment: mathematics trails.
4. Using stones and rhymes in mathematics education.
5. The National Curriculum Game: becoming familiar with the National Curriculum
6. Using practical apparatus to address number in the National Curriculum.

Whilst the approach to each session varies, there are some features common to each session and which provide the foundation for the work undertaken. These are as follows:

1. A workshop element.

Time is allocated in four of the six sessions for the students, in pairs or groups, to produce something which can subsequently be used in school; subsequently they report back on the school experience. This relates to the topic under consideration in the session. This workshop-based approach is incorporated into the sessions in order to provide an active learning experience for the students and an opportunity for them to work co-operatively in both pairs and groups. Some examples of such activities are: the making of a mathematical game; the production of a mathematics trail; the invention of a story or rhyme which can be used to encourage mathematicalleaming.

2. Reading relevant to the focus of the session.

Students are required to undertake directed reading linked to each session. They are asked to collect a file containing summaries of the reading, from which issues have been identified. Discussion of the reading forms a part of each session and informs the assignment. At the beginning of each session there is a discussion of the reading given at the end of the previous session and relating to the focus of that session. Readings have deliberately been chosen from a variety of different sources: the Cockcroft Report; Strategies; Child Education; Junior Education; Mathematics Teaching; Mathematics in Schools; Children and Number by Marlin Hughes; Non-Statutory Guidance and teacher-produced material. The intention is to enable students to recognise and become familiar with a wide range of sources for reading and to highlight and discuss the various issues arising in the readings.

A variety of mechanisms is used to facilitate the feed-back. One strategy has been to divide the group into sub-groups. Whilst all students are expected to undertake the reading, one member from each sub-group is responsible for leading the discussion in that subgroup each week. They then report back, in various forms, to the whole group (using, for example, posters, overhead transparencies, verbal reporting).

3. Feedback from work in schools.

Students report back on their mathematics work in school and the use of the activity they have produced. Part of each session is devoted to a 'show and share' activity, in which they show (where appropriate) what they have made and used and share what happened in its use in school.

The combination of workshop sessions and seminars on directed reading aims to develop the students' ability to relate their reading to their experience of trying something out in school and to reflect upon the relationship between the two: highlighting the mutual interdependence of theory and practice and encouraging the students to begin to develop their own philosophy of mathematics education.

The assessment of this block is an integral part of the work, informed by: the work in the sessions; the readings; and the experiences in school. Students are asked to write about their personal perception of mathematics, taking into consideration:

- the ways in which this perception has been influenced by their experiences (prior to and during the course)
- relevant reading

They are also required to discuss the possible implications for mathematics teaching and learning in the primary school.

The readings and the student assessment have been designed to form, with the taught sessions and experience in school, what is, we hope, a tightly coherent whole.

The First Three Sessions

A brief outline of the first three sessions is given below in illustration.

1. An introduction to the course: what is mathematics?

'This session focuses on attitudes to and perceptions of mathematics. Students consider their own and children's perceptions of and attitudes to mathematics and reflect upon the relationship between experience and attitudes.

The reading for the first session is an extract from the *Cockcroft Report* (Cockcroft, 1982, part one) about the nature of mathematics. This links to the focus and activities of the first session and is discussed at the beginning of the second session.

2. An introduction to counting and number: making a number game

The purpose of this session is to consider and analyse what is involved in the counting process and to examine some activities related to counting. Ordinal number is focused upon, and activities using number lines and number tracks are undertaken. Board games are then considered as a context for work on number lines and tracks. The students then design and make a number board game for use in school.

The students also discuss, in small groups, the previous week's reading and report back to the full group the issues they have identified. The reading given at the end of this session is Martin Hughes's *Children's Invention of Written Arithmetic* (Hughes, 1986, chapter 5).

3. Mathematics in the environment: mathematics trails

The context for mathematics examined during this session is that of a mathematics trail. The workshop activity is to produce a mathematics trail, to be used in school and the related reading is an article about mathematics trails from *Child Education.* The students are given examples of previously produced trails, both written and on video, which they can use to help them to produce their own mathematics trail. The trails produced are presented and discussed at a future session.

Conclusion

During the six sessions, a variety of teaching styles is employed and this is made explicit to the students. They are encouraged to reflect upon their own learning and the possible parallels with children's learning. Throughout the block it is emphasised that positive attitudes and confidence grow from success and from interaction with an enthusiastic teacher. It is further stressed that it is an intending-teacher's responsibility to be pro-active in developing such enthusiasm, and that the willingness to set aside preconceptions and undertake a positive re-appraisal of mathematics is a significant step towards professional maturity.

We hope that we are offering our students a sequence of learning situations which support them in beginning this process.

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Responses to Mathematics in Primary Initial Teacher Training

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This article arises out of concerns about specifying the mathematical content of the new six-subject BEd degree. I have used a series of small-scale surveys among students and colleagues to explore perceptions of the nature of the subject and its core content. Some of the issues raised may well be relevant to discussions on the new degree in other institutions.

Context: National Debate

Primary Initial Teacher Training institutions are at present rewriting BEd courses to come into line with Department for Education Circular 14/93. One of the many difficulties in doing this relates to the mathematics content. It is stipulated that there should be 150 hours devoted to mathematics, but this can be interpreted in different ways. The circular does not make a clear distinction between mathematics as an academic discipline and mathematics education as an aspect of curriculum studies. I want to make the distinction because there often seems to be difficulty in separating them in both lecturers' and students' minds and this can result in one being pursued to the detriment of the other. Do students need to study both and, if so, what should be the balance and the relationship between them?

The first set of issues this raises relates to the mathematics required by all primary teachers as classroom generalists. I would endorse the point made by Billington et at (1993):

"The prerequisite of a Grade C in GCSE mathematics, whilst welcome, is certainly no guarantee of the level of competence needed by primary teachers. Those who teach the subject require a grasp of the conceptual structures of mathematics they must teach, an awareness of the ways in which elementary ideas connect together, and an understanding of mathematical processes and routines, in order that they have a sound basis of personal knowledge from which to formulate their explanations to children and to plan appropriate sequences oflearning experiences."

The clear implication of this is that all primary teachers will require, to a greater or lesser extent, a confident grasp of the conceptual structure of mathematics above and beyond that implied by a grade C at GCSE.

Furthermore, what of the effect of current changes on the training of primary mathematics specialists? Here there is concern that the new six-subject BEd makes the training of specialist teachers more difficult, even though junior schools are tending to increased specialisation. The report on the teaching and learning of number in primary schools (OFSTED 1993) states that the "influence (of mathematics coordinators) was most effective where they had good knowledge of mathematics," and that 13% of schools "had difficulties in recruiting coordinators with the requisite expertise."

I have been interested for some time in the relationship between mathematics and mathematics education . As a practitioner I am aware that there is no "Chinese wall" between them in the classroom, that in teaching mathematics I touch on topics in mathematics education and vice versa. I am concerned, however, that there often seems to be confusion between them in students' minds.

In this context I am often aware that primary Initial Teacher Training students will seek to avoid mathematics that they find challenging. If given a choice, they will frequently opt for activities towards the lowest level of the relevant age range so as not to show any ignorance. Many of them seem to lack confidence in their own learning of mathematics, and to have a real fear of showing ignorance or stupidity. Similarly, many in-service primary teachers tend to be so nervous about their own mathematical competence that they will try to avoid talking about it. For both groups, discussion of curriculum issues may well seem a safer activity than exploring mathematical concepts.

Rationale for the Study

I was interested in looking at the images that trainee teachers and their lecturers have of the relationship between mathematics and mathematics education. I decided to carry out a series of smallscale surveys within my own institution, through which I hoped to explore these perceptions and the issues they raised. These were not intended to be rigorous opinion-surveys, but to highlight the issues and perceptions in the minds of participants, who were invited to comment freely on the issues raised. I have drawn on these comments in what follows.

Student Responses

My initial focus of enquiry was on the mathematics specialist students. In their responses to a questionnaire, first year BEd students who had elected to take the MCT (Mathematics, Computing and Technology) specialist cluster in their second and third years consistently interpreted mathematics as mathematics education. While they generally agreed that primary mathematics teachers should have a strong knowledge base, most of them did not expect that the specialist subject course would help their own mathematics. In general they did not have clear ideas about the mathematics they needed. Their suggestions included practical and real-life mathematics, fractions, algebra, investigations, statistics, and "basics", but no clear picture emerged as to why these were considered important.

An amended version of the questionnaire was administered to a group of second year BEd students who were currently taking the MCT specialist cluster. All of them said that the specialist subject had usefully increased their knowledge and understanding of mathematics. They felt more confident in their own mathematics as a result of the course, and also more confident to teach it . One student reported:

"One of the main reasons I chose this specialist subject was because it was the area I felt would let me down in the classroom. Although I could have taught primary maths, I couldn't have done it with much confidence, and more things 'link' now which I never thought of before. I now could teach maths with more idea of where I wanted the lesson to go and what I was looking for. I don't feel anxious now, but I did before."

Not surprisingly, there was a wide range of responses to the question, "What aspects of the course have you found most useful?" Alongside such general benefits as increased confidence, logical thinking, deeper understanding of mathematical processes and a greater familiarity with the language of mathematics, several respondents stressed the importance of discovering the links between discrete topics, using investigative methods to look at the "why" and not merely the "how" of the subject, and "learning the subject from a different perspective". Interestingly, the specific area within mathematics singled out most frequently in responses to this question was algebra.

Some respondents clearly valued the opportunity to reflect on their own learning, and implied that this would in turn increase their understanding of pupils' learning in the classroom.

Others felt the most useful aspect of the course had been their increased understanding of the National Curriculum, but in general the balance of these responses was clearly towards the greater understanding of mathematics as a discipline in its own right, rather than the curriculum aspects.

However, more than halfthe respondents felt that GCSE grade C gave a sufficient grounding in mathematics for a primary school teacher. While there was general agreement with the statement that "all primary students should study mathematics," some responses suggested that this was being interpreted as mathematics education rather than mathematics as such.

It is striking that students who seem to be having a fairly positive mathematics experience are willing to admit the need for studying mathematics but will still sometimes feel more comfortable talking about mathematics education, especially in relation to the mathematics that all primary trainees require.

Lecturer Responses

In an attempt to explore more deeply the issues raised by these perceptions I engaged in structured discussions with two groups of colleagues. The questions they were asked to consider included whether and why it was important for BEd students to study mathematics "at their own level" (i.e. beyond GCSE), what sort of mathematical content would be appropriate, and how these concerns should be reflected in the new degree course. One group, whose background was in secondary and higher level mathematics, taught mathematics to the specialist subject students. The other group, whose background was in primary schools with higher study of mathematics education, taught curriculum mathematics.

Both groups thought it important for students to study mathematics at their own level; both were aware of the need to develop the students' confidence in mathematics if they were to be able to teach it, of the importance of students' attitudes towards the subject, and of the equal opportunity issues raised.

The specialist mathematics lecturers were very conscious of the limitations of the GCSE as a qualification. They were concerned that from this background students were familiar with techniques, but often had no real sense of what mathematics was about. They

felt that the students tended to see the subject as a fragmented body of routines, and their perception of the National Curriculum reinforced this view. One lecture~ commented that mathematics is sometimes seen as just "harder and harder sums". Therefore they thought students needed to be given an overall perspective showing how topics fit together and develop. They needed to be able to generalise in order to get a sense of the underlying concepts, and to study topics that gave them a sense of this underlying pattern. Real life applications were important, as long as this was understood as working towards a mathematical understanding of the world, and not in a narrow or self-limiting sense. Their conclusion seemed to be that students needed to study subjects that would develop a wider vision of mathematics and not be topic bound. Relevant areas were thought to include algebra, functions and graphs.

The curriculum lecturers were less concerned about the limitations of the GCSE. They also thought that mathematical processes were important and laid great stress on algebra and real life applications, but they tended much more to emphasise the cross-curricular, historical and creative aspects of mathematics. They felt that the mathematical content should be based on investigation and problem solving, and should to a large extent be driven by students' own identification of their interests. It should include consideration of recent changes in the mathematics curriculum, and of issues arising from this. At the same time, a very clear emphasis on the centrality and importance of algebra was evident, coupled with a recognition that this was one of the areas where students showed greatest weakness and difficulty. In this context, developing confidence and a positive attitude to mathematics was seen as being at the heart of the specialist study of the subject.

Conclusion

Compared to the students, both groups of lecturers laid stress on students studying and developing their understanding of mathematics, whereas the students, particularly those in the first year, tended to lack confidence in their mathematics and to be much more willing to talk about mathematics education.

Both sets of lecturers argued strongly for specialist students studying mathematics but there was a difference in emphasis. The curriculum lecturers wanted the higher level study to some extent to resemble the primary curriculum with the emphasis on cross-curricular links, whereas the specialist subject lecturers emphasised connections within mathematics.

I intend next to look into the views on the mathematics content of the BEd firstly of non specialist students and then of lecturers who teach other subjects.

It may seem fairly obvious that teachers should have a good grasp of mathematics in order to teach it, but what this means in practice is open to interpretation. We are in the process of writing a new degree, which has to encompass both subject knowledge and application. It remains to be seen whether this is best done separately or by integrating them. One area of concern is that, if they are integrated, the pressure of both students' and lecturers' perceptions of the subject will lead to a tendency for the mathematical content to be squeezed out, and the mathematical level to be forced downward. If they are to be taught separately, how should we arrive at a consensus on the mathematical content, and which sets of views should be given more weight? We will still need to resolve issues about the nature and interrelation of these areas.

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Secondary Mathematics PGCE Students and their Mentors

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This paper summarises the initial results of a local survey of (i) secondary PGCE mathematics and two year PGCE conversion course (mathematics) students, who were asked to comment on their mentors in terms of "the good things that your mentor did, " and "areas where you would like to see things improved, " and (ii) their mentors replies to the question "what makes a good student?"

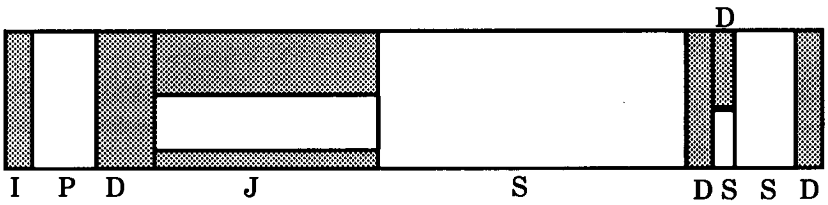
Background

Much of the literature currently available on mentoring looks at the role and responsibilities of the mentor, and suggests ways in which mentor and student might work together in school. However, in most cases the role of the mentor is considered from the perspective of the mentor. There is much less information available from the viewpoint of the student teacher. It is likely that such information is available in many institutions hidden in course evaluations, however much of it will not be clearly identifiable.

It was therefore decided to collect information from mathematics students which would be seen as most helpful and supportive to the PGCE partnership, and also to collect something similar from the mentors. The intention was to use this information to help inform the work of the partnership, by circulating it to all the parties concerned. This would let mentors see which of their practices had been considered helpful by the students, thereby providing all tutors and mentors with the opportunities to consider their own practice. It also allowed all mentors to see the variety of activities (assumed successful) undertaken by other mentors.

From the students' viewpoint, questioning the mentors, would identify those areas which were considered important by their mentors. These areas, often unstated, are likely to influence the ways in which performance is evaluated and assessed, particularly since students often assume that they are expected to conform to departmental norms. It was therefore considered important not only to identify these areas, but also to make them explicit to new students.

The secondary PGCE course has one main school experience combined with two shorter experiences at different schools. The course begins with an induction week (I) which is followed by three weeks in a primary school (P); three department weeks (D) in the University; nine joint (J) weeks (2 days school, 3 days department); 14 school weeks (8); 8 department days; 17 school days and one department week. The course provides a long experience in one main partnership school, with two other shorter school experiences (one of which is usually in a feeder primary school).



The two year conversion course (mathematics) has the same pattern, but there is a one year break for mathematics enhancement at the end of the 'J' weeks. There are slight variations in the timing of department work and an additional three week school practice in the summer of the first year, otherwise the pattern is exactly the same as for the one year course.

The Students' Views

At the first University session after the end of the main school practice in May 1994, the 1993-4 secondary PGCE mathematics and the 1992-4 two year PGCE conversion course (mathematics) students were asked to write down their comments on their mentors in terms of 'the good things that your mentor did', and 'areas where you would like to see things improved'. Both groups of students were together for this session. Comments were written down, initially without discussion with other students. The replies were free response items, so no specific headings were given to students. The names of students were attached to their own comments, so students were aware that their comments would be identifiable. These comments were then discussed in small groups, before being collected at the end of the session. This did allow students time to reflect on their comments and add (or subtract) things after the discussion.

The good things

The comments below, which refer to “good things”, in the student teachers' views, have been grouped into a number of broad areas, which appear to arise "naturally" from the comments. Unless stated otherwise, each individual made only one comment in each area. The groupings have been classified in order of importance, determined by the number of responses. All 9 two year students and 18 of the 22 one year students were present and they "represented" 18 different schools. A total of 118 comments were made, with a minimum of two and a maximum of seven from the students (this was after the comments had been put into broad categories).

The importance given by students, as determined by the number of students making comments is indicated in rank order as follows, with the most important area given first:

- advice and encouragement
- development and support
- classroom management
- meetings with mentors
- subject knowledge
- teaching methods and teaching strategies
- planning and preparation
- observation
- application and interview
- information technology

Advice and encouragement

A total of 16 students (59%) made comments. Six of them welcomed the use of positive comments or constructive criticisms, including one on the use of review sheets. Four students liked the encouragement given, with two comments that the mentor was eager to listen. Four students liked the advice which was described as either general, or open, or useful, or always ready. The final comment welcomed that advice was only given when asked.

Development and Support

There were 15 (56%) students who made comments in this area. Eight of these mentioned the freedom or flexibility given, with two others commenting "supportive". One student noted the mentor's high expectation, honesty and frankness and another felt that training was important to mentor. The other three comments were opportunities to discuss needs; provision of simple equipment; and ensuring feedback on progress from all teachers.

Classroom management

A total of 14 (52%) students made comments. Of these, eleven referred either to support and help (5 in total) or advice with regard to difficult pupils and/or classes (the other six) with five of the eleven using the general term "discipline". Two of the others noted the helpfulness of constructive criticism on classroom management and the final student teacher remarked on suggestions for room layout to support children's learning.

Meetings with mentors

Twelve student teachers (44%) made 14 statements about meetings with mentors. Eight of the students commented on the availability of the mentor, which included mentors who were always available (two); mentors who always made time (three); a mentor happy to stay behind; and mentors who would find time if needed (two). Four specifically mentioned weekly meetings, though one was only "if needed", and two named Monday. Commenting additionally on the work ofthe mentor, examination of the teaching practice file was liked by two - one on a weekly, the other on a regular basis.

Subject knowledge

A total of eleven students (41%) commented. Five comments were related to help with resources and four to help with ideas. One student noted specific support for work in Mathematics Attainment Target 1 and one noted advice on appropriate level of work for classes.

Teaching methods and teaching strategies

Eleven (41%) of the students made comments. Four students mentioned discussion of alternative methods while four mentioned suggestions of other methods. Ofthese eight replies two referred to pre-lesson and three to post-lesson comments, with one mentioning how to get the best from a group. Two of the others commented on the freedom to experiment and the other, in a similar vein, on encouragement to develop one's own style.

Planning and preparation

There were comments from ten students (37%). Three of these highlighted the element of choice about topics to teach. Two were concerned with advice on structure - one of the file the other of lesson plans. One student listed the comments on the teaching file, whereas the other four concerned advice about plans, with one of these noting the comments on possible problem areas.

ObservatIon

One of the comments received from the eight students (30%) welcomed the lack of pressure on observation and examination of the file. Six of the others found the supportive comments received after lesson observation a good feature, with one of these highlighting the helpful written analysis. The final student appreciated the mentor understanding that observed lessons were different.

Application and interview

Of the four replies (15%) received from the students, two welcomed the interView advice and one the help with job applications. The other response concerned general discussions about employment and teaching.

Information Technology

Two students (7%) in different schools commented on how their mentor had helped enable them to undertake an information technology programme in mathematics.

Other comments

The 13 other comments received mostly related to: social involvement at a personal (two) or school level (one); school atmosphere (one); and integration within school/department (four). The other five commented on: workload; ability of mentor to solve problems; the wide experience of the mentor; the awareness of needs; and co-ordination.

Areas where you would like to see things improved

A total of 13 student-teachers supplied twenty comments. Six of them wanted regular timetabled meetings, though two of these (both in the same school) did not want to see such sessions regularly cancelled due to cover. One student would have liked greater contact with the mentor, which had been reduced due to the mentor's other school responsibilities. It was suggested that a split school practice should include a mentor in each school. Three students would have preferred more observation, two of these wanting it throughout the practice. Two of the students wanted practice lessons, with one of them requiring examination of lesson plans in advance. One student wanted the mentor to be better organised, another had wanted more contact with the named mentor and another wanted topics to be suggested, rather than complete freedom of choice. Two students (both from the same school) wanted a mark book. The other three comments, on subject issues were more discussion of teaching strategies, more information on how to write schemes of work, and help with resources.

The Mentors' Views

At the mentor meeting in June 1994, mentors were asked to write down, without discussion, five things under the heading of "what makes a good student teacher?". These comments were discussed in small groups, followed by a general discussion. It was evident that there was a consensus on the factors which make a good student, with many of the written comments containing similar views. There were 14 mentors at the meeting, and replies were later collected from three other mentors, representing 70% of the total, accounting for about 70% of the students.

All the statements were then read and classified into one of seven 'natural' groups which came from the data. As is often the case with qualitative data, it was necessary at this stage to rearrange, combine or separate some of the statements, resulting in 67 new statements (instead of the original 85).

Although it was intended that the comments on a good student should be general statements, many mentors were, quite naturally, influenced by the memories of their most recent student(s), with about 20% of the comments arising like this. In most cases, it is the shortcomings of those students that are evident. It is also impossible to determine which factors might be assumed as a prerequisite by some mentors, until a student might lead them to think otherwise. As a result there could be some even more important, possibly basic, features that have not been mentioned because of certain assumptions made by individual mentors.

The classification of statements resulted in the following groups,

ranked in order of the number of statements made:

- personality/personal skills
- relationships
- advice
- classroom management
- planning and preparation
- teaching strategies and methods
- subject knowledge

Personality/Personal Skills

A total of 15 mentors (88%) made comments of this kind, with eight mentioning enthusiasm or interest. Three noted enthusiasm in general terms; two enthusiasm for the subject; one for teaching the subject; and two for teaching and the subject. Some comments referred to more specific personal skills and qualities including patience; good communication skills; good attendance record; willingness to discuss teaching; not being afraid to admit when things go wrong; realisation that teaching is difficult; being willing to participate in lessons; having good body language; the ability to raise one's voice; having a sense of humour; having endless energy and commitment; being well organised; allowing time to reflect (two comments); the ability to cope with the unexpected; recognising weakness and being able to work on it; having confidence to deviate from plans; able to respond in different ways, and being reliable. **In** some ways this section might be seen as a "catch-all" for information not classifiable in other ways.

Relationships

There were 12 mentors (71%) who made comments in this category. Five of the them considered that establishing good relationships with both pupils and staff (or the department) was a feature of a good student, with three others mentioning this linked only with pupils, and two others just with staff. The willingness to become involved and volunteer time to the department was mentioned by four mentors.

Advice

In terms of advice, ten of the mentors (59%) thought that a good student should listen to advice, and six of these mentioned students acting on such advice.

Classroom management

Of the nine mentors (53%) commenting about classroom management, four specifically mentioned good organisation and/or management, of these two provided further elaboration (expect high standards and finish lessons carefully). Other comments were more specific with some being seen as related to the most recent students, for example, good discipline, know names of pupils, be firm (two comments), create a good learning environment and involve children by careful questioning.

Planning and preparation

Seven of the comments received from the nine mentors (53%), mentioned that a good student should have good lesson preparation skills. The other two mentors wanted students to know what is expected of them in planning terms, and to be prepared to research their own material.

Teaching methods and strategies

All seven mentors (41%) thought that a good student should be flexible, willing to learn and try out different approaches to lessons.

Subject knowledge

Four mentors (24%) expected good subject knowledge. These were probably influenced by gaps which became evident in their most recent student(s).

Conclusions

I t is to be expected that students' main concerns should be centred on themselves, and how they might see the role of mentor (as an advisor), hence the importance of advice and encouragement given, followed by features of development and support offered. The importance of classroom management is recognised, as it is the main area where advice or support might have been given. Combining the "good things", together with "areas of improvement", suggests that regular meetings with mentors are considered important; it is not sufficient just to have a mentor available.

In terms of mentors' views there appears to be much more agreement about which things are important, and here there was little emphasis on mathematical skills or competence. Personal skills and the ability to form relationships was seen as much more important, as too was the willingness to accept advice. It could be that these facets are considered as "immovable", and therefore necessary, whereas others, although desirable, might be developed through the process of training.

These are no more than initial, brief thoughts based on a small sample of students and mentors. As such, they might be of interest to others. I hope to repeat the exercise at the end of this year. I would welcome comments.

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Mathematics Education in the Mari-El Republic of the Russian Federation

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During the first half of August 1993, *before the troubles which occurred in Moscow during that Autumn, I had the opportunity to visit the Mari-EI Republic of the Russian Federation. The trip was organised in conjunction with an English Language INSET Summer School for teachers and lecturers of that region, coordinated by my College, the Yoshkar-Ola Pedagogical Institute and held at the Mari Institute of Education. This provided me with* a *chance to visit some educational establishments and to compare the local system with that of the UK I also gave* a *lecture on the National Curriculum for England and Wales to people attending the Summer School. In November* 1993 *my own institution was visited by some secondary teachers from school number* 61 *in Moscow. This enabled me to discuss my experiences from the Mari -EI with educationalists from* a *different part of the Russian Federation. As might be expected from the strict state control of the Soviet/Russian educational system up to now, many of my observations from the Summer seem also to apply to the whole of Russia, and probably to many of the other republics of the former Union of Soviet Socialist Republics.*

Yoshkar-Ola and the Mari-El Republic

The old Soviet Union (USSR) comprised 15 full republics, or SSRs such as the Russian Federation, the Ukraine, Lithuania and Uzbekistan. Contained within some of these were "Autonomous Republics", or ASSRs, such as Chechnya, Dagestan, Tatarstan, and the Mari-El, with rather less status. The SSRs reflected the distribution of the major distinct nationalities, linguistic and ethic groupings of the Soviet peoples, and the ASSRs provided homelands for the minor nationalities. In addition, there were also "autonomous regions" and "national areas" (with less autonomy than the ASSRs). Many of these ASSRs , autonomous regions and national areas were contained within the Russian Federated SSR (now the Russian Federation). Many of these have now acquired

the status of "Republic within the Russian Federation", although some (such as Chechnya) would like full independence.

Yoshkar-Ola (literally "Red City" in the local language) is a city of some 280 000 inhabitants and capital of the Mari-EI Republic, situated in the Central Volga Region of the Russian Federation, some 650 km east of Moscow. Although founded in 1584 by Tsar Ivan the Terrible and called Tsarevakaksheis until the revolutions of 1917, Yoshkar-Ola is a mainly modern city, now on both banks of the Lesser Kakshava river (a tributary of the Volga), having developed rapidly during the Second World War (when many factories in the western USSR were moved east to be saved from the Nazi invasion) and thereafter. It is a sizeable industrial centre, manufacturing such items as bicycles and electrical goods - notably radios and refrigerators. The local industrial economy is somewhat in recession, partly due to a decline in demand for military electronics, but local agriculture and forestry continue to thrive.

The Mari-EI Republic, bordered by Tatarstan to the south-east and the Chuvash Republic to the south-west, has a population of around 780 000 - mainly a mixture of ethnic Russians (around 47%) and native Mari people (43%). The Mari are a Finno-Ugric race, and have their own language (closely related to Finnish, Hungarian and Estonian) and culture. Much of the region is covered by forest (largely pine and birch), but there is also much cereal and livestock farming. Until around four years ago, the republic was used for internal exile, and was closed to foreigners (in a similar manner to the city of Gorkiy, now Nizhni-Novgorod). Thus, the presence of western visitors proved something of a novelty to the local inhabitants.

School Education

During my stay, I was able to visit secondary school number 18 in Y oshkar-Ola. This is part of an experimental complex also

including a primary school. The secondary school is designated a centre of excellence for Art, Music, Dance and Sports. [In Russian towns, it is common for pupils to spend the whole of their 10 years of compulsory education in the same school complex, or "ten year school". Smaller villages, however, may only have a primary, or "eight year" primary plus junior secondary, school.] It has a total of 1200 pupils, making it the largest school in the Mari Republic. The school is mainly intended for children living in the immediate neighbourhood, but pupils from other districts of the city can attend if they succeed in an entrance exam.

The old school building - still in use, but not included in the official guided tour - was rather dingy (like older schools in the UK) and sported a notice-board still decorated with the emblems of the former "Young Pioneer" and Comsomol (Young Communist League) movements, along with photographs of "model pupils" in their Pioneer uniforms. Not a lot of change here, it seemed! However, the new buildings, already in use despite not yet being fully fitted-out, were very impressive. We were informed that many pupils had participated in the latter stages of fitting-out and decorating classrooms, and that this had given them a true sense of "ownership" of the school, with a greatly beneficial outcome. There were impressive dance studios and a concert hall, and wellequipped science laboratories and language rooms. The school has also recently acquired a classroom of IBM286 personal computers, and "Informatics" (computer studies) is offered as an optional subject to senior pupils.

This school has clearly received a lot of investment recently. I spoke to several teachers from other schools, who welcomed the experiment of a more modern curriculum offered at school number 18. However, they felt that this school was being given priority for resources much-needed by other, less glamorous, schools.

In the Russian Federation, children can attend kindergarten (or nursery school) from age 2, but they do not start primary school until 6 or, more usually, 7. The final year at kindergarten acts as a "school preparation year" where pupils are introduced to the basics of reading, writing and counting. Three years at primary school, then seven years secondary schooling follow, so that most students complete their compulsory education at 17.

Russia has a "National Curriculum" - still rather rigid and very much based on the old Soviet system. The last major reforms, increasing the duration of compulsory education from 8 to 10 years, were introduced around 1964 and fully implemented by about 1970. However, there is now considerable pressure to make it more flexible. At present, pupils follow a broad curriculum up to age 15, including Russian, History, Geography, Mathematics and, normally from age 10 or 11, a foreign language and Science. Most secondary schools teach separate Physics, Chemistry and Biology throughout. However, as part of its experimental curriculum, school number 18 in Yoshkar-Ola offers General Science (2 lessons per week initially), with specialist Physics, Chemistry and Biology only for those aged 15 and over. In their last two years of compulsory education, students can bias their studies towards their own particular interests, either continuing at school, or transferring to a college specialising in vocational training.

The number of hours spent on each subject, year by year, in a normal school teaching in Russian, are shown in Table 1 below. In many former ASSRs, pupils may instead attend a school where the teaching medium is the local language. In such cases, the curriculum is similar to that of the "Russian language schools", but some time is used studying the local language and literature, as well as Russian. However, in the Mari-El (which has a rather small population of Mari speakers) only primary education is available in the Mari language. This is largely due to the lack of availability of (and wider demand for) textbooks in Mari. In some towns there are also schools where the teaching medium is a foreign language, such as English.

Table 1

Year number :	1	2	3	4	5	6	7	8	9	10
Hours per week										
Mathematics	6	6	6	6	6	6	6	6	6	5
Russian Language	12	10	10	7	5	3	3	2	-	-
Russian Literature	-	-	-	3	2	2	2	3	3	3
Foreign Language	-	-	-	-	5	4	4	2	2	2
Sciences (Nature)	(1)	(2)	(2)	(2)	2	4	6	7	8	10
Geography	-	-	-	-	2	3	2	2	2	-
History	-	-	-	2	2	2	2	3	5	3
Sociology	-	-	-	-	-	-	-	1	-	2
Art	1	1	1	1	1	1	1	-	-	-
Music	1	1	1	1	1	1	1	1	-	-
Technical Drawing	-	-	-	-	-	1	1	1	-	-
Physical Education	2	2	2	2	2	2	2	2	2	2
Work Training	2	2	2	3	3	3	2	2	4	4
Total Compulsory Studies	**24**	**24**	**24**	**27**	**31**	**32**	**32**	**32**	**32**	**32**
Optional Studies	-	-	-	2	2	2	2	4	4	4

Most pupils take the state-organised final school leaving examinations at 17. Partly because of shortages in printing and duplicating facilities (photocopiers are rare in the provinces, and carbon-paper is still commonplace), a very high proportion of these exams - even in Science subjects - are oral. Candidates are given a set of questions and allowed about an hour to prepare their responses. They are then tested by a board of examiners, some of whom will be from the student's own school, others external. Students who are successful in these exa inations can proceed directly to higher education, although many institutions also require a satisfactory performance in their own entrance exam. Furthermore, some young men choose to first fulfil their obligation to complete 18 months military service. However, there is a possibility that this requirement may be removed in the near future, so many now defer going into the armed forces until after completing their studies.

Initial Teacher Training

The Mari-EI Republic has three institutions of higher education, all in the capital, Yoshkar-Ola. The newest of these is the Mari University, which has strengths in Law and Finno-Ugric Studies (for which it is an internationally recognised centre). The Poly technical Institute was originally founded as an external college of Kazan University, and specialises in Sciences, Engineering, Economics and Forestry. The oldest of the three is the Pedagogical Institute, or teacher training college, educating both primary and secondary student teachers with a wide range of subject specialisms. A very high proportion of Higher Education students at these three institutions are from within the Mari-El. In addition, the Mari Institute of Education offers INSET courses for practising teachers and is the local agency for Moscow's new Open University. I was fortunate to be able to visit the Faculty of Physics and Mathematics at the Pedagogical Institute during my stay, and to discuss the initial teacher training (ITT) programmes with some of its staff.

The Faculty of Physics and Mathematics offers a five-year programme leading to a "Teacher's Diploma" (roughly equivalent to a BEd or BA with QTS in the UK) with three types of specialism:

Mathematics with Physics, Physics with Mathematics and Physics with Informatics (Computer Science). Each of these pathways has an intake of 25 students each year. These are quotas set by the Government and, at present, there is no pressure to increase student numbers in this faculty. Indeed, staff stated that it was now difficult to recruit good students - partly because school teaching has a low social status and is poorly paid (junior teachers in the region currently receive around £40 per month). As a consequence, fewer than 50% of some intakes successfully complete the five year course. The Teacher's Diploma is the usual method of entry into school teaching in Russia - there is no equivalent of our PGCE - and it is really quite difficult for a graduate in, say, Mathematics (without any "pedagogical" content in the course) to become a secondary school teacher.

The Pedagogical Institute in Yoshkar-Ola does not, at present, offer higher degrees. In order to improve their qualifications, teachers of some years experience [Post-graduate students in most other disciplines have also usually worked for several years between completing their "diploma" (or first degree) and returning to their studies.] may go to an institution specialising in postgraduate study, usually outside the Mari-El. Normally, the first post-graduate qualification for teachers is the degree of Candidate of Pedagogic Sciences (roughly equivalent to a UK PhD) [The Doctorate degree in the former Soviet Union is quite rare, being comparable to the higher doctorates (e.g. D.Sc. or D.Litt.) in the UK], which takes at least 3 years of full-time study, or about 5 years part-time, and includes a substantial research project.

The Faculty is comprised of three Departments: Physics (19 lecturers, including the current Dean, and 8 technicians), Mathematical Analysis (8 lecturers and 1 technician), and Algebra & Geometry (12 lecturers and 1 technician). "Informatics" courses are mainly taught by staff from the Department of Algebra & Geometry. Despite what might appear to be a high staff to student ratio, contact teaching loads are quite heavy. Senior Lecturers may be required to teach up to 600 hours per year (up to 18 hours in anyone week), with Assistant Lecturers doing up to 750 hours per year (maximum about 22 in one week). Nevertheless, the academic staff do manage to take on industrial consultancy work and pursue their research interests which include Mathematics Education, Partial Differential Equations, Probability Theory, Topology and Abstract Algebra. The Physics Department also has a thriving research program. The Faculty has two computer classrooms, each with twelve Yamaha microcomputers, plus several IBM PCs (or Russian copies) which are mainly for staff use.

Each of the five years of the Diploma programme is divided into two semesters, of about 20 weeks each. Internal examinations are held at the end of each semester, but only the final examination is externally assessed - it is set by the state and, like the School Leaving Certificate, consists largely of oral exams. Teaching practices are assessed both by one in-school and one college-based tutor, although on occasions an external inspector may also visit, just as is the case in the UK.

Many aspects of the programme are similar to tose at my own institution. However, the course in Yoshkar-Ola seems to be considerably more academic - particularly in the early part of the programme. Whereas, in the UK, BA(QTS) or BEd students typically have two weeks of observation in schools before even starting their course, a short 3 week teaching practice in their first year and one of 6 weeks in their second, the Russian trainee teachers only have 4 single weeks of "school observation" in their first three years, followed by a four-week block at the start of the Summer between the third and fourth years of the course. They only have two long teaching practices. The first, in semester 7 (year 4), lasts 5 weeks and is carried out at junior secondary level (with pupils aged between 10 and 14), covering both the subject areas being studied by the trainee teacher. The final practice, in semester 9 (year 5), is for 6 weeks covering the senior years (ages 14 and above) of secondary school in the trainee's main academic subject only. This contrasts with 6 weeks and 7 weeks (covering both of the student's subject areas) in years 3 and 4 respectively of the course offered by my own college. Furthermore, Mathematics students in Yoshkar-Ola are required to complete courses in advanced topics, such as differential and abstract axiomatic geometry and measure theory, which I would not dream of teaching to my students! .

Practical laboratory work forms a substantial component of the academic physics programme: students doing physics as their minor subject have about 40 hours of experiments per semester, whereas those majoring in physics have 50 hours per semester, plus a special practical project in their final year.

An outline of the academic and pedagogical courses taken by students in the Faculty of Physics and Mathematics is given in the appendix below. Broadly speaking, these are similar in content to those which would be found in many Mathematics and Physics degree programmes in the UK. However, I believe that the Russian courses tend to be of a more abstract and/or academic nature, both in content and difficulty, than would be found in typical BEd or BA(QTS) schemes in this country. The Russian educational studies (or "pedagogical") courses are also of a rather more formal & theoretical type than those typical in the UK, with educational theory, psychology and "didactic studies" being prominent.

Conclusions

It can be seen that many aspects of both secondary education and ITT are similar in the Russian Federation and the UK. However, the Russian programmes are, at present, more rigidly defined, restricted by state examinations and, at 11'1' level, are rather more academic than their UK equivalents. Nevertheless, unlike the UK, the trend currently predominant in Russia seems in favour of making education less formal and more flexible. Unlike recent Government policy in the UK, there does not yet appear to be any move to make Russian teacher training mainly based in school.

Further Reading

A highly informative and readable account of how the system was during the Soviet period can be found in "Soviet Education" by Nigel Grant (revised edition, Pelican Books, 1968). Remarkably little seems to have changed in the course of over 25 years!

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APPENDIX

Compulsory and Optional Courses Offered by the Faculty of Physics & Mathematics, Yoshkar-Ola Pedagogical Institute

The basic structure of the courses taken by students following each of the three pathways is the same. All follow a broad range of Physics, Mathematics & Computing foundation courses in the early semesters, then bias their studies towards their main subject later on. The students which graduated in 1993 had also had to take courses in Economic and Political Theory (from a Marxist-Leninist viewpoint) in earlier years, but such courses are becoming less emphasised than in the past. The scheme shown below is that for students majoring in Mathematics.

Semester 1 (Year 1)	Mathematical Analysis (Theory of Limits)	(126 hours)
	Algebra (Matrices & Systems of Equations)	(72 hours)
	Geometry (Vectors, Euclidean Plane)	(72 hours)
	Pedagogical Studies	(45 hours)
	Psychology	(27 hours)
	Foreign Language	
Semester 2 (Year 1)	General Physics	(100 hours)
	Mathematical Analysis (Applied Calculus)	(90 hours)
	Algebra (Congruences, Diophantine Equations)	(90 hours)
	Geometry (Euclidean Three-Dimensional)	(90 hours)
	Computer Programming (in PASCAL)	(36 hours)
	Psychology	(54 hours)
	Foreign Language	
Semester 3 (Year 2)	Mechanics	(100 hours)
	Analysis (Functions of Several Variables)	(72 hours)
	Algebra (Polynomials)	(72 hours)
	Geometry (Projective & Representational)	(54 hours)
	Computing	(72 hours)
	Pedagogical Studies	(36 hours)
	Psychology	(36 hours)
	Foreign Language	
Semester 4 (Year 2)	Molecular Physics & Thermodynamics	(100 hours)
	Materials Technology	(90 hours)
	Analysis (Multiple Integrals with Applications)	(90 hours)
	Algebra (Groups, Rings & Fields)	(54 hours)
	Geometry (Foundations & Axiomatic Systems)	(72 hours)
	Computing	(36 hours)
	Pedagogical Studies	(36 hours)
	Didactic Studies	
	Foreign Language	
Semester 5 (Year 3)	Electricity & Magnetism	(100 hours)
	Theoretical Mechanics & Special Relativity	(50 hours)
	Analysis (Ordinary Differential Equations)	(68 hours)
	Analysis (Measure Theory)	

	Algebra (Linear Algebra & Eigen problems)	(68 hours)
	Numerical Methods & Computing	(68 hours)
	Methods of Teaching Physics	(72 hours)
	Educational Psychology	(36 hours)
Semester 6 (Year 3)	Probability & Mathematical Statistics	(63 hours)
	Geometry (Differential Geometry)	(68 hours)
	Analysis (Complex Variables)	(36 hours)
	Electrodynamics & Relativity	(50 hours)
	Methods of Teaching Physics	(72 hours)
	Teaching Practice Preparation Workshop	(26 hours)
	Pedagogy	(36 hours)
Semester 7 (Year 4)	Mathematical Logic & Algorithm Theory	(44 hours)
	Quantum Physics	(100 hours)
	Methods of Teaching Mathematics	(44 hours)
	Methods of Teaching Computing	(30 hours)
	Teaching Practice	(5 weeks)
Semester 8 (Year 4)	Statistical Physics & Thermodynamics	(50 hours)
	History of Maths & Physics	(40 hours)
	Methods of Teaching Maths/Computing	(72 hours)
	Pedagogy	
Semester 9 (Year 5)	Nuclear & Elementary Particle Physics	(40 hours)
	Specialist Option (Mathematical or Educational)	(40 hours)
	Mathematics Education Workshop	(48 hours)
	Teaching Practice	(6 weeks)
Semester 10 (Year 5)	Axiomatic Number Systems	(36 hours)
	Mathematics Education Workshop	(36 hours)
	Specialist Option	(70 hours)

The specialist options in Mathematics are only taken by students for whom Mathematics is their main subject, and are chosen from Partial Differential Equations, Topology, Abstract Algebra, Modern Methods of Teaching Mathematics. Students majoring in Physics have a similar range of specialist Physics options available.

Note: The statements of how many hours are devoted to each course are as told me by Faculty staff members. These make the students' total semester lecture time seem unfeasibly high. However, it is certainly true that Russian students have much heavier lecture loads than most UK undergraduates.

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The Use of Computer Algebra Systems in Initial Teacher Education

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This paper discusses the use of Computer Algebra Systems (CAS) in initial teacher education on a primary undergraduate degree. To avoid a piecemeal approach we have tried to identify principles which will guide decisions about the effective use of such software. It is shown that students need to develop a questioning approach to the use of CAS and not blindly accept results if understanding is to take place. The concept of validating results is shown to be crucial.

Introduction

With the development of powerful Computer Algebra Systems (CAS) and greater opportunities for their use there is a need to consider the most appropriate way of planning and integrating the technology into the curriculum. Some investigative work on one such

package, DERIVE (Version 1) [Please note that the comments on Derive refer to Version 1 which has since been superseded by Version 2.], involving the analysis ofa problem involving x tan (l/x), initiated a discussion about the best ways of using Derive and similar packages with our students. It was felt that if an ad-hoc, piecemeal approach was to be avoided then a deeper understanding of the issues involved in using such software was required. Accordingly this discussion paper is our first attempt to identify suitable strategies for the use of computer algebra programmes such as Derive.

In our experience the students following the BA(Ed) course who specialise in mathematics have a varied background. While most will have recently followed an A level course and obtained grades throughout the A to E range, others will be mature students with relatively limited recent mathematical experience. The teaching styles adopted try to reflect this variation by the use of several approaches including exposition, small group and individual work, lectures workshops and tutorials. Derive is also used as an analytical and exploratory tool, but in a relatively limited, unstructured way at present. Our discussion concerning the best use of Derive focused initially on solving particular problems, but quickly developed to wider questions. We felt that some overall principles needed to be established otherwise any further use of Derive or similar software would lack coherence.

Williamson (1992) states that: "In the UK, it [Derive] raises questions about the Sixth Form mathematics curriculum similar to those raised by electronic calculators." He goes on to state that by shifting the emphasis from technique Derive enables the focus of tuition to be placed upon mathematical reasoning. Whether one accepts this view or not there are many issues related to using such software. Clearly such powerful systems must influence the teaching of mathematics, especially with the advent of multimedia computing, but exactly how is open to debate. For example, in the school context, Kershaw et al (1994) refer to software packages which will "introduce children to new mathematical ideas at an earlier age and make obsolete many elements of current curriculum". Similar arguments would appear to apply at an undergraduate level and the exact use of such software to enhance learning needs consideration. French(1994) refers to the need to "be clear what understanding and skills are necessary to use the tool [CAS] intelligently and how they can be developed".

Within the BA(Ed) context the following questions were raised during our initial discussions:

- How should the content ofthe curriculum change, if at all, given that a package such as Derive can be used to carry out much or all of the technicalities of algebraic manipulation?
- How could Derive be used best within our present mathematics course structure?
- Should the structure of the course be based more closely on software such as Derive, possibly being built around it rather than incorporating it into the present system?
- What is the best balance of the teaching of mathematical techniques and reasoning at this level?
- What account should be taken and how, of any previous

experience the students may have of using such software?

These are far reaching questions leading to the central issue of what constitutes mathematics and how it is best learnt! They cannot be answered simply, but the following is a first attempt within the context of the BA(Ed) degree.

Parallels With the Use of Calculators

As quoted above from Williamson (1992), we could possibly draw on the experience of using calculators to inform our use of Derive. Shuard et al (1991) reported in the PriME project that calculators can be used:

- to check mental calculations
- for calculations too complex for children to do in their heads
- as a resource for generating and developing mathematical ideas and processes
- to explore the calculator's keys and operations

Their use removes the drudge of doing many calculations by hand, and allows a focus on problem solving and concept building. However as Shuard (1986) states the calculator can only be useful "carrying out the calculation". Formulating the calculation and interpreting the results "call on further mathematical processes and skills". Formulating requires the making of decisions on what data to use and collect, paying attention to units, estimating results and planning calculations. Interpreting requires checking, deciding on accuracy, paying attention to units and stating the solution.

In a similar way Derive could be used:

- to check calculations and algebraic manipulations
- to carry out complex manipulations beyond the paper and pencil ability of the user
- as a resource to generate mathematics through the exploration offunctions, pattern recognition, standard integrals, etc

Williamson (1992) refers to four themes in using Derive:

1. As an algebraic calculator to aid reasoning.
2. To increase mathematical modelling power and problem-solving toolkit of students.
3. To help develop insight through exploratory approaches.
4. As a means of conveying new mathematical issues through its mode of operation.

Derive could also be used as a check mechanism and so the themes parallel the calculator usage listed above. These themes could form a basis in developing a more informed approach to using Derive. However this comparison ought not to be identical since, as Morris (1994) states, "if the number of pupils at the end of Key Stage 2 who are learning to do long division and long multiplication by the standard algorithm is replicated post-16 we are in for several frustrating and wasted years of teaching A level!" This argument can be extended to undergraduate work.

One of the necessary requirements for intelligent use of calculators is the need to estimate results and not rely blindly on the given results. Sparrow et al (994) believe that many children do not in fact use estimation skills. Drijvers (1992) also considers that teachers can " tend to jump to abstractions that are made too fast while the handwork phase of simple examples is skipped." He carries on to say that use of the software may therefore be too powerful for students, and not aid understanding or reasoning.

In the next section an example is discussed in order to determine how the above principles can be used in practice, as well as developing some further principles in the use of CAS.

The Function x tan(1/x)

A CAS program such as Derive can be very useful to illustrate why some theory is true. For example, a standard application of the sandwich theorem for limits of functions is to show that the function x sinO/x) has a removable singularity at x = 0. Derive will not only confirm algebraically that the limit of this function as x tends to zero is zero but can also be used to plot the functions graph and demonstrate how it is sandwiched by the functions 'plus or minus the modulus of x'. A natural extension to this problem is to explore the functions x cos(l/x) and x tan(l/x) and to use Derive for the investigation. To our surprise Derive (Version 1) gave the following result: *limit (as x tends to 0)* x tan (l/x) = 0.

We decided to ask Derive (Version 1) to plot this function and to examine closely what happens near the origin. Figure 1 suggests that x tan(l/x) is sandwiched by the functions 'plus or minus the modulus of x'. However, analysis of this function reveals that for x = 2/[(2n + *1)1t],* for n any integer, x tan(l/x) has an infinite singularity. Clearly Derive has made an error! We decided to see what other software would produce. Figures 2 and 3 show a plot of the function produced by Supergraph on the BBC and Excel close to the origin, both of which are in keeping with our expectations.

Our realisation that Derive (Version 1) was actually incorrect meant that we had, at least subconsciously, questioned the results, drawing on our experience. The further exploration then allowed us to be confident about the behaviour of the function. This process of what was essentially validation, parallels the need for estimation when using calculators. A questioning approach is required when using CAS, as for other mathematical work, but any handwork or validation relies on insights gained through previous mathematical experiences. We wondered whether our students would question the use of Derive in this context and indeed make some attempt to validate their results. Accordingly, we set up an activity with our third year group.

Figure 1: *xlan(llx) as ploned by Derive (Version 1)*

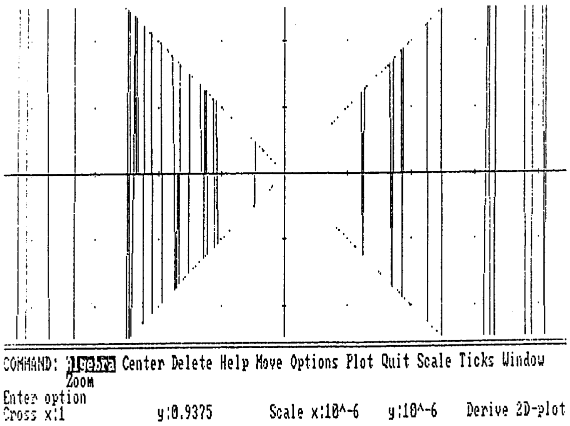
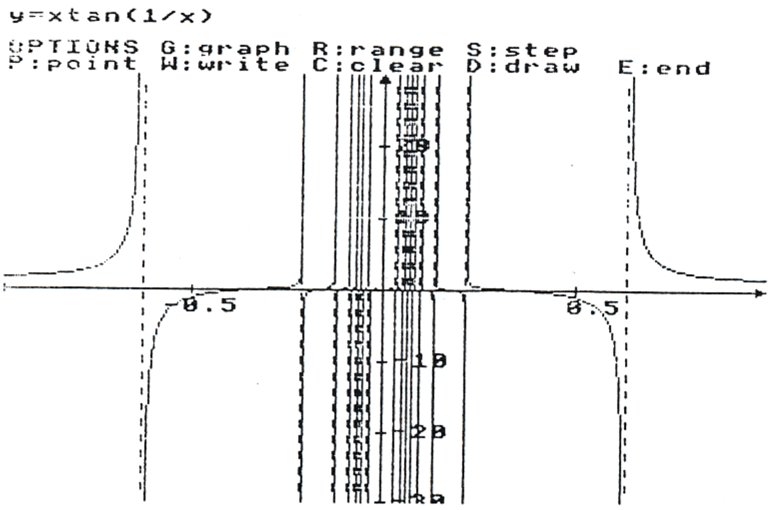

COMMAND: Algebra Center Delete Help Move Options Plot Quit Scale Ticks Window
Zoom
Enter option
Cross x:1 y:0.9375 Scale x:10^-6 y:10^-6 Derive 2D-plot

Figure 2: x tan(1/x) as plotted by Supergraph



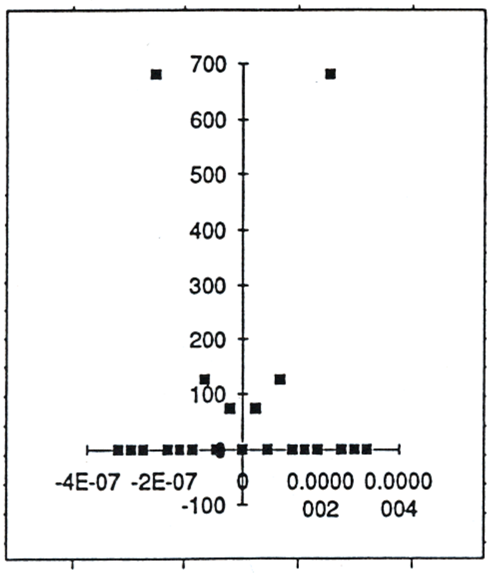


Figure 3:

x tan(1/x) as plotted by Excel

The Activity

The 21 students were divided into four groups and each of these was further sub-divided into three sub-groups. Each sub-group was asked to explore the properties ofx tan(l/x) using Derive (Version 1) on a PC, Excel (a spreadsheet) on a PC, and Supergraph on a BBC micro, respectively. After 20 minutes they then reported back to their original group comparing results. Following a 20 minute discussion they were given a short questionnaire to complete. We were interested in finding out:

- whether they would automatically try to validate the computer result, either when working with the software or during the ensuing discussion
- any reasons for not estimating the function behaviour or checking results afterwards
- whether they would question Derive as opposed to the other software, since it could be seen as being the most powerful programme

The results are summarised in the table below.

Table 1: Responses of students to questionnaire

	YES	NO
Estimated before using the software	0	21
Checked after using software:	15	6
by calculator	6	
by analysis	4	
by group discussion	5	

Five students considered the group discussion to be a check on results. Since this was part of the activity, we cannot say with any confidence whether they would have used discussion with others as a check on their own initiative. Only ten, less than half, used handwork of some description, whilst six considered that no checks had taken place (in spite of their discussions)! No one "estimated" beforehand, which could support and extend the classroom view held by Sparrow et al (1994) of having suspicions that many children do not use estimation skills when using calculators. However, in a plenary discussion about one third of the students stated that they would have used handwork first to estimate results, but considered that they had been directed to using the software immediately. Even so, about two-thirds had not thought of estimating results.

The discussion also indicated that students considered Derive to be more accurate and powerful than the other programmes, as we expected.

The students were generally surprised about the inaccuracy of Derive (Version 1) and the accuracy of Supergraph!

We then asked them to briefly state what they had learnt through the exercise as the final part of the questionnaire. All stated that computers were not to be trusted implicitly!

"not to blindly trust technology"

"wise to estimate and check results rather than simply trust technology"

"don't trust computers, try other methods"

"not to assume that computers are always right, and that the more powerful the technology the more accurate the answer" "super graph is not as useless as I assumed"!

Conclusions

We were surprised that after using the software and discussing the different results achieved relatively few tried to check the results by other methods. It means that in future we must push more strongly the principle of validating results and questioning findings. Students need to be aware of the necessity of reflecting on and thinking about their mathematics! However, this does raise another issue. We originally suspected the results provided by Derive (Version 1) due to our previous insights and experiences. Does this mean that students need to have the traditional handwork approach to skills and techniques before they can question and consider CAS results effectively? There is no easy answer to this, but there is a clear need for students to be able to interpret results and manipulate algebraic expressions to a suitable level in using CAS packages effectively. A balanced curriculum is therefore required where reasoning, skills work, exploration and consolidation take place, within a questioning framework, where results are automatically validated. The nature of the balance is of course open to further debate!

We have only just started to explore the issues raised in the introduction but we will use CAS : (i) as an algebraic calculator, (ii) as an explorative tool to aid reasoning and concept development, (iii) in problem solving and modelling; within the framework of an emphasis on the links between CAS use and the more usual handwork, and the need for validation of results.

We feel at present that we will continue to integrate the use of CAS packages within the present curriculum rather than build the students' experiences around CAS. However, the nature and balance of that integration will change, and even as we require our students to question outcomes, we ourselves must adopt the same questioning approach.

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