

## CONTENTS

- Page 1 Kenneth Ruthven  
*Pedagogical Knowledge and the Training of Mathematics Teachers*
- Page 11 Laurinda Brown & John Hayter  
*Illumination by Reflection under Pressure*
- Page 17 Lindy Furby & Val Heal  
*Evaluating an INSET Course: a Case Study*
- Page 21 Henk Schuring  
*Assessment in Secondary Mathematics in the Netherlands*
- Page 32 Ian Thompson  
*An Investigative Approach to the Issue of Attitudes to Mathematics*
- Page 39 Derek Haylock  
*The AMET Response to the Review of the National Curriculum & Assessment*
- Page 46 Derek Woodrow  
*The JMC Response to the Draft Circular on Primary Teacher Training*

## Pedagogical Knowledge and the training of mathematics teachers

**Kenneth Ruthven**

University of Cambridge

*Current change in the initial training of teachers centres on moves to base training substantially - even exclusively - in schools, and to give school staff - particularly classroom practitioners - greater responsibility for it. In this paper I want to focus on one aspect of this shift which has not yet received much discussion or analysis: on the forms of professional knowledge through which the practice of teaching can be illuminated; and correspondingly, on the ways in which such knowledge might be developed through school-based training. As a heuristic device, I will take a rather specific but loosely-defined issue - teaching fractions to a lower secondary class - and explore different forms and sources of pedagogical knowledge on which student learning might draw.*

Tacit expertise

Recent research has explored the thought and action of classroom practitioners (summarised in Bromme & Brophy, 1986; Calderhead, 1987; Clark & Peterson, 1988; Fennema & Franke, 1992) highlighting the role of 'tacit knowledge' (Polanyi, 1967) or 'knowing-in-action' (Schon, 1987) in guiding teaching. This research draws attention to the skilful, but rarely articulated, ways in which teachers make sense of, and act in, teaching situations. Although the major focus of such research has been conceived in general pedagogical terms and focused on lesson planning and classroom management, some studies have examined teachers' handling of particular subject matter, what has been termed their *pedagogical content knowledge* (Shulman, 1986).

Lehrer & Franke (1992) examined the constructs used by two experienced teachers in relation to the teaching of fractions. There were some important constructs in common between the two teachers: in content, the idea of fraction as part of a whole, and the equivalence of 1 with a corresponding fraction; in pedagogy, the

equivalence of 1 with a corresponding fraction; in pedagogy, the differences between constructs also proved striking. Whereas one teacher focused on standard routines, reflected in constructs such as 'teach a procedure' and 'break into steps', the other emphasised developing different elements of pupils' thinking and building relationships between them, reflected in constructs such as 'work on conceptualization', 'specific skill needed', 'play with a related problem' and 'create a related word problem'. Similarly, whereas the pedagogical constructs of the first teacher - such as 'fractions given are equivalent' and 'use smaller numbers' - reflect a solely epistemological analysis of teaching, corresponding constructs from the second teacher - 'build relationship between fraction numbers' and 'work with a fraction the children understand' - incorporate a cognitive dimension. Significantly, the interconnections between constructs were much more developed in the second teacher's thinking; in particular, connections relating characteristics of mathematical problems to pedagogical strategies.

Leinhardt (1988, 1989) analysed the mental lesson plans of novice and expert teachers. Their nature differed in a number of important ways. First, the experts were able to draw on a rich repertoire of standard lesson segments, often labelled in idiosyncratic terms: 'a pizza party'; 'working with the muffin tins'. These corresponded to images or prototypes of the patterns of action and interaction characterising such lesson segments. By comparison, novices lacked both breadth in their repertoire of lesson components, and clarity and depth in their images of them. Second, the experts' agendas were less teacher-centred, referring to pupils' activity far more frequently, and building in checkpoints for pupil understanding. Finally, there was considerably more evidence in the experts' plans of an underlying instructional logic and flow. Leinhardt also examined explanations of the idea of equivalent fractions offered to pupils by

experts and novices. She found that experts were considerably more likely to employ a representation already familiar to their pupils in introducing the new idea, and to anticipate the prerequisite skills required by, and available to, their pupils. They were also considerably more successful in coordinating and completing their explanations satisfactorily. Finally, only experts (and then only some of them) clarified the problem in terms of underlying principles.

These studies illustrate the pedagogical prototypes and images which expert teachers use to make sense of their experience and guide their actions. To acknowledge its largely private and unarticulated character, I will term this knowledge, *tacit expertise*.

### **Pragmatic wisdom**

Within the profession of teaching, there is a longstanding tradition of attempting to distil and articulate the 'good practice' of expert teachers. This tradition is embodied in the production of reports and guidelines by working groups, and in the development of textbook schemes and teaching resources.

Where fractions are concerned, the National Curriculum, for example, indicates the kinds of knowledge expected at different stages in learning: from "understanding the meaning of 'a half and 'a quarter'" at NC Level 2; through "recognising and understanding simple fractions in everyday use" at NC Level 4 and "calculating fractions and percentages of quantities" at NC Level 5; "understanding and using equivalent fractions" at NC Level 6; to "calculating with fractions" at NC Level 8. But the teacher needs to translate the National Curriculum orders into appropriate classroom activities and judgements about pupils' responses.

A textbook series, such as *SMP 11-16* (SMP, 1983/4) indicates of the kinds of classroom activities that pupils might tackle at different stages in their learning: at SMP Level 1, interpreting 'striped flags' in fractional terms, and dividing 'herds of animals' into complementary fractional parts; at SMP Level 2, working with times expressed primarily in halves and quarters of an hour, and using 'fraction strips' to compare fractions; as an extension activity at SMP level 4, meeting fractional multiplication through subdividing squares into grids and traffic flows into component streams.

Alternatively, an 'investigative' resource book, such as *Starting Points* (Ban well, Saunders & Tahta, 1972) provides a range of more open-ended activities, suggesting questions that the teacher might ask, and indicating important ideas that might be brought out of the activity: making fractions out of pairs drawn from four numbers; arranging fractions in some form of table; plotting fractions as number pairs. But neither source (including the *SMP 11-16* teacher's guides) indicates the way in which pupils are likely to respond to them.

One valuable source of such information is, of course, teachers' classroom accounts of such activities. The *Passages* section of *Mathematics Teaching* illustrates the kind of observations that a perceptive teacher is likely to be able to offer. Collyer (1986), for example, describes classroom discussion of a fractional problem adding a half to a sixth - between herself and members of a lower secondary class.

Essentially, these sources articulate what is taken to be good professional practice. The central characteristics of this form of professional knowledge are well captured by the National Curriculum Working Group for Mathematics in its discussion of good practice (DES, 1987).

"Various models and theories of learning mathematics have been formulated but it is not the purpose of the working group to come to a considered opinion on these. Instead the working group would like to emphasis observable things that pupils .. are doing when they are learning mathematics. It is the view of the working group that mathematics is learned by .. [the following list of activities]."

By referring to such knowledge as *pragmatic wisdom*, I acknowledge its concern to distil and articulate experience in terms of commonsense judgements about practice.

#### Grounded science

The last thirty years have seen the growth of research in mathematics education, largely driven and conducted by individuals and institutions professionally involved in the teaching of mathematics and the training of mathematics teachers. An important acknowledgement of this growth was the commissioning of reviews of research (Bell, Kiichemann & Costello, 1983; Bishop & Nickson, 1983; Howson, 1983) at the time of the Cockcroft Committee, and its current state can be seen in the recent *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992). By referring to such knowledge as *grounded science*, I acknowledge the practitioner influence on its concerns, and the notions of rigorous enquiry which shape it.

In relation to fractions, the results of diagnostic research (Hart, 1981; Kerslake, 1986) indicate the nature of children's difficulties (such as the dominance of the 'part of a whole' visual model; lack of coordination of the fraction with the division concept; and the lack of integration of the concepts of fraction and number). Equally, teaching experiments evaluate speculative methods of addressing such difficulties (Kerslake, 1986; Streefland, 1991).

The dauntingly named, but highly accessible, didactical phenomenology of fractions examines the everyday language of fractions; clarifies the senses of fraction as 'fracturer' and 'comparer'; relates these to the canonical concept of rational number; and provides accounts and analyses of relevant classroom activities and pupil reasoning (Freudenthal, 1983; Streefland, 1991).

By drawing on both these kinds of primary analysis, it has proved possible to develop structural and developmental analyses of the fraction concept (Dickson, Brown & Gibson, 1984; Kieren, 1988).

#### Overview of pedagogical knowledge

These three constructs - tacit expertise, pragmatic wisdom and grounded science - are offered as prototypes rather than categories; as a heuristic device to capture the range of pedagogical knowledge that one might seek to make beginning teachers aware of as a resource for their professional learning. Moreover, there is interaction between these types of knowledge. Pragmatic wisdom, for example, borrows ideas both from tacit expertise and grounded science: in the former case it articulates and codifies ideas, but at the same time uproots them from a functional system of intelligent action; in the latter case, it gives ideas practical form, but in doing so discards the argument and evidence underpinning them. This highlights the important differences between the types of knowledge which might be summarised as follows: tacit expertise is the enacted expression of refined experience; pragmatic wisdom is the commonsense articulation of favoured practice; grounded science is the systematic and codified analysis of practice.

I do not seek to privilege any of these forms of knowledge. Each is, in its own way indispensable to developing the waulity of teaching, and I regard each as making an important contribution to initial training, not only in terms of classroom effectiveness, but as influences on the formation of a professional identity.

#### Learning from expert practitioners

For Berliner (1986), the performance of experienced practitioners offers the temporary scaffolding from which novices may learn to be more expert. If expertise resides in the situated thinking and performance of experienced practitioners and is best developed through exposure to this, then where better for novices to learn to teach than in school, through observation and

experience under the guidance of experts in the craft of teaching. This provides a powerful case for giving expert practitioners a greater role in initial training, and for practice-centred forms of learning.

The first challenge, however, is to persuade teachers and students to accept this view of professional learning. There is a widely reported perception amongst teachers (Lortie, 1975; Feiman-Nemser & Floden, 1986) and amongst student teachers (Lanier & Little, 1986; Calderhead, 1991) of mastery of the craft of teaching as an idiosyncratic process of learning from personal experience which generates a correspondingly limited conception of the formative role of the mentor.

The problem is pinpointed in a recent evaluation of the well-established system of strongly school-based training for secondary teachers which was operative in France until very recently (Leselbaum, 1987). This system gave a central role to the 'conseiller', an experienced teacher chosen by the authorities to mentor the student teacher.

'The great majority of mentors, chosen and designated to carry out .. the professional training of future teachers, "paradoxically" do not see themselves as "trainers" but as "ordinary teachers entrusted with a student teacher" .. Deeply convinced that "there is no true training" for "the difficult job of teacher", that it must be learnt "on the job" and that experience is difficult to pass on, the majority agree that they cannot offer any recipes and that they themselves do not always know why they succeed with certain classes and fail with others! All that they consider themselves able to do with their students is to let them attend their classes without either feeling capable of theorising their professional practice or claiming to offer a "model" to imitate.'(Leselbaum, 1987; pp. 59-60; author's translation).

A second challenge is to find appropriate ways of articulating and developing professional knowledge through practice-centred activities. McIntyre (1988) is critical of traditional forms of observation and experience in initial training. He suggests that observation is largely unsuccessful for two reasons: because students do not know what to look for, and teachers do not know what to highlight; and because the practice that students observe is not submitted to critical analysis. Similarly, McIntyre suggests that experience is ineffective because, in the absence of systematic diagnostic appraisal and supportive discussion from more expert practitioners much student learning is at a level of semiconscious trial-and-error with responses being shaped and reinforced by pupils.

Both observation and experience, then, need supportive frameworks, to help students to see and make sense of what is going on in the classroom. Take the typical example of a student assigned to observe and assist a group of pupils working on fractions with the SMP 11-16 booklets. Without all appropriate framework, the student may not progress beyond feeling at a loss for much of the time, noting only the extent to which pupils produce correct results, and intervening occasionally in response to pupil requests by demonstrating a method or clarifying an instruction.

The quality of student learning from such an activity could clearly be improved by familiarity both with general ideas of pedagogy and more specific ideas of pedagogical content knowledge. Through prior reading and discussion, the student could formulate much sharper foci for observation, enabling the recording of evidence and incidents for subsequent discussion. Equally, such preparation would provide the student with a more varied range of modes of intervention, on which the class teacher might be able to offer further advice in the light of their particular knowledge of the pupils concerned, the evidence of pupils' work, and passing observation of the student's interventions. Similarly, by observing the class teacher in a similar situation and then using eliciting techniques (McAlpine et al., 1988) the student can gain access to pedagogical reasoning underlying the class teacher's expert performance.

An innovative programme of initial training in which the process of professional learning has been carefully conceptualised and the role of the mentor clearly articulated is the Oxford internship scheme (McIntyre, 1988; Haggarty, 1992). A particular strength of the scheme is its recognition of the distinctive role that the mentor/practitioner can play in articulating tacit expertise, and showing how contextual features shape the use of different forms of professional knowledge. Consequently, the scheme incorporates a range of strategies intended to help student teachers and their mentors engage in a contextualised exploration of professional knowledge.

Nonetheless, despite the careful preparation and the considerable commitment of all partners, an evaluation of the scheme in operation describes the implementation of the role of mentor as disappointing (Haggarty, 1992). The evaluation reports that, rather than opening up their contextualised thinking about professional tasks the mentors tended to offer their own decontextualised theories to interns with little reference to the realities of practice. Even within this frame, the range of ideas deployed was relatively restricted, largely reflecting the mentor's personal experience and preferences, and making little reference to any codified body of professional knowledge.

Equally, higher education lecturers may fail to recognise and fulfil what, at least in the Oxford scheme is seen as their distinctive role. Although there was no evidence of this in the Oxford evaluation, a recent study of perceptions of professional learning in initial training (Squirrel et al., 1990) found lecturers "keen that students should rely .. on knowledge derived from personal experience rather than books, particularly in courses moving towards a more school-based structure".

*This article is based on a research communication presented at the Second British Congress of Mathematics Education which took place at the University of Leeds in July 1993.*

#### References

- Hanwell, C., K. Saunders & D. Tahta (1972) *Starting Point*. Oxford: Oxford University Press.
- Bell, A., D. Kiichemann & J. Costello (1983) *A Review of Research in Mathematical Education: Research on Teaching and Learning*. Windsor: NFER-Nelson.
- Berliner, D. (1986) 'In Pursuit of the Expert Pedagogue'. *Educational Researcher*,
- Bishop, A & M. Nickson (1983) *A Review of Research in Mathematical Education: Research on the Social Context of Mathematics Education* Windsor: NFER-Nelson.
- Bromme, R. & J. Brophy (1985) *Teachers... Cognitive Activities*. In B.
- Christiansen, A.G. Howson & M. Otte (eds) *Perspectives on Mathematics Education*. Dordrecht: Reidel.
- Calderhead, J. (ed) (1987) *Exploring Teaching... Thinking*. London, Croom Helm.
- Calderhead, J. (1991) 'The Nature and Growth of Knowledge in Student Teaching'. *Teaching & Teacher Education*, 7, 5/6, 531-535.
- Clark, C. M. & P. L. Peterson (1986) 'Teachers' Thought Processes'. In M. C. Wittrock (ed) *Handbook of Research on Teaching*. New York: Macmillan.
- Collyer, S. (1986) 'Adding Fractions', *Mathematics Teaching*, 116. Department of

Education and Science (1991) *National Curriculum MathematicB Workinll Group Interim Report*. London: HMSO.

Department of Education and Science (1991) *MathematicB in the National Curriculum*. London: HMSO.

Dickson, L., M. Brown & O. Gibson (1984) *Children Learninll MathematicB: A Teacher'B Guide to Recent ReBearch*. London: Cayell.

Feiman-Nemser, S. & R. E. Floden (1986) 'The Cultures of Teaching'. In M. C. Wittrock (ed) *Handbook of ReBearch on Teachinll*. New York, Macmillan.

Fennema, E. & M. L. Franke (1992) 'Teachers' Knowledge and Its Impact'. In D. Grouws (ed) *Handbook of ReBearch on MathematicB Teaching and Learninll*. New York: Macmillan.

Freudenthal, H. (1983) *Didactical Phenomenology of Mathematical StructureB*. Dordrecht: Reidel.

Grouws, D. (ed) (1992) *Handbook of ReBearch on MathematicB Teachinll and Learning*. New York: Macmillan.

Haggarty, L. (1992) *[nveBtillatinll a new approach to mathematicB teacher education: an action reBearch Btudy*, Unpublished DPhil thesis, University of Oxford.

Hart, K. (1981) *Children'B UnderBtandinll of MathematicB 11-16*. London: John Murray

Howson, G. (1983) *Curriculum Development and Curriculum Research*. Windsor: NFER Nelson

Kerslake, D (1986) *Fractions: Children's Strategies and Errors*. Windsor: NFER-Nelson.

Kieren, T. (1988) 'Personal Knowledge of Rational Numbers: Its Intuitive and Formal Development'. In J. Hiebert & M. Behr (eds) *Number Concepts and Operations in the Middle Grades*. New York: Lawrence Erlbaum Associates.

Lanier, J. E. & J. W. Little (1986) 'Research on Teacher Education'. In M. C. Wittrock (ed) *Handbook of Research on Teaching*. New York, Macmillan.

Leselbaum, N. (1987) *La formation des enseignants du second degre dans les centres pedagogiques regionawc*, Paris: Institut National de Recherche Pedagogique.

Lehrer, R. & M. L. Franke (1992) 'Applying Personal Construct Psychology to the Study of Teachers' Knowledge of Fractions'. *Journal for Research in Mathematics Education*, 23, 3, 223-241.

Leinhardt, G. (1988) 'Expertise in Instructional Lessons: An Example from Fractions'. In D. Grouws & T. Cooney (eds) *Effective Mathematics Teaching*. New York, Lawrence Erlbaum Associates.

Leinhardt, G. (1989) 'Math Lessons: A Contrast of Novice and Expert Competence'. *Journal for Research in Mathematics Education*, 20, 1, 52-75.

Lortie, D. C. (1975) *Schoolteacher*. Chicago: University of Chicago Press.

McAlpine, A. , S. Brown, D. McIntyre & H. Hagger (1988) *StudentTeachers Learning from Experienced Teachers*. Edinburgh: Scottish Council for Research in Education.

McIntyre, D. (1988) 'Designing a Teacher Education Curriculum from Research and Theory on Teacher Knowledge'. In J. Calderhead (ed) *Teachers' Professional Learning*. Lewes: Falmer.

Polanyi, M. (1967) *The Tacit Dimension*

Schon, D. (1987) *Educating the Reflective Practitioner*. San Francisco, Jossey Bass.

School Mathematics Project (1983/84) *SMP 11-16: Levels 1-4*. Cambridge: Cambridge University Press.

Schulman, L. (1986) 'Those who Understand: Knowledge Growth in Teaching'. *Educational Researcher*, 15,2, 4-14.

Squirrell, G, P. Gilroy, D. Jones & J. Rudduck (1990) *Acquiring Knowledge in Initial Teacher Education*. Wetherby: British Library Board.

Streefland, L. (1991) *Fractions in Realistic Mathematics Education*. Dordrecht: Kluwer.

© The Author

**This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)**

**Articles available online at [www.amet.ac.uk](http://www.amet.ac.uk)**

**Original pagination of this article – pp1-10**



## **Illumination by Reflection under Pressure**

**Laurinda Brown & John Hayter**

University of Bristol

*This article describes how a model for describing professional competence emerged from discussions between university tutors and teachers from partnership schools. A feature of this model is the notion of a dimension of professional behaviour along which behaviours at both extremes may be seen as undesirable, but with a range of acceptable behaviours between these.*

The subject committee was progressing in planning the new PGCE course by asking a subset of experienced associate tutors (ATs) to look at particular areas and to make proposals. Such a group of tutors met with us to face the list of professional competences. It was not a novel task, for at NCC conferences and AMET weekends some of us had already wrestled with this daunting list. It was all too tempting to question relative significance, to debate perceived ambiguity and to make clear unexpected contradictions. Apart from some glow of self-righteousness at not falling for such simplistic formulae, there was little sense of satisfaction from making progress in seeing how to apply the list.

The subject committee had themselves wrestled with the task but were not able to give a clear brief to us with which to work with the group of ATs. It had taken a long time just to get to grips with *Subject Knowledge*, but the list in 9/92 is so long. The following paper provided a summary of the thoughts of the Subject Meeting and was circulated before the meeting:

*The Subject Committee considered the section of the document Circular 9/92 on competences and came up with the following general guidelines to tackle the task of producing a list of criteria which we could all use to help our work with and ultimately use to assess student teachers.*

*General criteria:*

- *Evidence that the student is able to develop professionally; they are able to reflect on their experiences and develop strategies to overcome problems that they meet.*
- *That there is a movement away from simply considering classroom management or survival strategies to the consideration of issues such as assessment or differentiation by outcome or pupil misconceptions and ways of working with the pupil to overcome them.*

*We took one particular section from 9/92 and worked on it to produce a set of competences that seemed to fit for us within it - including what we might consider to be the fail criteria:*

*Subject Knowledge:*

- *revisiting the subject; working at the pupil's level; recognising diversity in pupils' methods; understanding their problems*
- *confidence to study and learn new mathematics for themselves*
- *recognising gaps in their knowledge and showing the willingness to work on them eg Mechanics, Statistics, Discrete Mathematics*

*Fail criteria: No confidence to learn mathematics and/or no willingness to revisit.*

*The suggestion is that, as a group, we come up with criteria for the other sections - including a fail criteria.*

*Hope this is enough to get us started.*

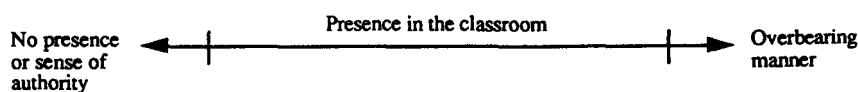
And so here we were again, but that much closer to a deadline. Few wanted to argue about items on the list but some began to question how they themselves would be assessed on such competences. Surely we should all still be making progress on these dimensions? Were we on the right side of some satisfactory performance line - probably not on some of these competencies on a bad (perhaps even a good) day! But how would we assess student teachers during an initial training course. Silence.

*Why don't we try to specify some behaviours we can all agree upon as good practice? The attractions of starting with the blank sheet were evident. Always more fun to create a list than try to perform to someone else's. No shortage of ideas for desirable behaviours.*

*But what's the point of making another list when we have to work to the one government has given us? Some grudging assent but no enthusiasm to engage.*

*Why don't we start with what we do at present? How do we decide whether someone is failing on teaching practice? Someone suggests that for them "no presence or sense of authority in the classroom .. would be grounds for serious concern.*

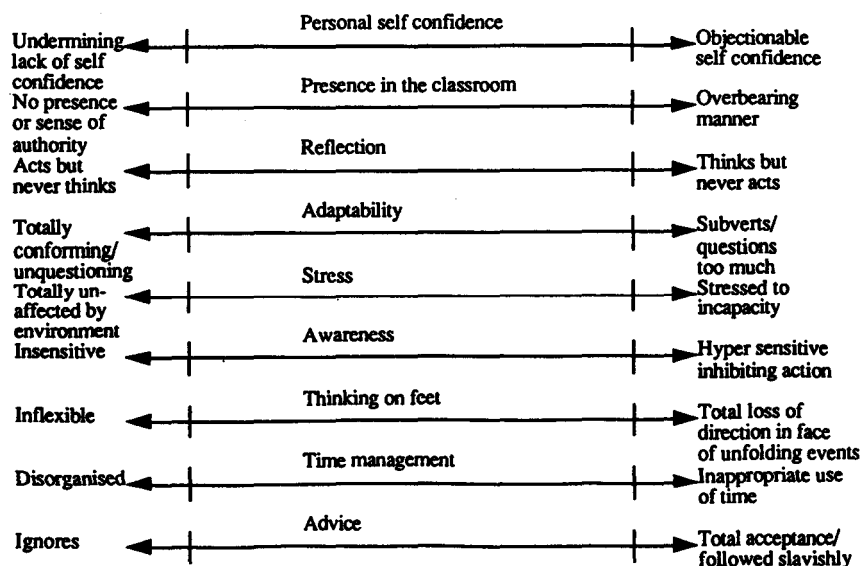
*But what about the person who has an overbearing manner in the classroom even after a term of counselling to modify their style? This seemed to concern people too. An axis Classroom presence was born with concern expressed about behaviours at either end.*



Another extreme was mentioned: *totally ignores all advice given*. Murmurs of agreement. *But what about those who follow advice slavishly?* Another dimension had appeared.

Over the next half hour a stream of contenders, derived from experience working with students over the years, emerged. To our surprise there were almost always two extremes relating to some aspect of behaviour and it was the extremes that caused concern. Seen as a continuum, each dimension had an acceptable range of variation but, moving in each direction, there was an awareness that extremities made for major problems in the classroom. This felt like the beginnings of a checklist that related to our concerns and our professional practice working with students. But how did it relate to the list of 9/92 competencies?

Back to the list. Links were often possible with particular competences. For example on the use of IT we were concerned by the extremes of *overused - obscures or antagonises* and *unable or unwilling to select and use resources* (Competence 2.3.7) On *general teacher responsibilities* (2.6.2) there were concerns about those *oblivious to responsibilities outside mathematics teaching* and those for whom *other work seriously interferes with subject teaching*. There was not a complete match but it felt like a workable idea; certainly something worth developing. Some dimensions which seemed vitally important to us were outside the competencies list. We labelled these *general* and include the list below. There seemed a reasonable prospect of finding dimensions to match the items in the competency list. Importantly, we seemed to have an emerging instrument for assessing the competence of the student teacher whilst retaining the original competence list as a set of vectors along which we would be moving throughout our careers. There was no need to find a magic satisfactory point on each for the teaching practice 'pass' decision.



It was realised that an extreme behaviour along anyone dimension would mean that the student was probably extreme on many others. There were links. The question, therefore, of how many fails make an overall fail is irrelevant. Any failing behaviour is a cause for concern. Not, of course, at the start of the year, but certainly if the behaviour never modifies away from the extreme to the centre.

So from late afternoon weariness and gloom to a sense of achievement and progress. The task of tidying and matching left to a member of the group, we could adjourn for a well earned meal.

The complete draft list (including the links with 9/92) was discussed at the subject committee and it was agreed to explore the list in practice during the coming year. This meant that there would need to be a session during the mentor training for those teachers working closely with us - the ATs. Some of the new ATs who are working with us in partnership have not worked with student teachers on a PGCE course during the first term of the year. Would the list make any sense to those not involved in its creation?

The early mistakes which students make are capable of being turned into 'cartoons' - strong visual images which are funny to practising teachers because of an awareness of the problems which can follow from such behaviour. An example would be:

Student teacher, with their back to the rest of the room, standing or sitting having an intense discussion with one pupil in the back corner of the classroom.

The cartoon would simply have the rest of the pupils engaged in various forms of outrageous behaviour with the teacher blissfully unaware. Such cartoons seem to highlight the behaviours at extremes of the dimensions.

For the session in the mentor training such a cartoon was described. The discussion focused on what strategies could be offered to the student so that they might be able to notice such behaviour and do something about it. A role play of talking to the class whilst writing on the blackboard without turning to face the group, provided another example.

The ATs then split into small groups and worked at identifying behaviours which they might meet during the early stages of the students teaching, using the dimensions to help them to focus on extremes. For identified behaviours they would then work together to consider what could be offered as strategies to overcome the problem, given that the student teacher could not come up with anything themselves.

The session was fun and yet productive, although some groups used the time to talk more generally about the list of dimensions and their work in schools. It is the wealth of experience of observing many student teachers over time which needs to be built up in the new ATs and the plenary discussion helped those who had not much experience. The list seemed to provide a useful tool to provoke discussions about practice.

We have a number of questions about the use of this list in practice. Is it more important to have simple but possibly unclear descriptors or try to find more detailed but hopefully unambiguous ones? When should the students be given a copy? We have decided for this year that this will not happen at the start of the course, but at the time of going to press have not yet decided when. Can we work with this model so that it becomes a tool for development and not only for assessment?

The value of all this was in the working together of the various groups of teachers to create a common culture and perhaps you might be able to come up with an equally productive but essentially different model.

Those interested in obtaining a full list of the dimensions should write to Laurinda Brown & John Hayter, University of Bristol, School of Education, 35 Berkeley Square, Bristol BS8 IJA.

© The Author

**This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)**

**Articles available online at [www.amet.ac.uk](http://www.amet.ac.uk)**

**Original pagination of this article – pp11-16**

## Evaluating an INSET Course: a Case Study Lindy Furby & Val Heal

Bradford and Ilkley Community College

*At Bradford and Ilkley Community College we have conducted a 20 day National Curriculum training course for Bradford LEA for three years. The teachers have come from mainly First and Middle Schools. This is a course offered to LEAs under the provision of the Training Grants Scheme. The course has had to conform to a prescribed framework with the stated emphasis being on developing teachers' confidence and ability in mathematical knowledge and understanding. When a heterogeneous group of teachers attend a highly intensive course our evaluation has shown that it is essential to target teachers' perceived needs. Obviously there are many other objectives, but by focusing teachers' perceived needs the course team have found that they can provide the foundation for developing and extending all of the teachers, knowledge, concepts and skills in mathematics.*

The first two years of the course were evaluated in writing at the end of the course by the students. During the second year a new course leader was brought in who was invited to attend an HMI conference for Mathematics and INSET on the National Curriculum. Here she became concerned that although the course had been deemed successful, there was no data to confirm that we were dealing with teacher's perceived needs in mathematics. Based on their previous experience of INSET, the course team had formed their own ideas of which areas of mathematics teachers would feel confident about. This subjective evidence was now recognised as being insufficient. The team needed more accurate data and set a rigorous methodology to confirm or refute their beliefs.

Prior to the course, the course leader visited each participating teacher and head teacher in their school to discuss the course. It was decided to ask the teachers to fill in a confidential but numbered questionnaire to ascertain just what the teachers considered were their weaknesses and strengths in mathematics. The same questionnaire was given to the course members at the end of the course. A comparison of teachers' perceived strengths and weaknesses in mathematics before and after the course was carried out.

The design of the questionnaire was based on the National Curriculum. There were six main areas, one for each Attainment Target, which were then further broken down into the strands as identified in the 1991 Non-Statutory Guidance. *Number* was subdivided into 'small numbers', 'place value' and 'decimals', and *Arithmetic* into the four operations:- addition, subtraction, multiplication and division. The sixth area, *Measurement*, was divided into separate kinds of measurement. Teachers were asked to identify on a five-point scale the level of confidence they felt they had with the mathematics.

In addition to the questionnaires which generated quantitative data there were two questionnaires designed to generate qualitative data. The first of these was given at the end of the first five days. Its function was to check on the course design to make sure that the teachers received a course that fitted both the needs identified in the quantitative data questionnaire and other needs perceived by them. The second questionnaire was given during the last days of the course and was designed to conform to the framework prescribed by the DFE. This was the same evaluation as had been carried out in the previous two years of the course.

For the team's own planning each day's input was evaluated by identifying areas of content and pedagogy covered and taking comments about the day from the teachers. This enabled the team to adapt the style of the following days to maximise the teachers' interest and involvement.

Results of the first questionnaire showed three areas of marked strength - number, arithmetic and measures - with decimals appearing less of a perceived strength. There were two ATs which had responses below average:- *Using and Applying Mathematics* and *Algebra*. The rest of the results were somewhat mixed with certain strands in an AT being perceived as a strength while others as a weakness. In the *Shape and Space* AT location and movement were considered to 'be particular weak areas whereas the teachers were reasonably confident with shape. Teachers pinpointed probability in data-handling and volume in measures as weaknesses.

The results tended to confirm the team's belief that emphasis would need to be placed on Using and Applying Mathematics, Algebra, Shape and Space and Data Handling. The course already contained a high proportion of time on these areas but the team were able to focus the time and the content on the specific strands identified by the teachers and re-work the materials to give the emphasis required.

The course appeared to be going well with the comments from the teachers affirming this particularly in the key areas of perceived weakness.

*"work on AT 4 most useful, clarified things." "understanding of algebra improved with last week and today."*

*"a really good day with good ideas that are useful for school" (AT 5)*

*"it was real, colourful, stimulating, made you think and made you want to do it." (AT 4)*

*"I feel 1 know a bit more about probability"*

It was also apparent to the tutors that the teachers were developing a high degree of autonomy both in working through the mathematics and in their growing confidence with the 1991 National Curriculum documents.

When the results of the second quantitative questionnaire were collected they were given to the present 3rd year major mathematics students on the Primary BEd who applied a 't' test for paired samples to the data. Their results indicated that in some areas there had been no change but in others there was fairly significant to highly significant improvement in the teachers confidence in mathematics. In only one strand - length in the *Measures* section - did the results go down, but this was very marginal.

In *Using and Applying* there was a highly significant improvement in the teachers perceived understanding across the three strands. In Number there was a smaller improvement with a fairly significant improvement in place value and decimals. Teacher confidence in arithmetic showed no significant change.

Algebra, on the other hand, showed a significant improvement in all three strands. *Shape and Space* results revealed a highly significant improvement within the movement strand and a significant improvement in location. In *Measures* there were two significant improvements in the areas of measurement we had worked on. These were in area and volume, with capacity showing a fairly significant improvement. In *Data Handling*, collection showed a significant improvement, representing and interpreting a fairly significant improvement, and probability a highly significant improvement.

The statistics from the data collection indicated that the teachers considered that they had improved their understanding in all areas and marked progress and development in the areas that they had previously considered themselves to be weakest and the areas that had been pin-

pointed from the first questionnaire for special treatment.

The course is highly intensive and tends to be attended by a disparate group of teachers and for this reason it is essential for the success that accurate information is collected about the perceived needs of the attending teachers. The whole process was invaluable to the team and the teachers. It enabled the teachers to both identify for themselves and inform us about their needs for the development of their mathematical knowledge and understanding. Not only were we able to adapt the course design to the teachers needs but evaluate the validity of the course design. The course design appeared valid and appropriate. The team had provided the foundation for developing and extending the teachers knowledge, concepts and skills in mathematics. The team believed that the evaluative process was so successful that they have decided to follow a similar pattern next year.

**© The Author**

**This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)**

**Articles available online at [www.amet.ac.uk](http://www.amet.ac.uk)**

**Original pagination of this article – pp17-20**

## Assessment in Secondary Mathematics in the Netherlands

Henk N. Schuring

CITO

(Netherlands National Institute for Educational Measurement)

*In 1985 a new programme was introduced in the last two school years of pre-university education in the Netherlands, divided into two subjects: Mathematics A with emphasis on applying mathematics in other subjects, and Mathematics B, pure mathematics. The curriculum for Mathematics B is a development of the old programme: analysis, including calculus and differential equations, and geometry with focus on 3-D space and solids. Mathematics A, mathematics in concrete realistic contexts, was entirely new and differed very much from what teachers and examiners were familiar with. This paper analyses data from the assessment of this new Mathematics A syllabus.*

Secondary Education in the Netherlands

Secondary Education in the Netherlands is divided into four welldefined streams:

- Lower Vocational Education (four years, taking 21% of the students),
- Intermediate General Education (four years, taking 34% of the students),
- Higher General Education (five years, taking 24% of the students),
- Pre-University Education (six years, taking 20% of the students).

Mathematics A and B

The Mathematics A curriculum differs fundamentally from the more traditional Mathematics B. It includes: applied analysis, including the derived function as a measure of change; applied algebra, including matrices and linear programming; probability and statistics, including hypothesis testing, and informatics, including simple programs and programming.

It is designed for those who did not see mathematics forming a substantial part of their future university studies, e.g. economy, psychology, etc.

At the end of the course, students have to take a final examination in seven subjects. Mathematics is not compulsory, so students can take mathematics A, or mathematics B, or both A and B, or no mathematics at all. In 1992 60% of all pre-university students opted for mathematics A, 47% for mathematics B, 18% for both A and B, leaving 11% of students opting for not taking mathematics at all.

Reform of the pre-university education curricula preceded those of the higher general education. In this stream students have to take a final examination in six subjects; from 1992 on students can choose out of two mathematics subjects: mathematics A and mathematics B. The examination syllabus for A consists of tables, graphs and formulae, discrete mathematics and statistics and probability; the syllabus for B of applied analysis and geometry in 3-D.

In August 1993 every school for secondary education will start with new programmes for mathematics for all school types in the age range 12 - 16, with more emphasis on applied



mathematics and 3-D geometry.

The assessment in the final year of secondary education consists for 50% of teacher-made tests, mostly written essay-tests, sometimes oral and sometimes individual pieces of work. As these tests differ from school to school and depend on the schoolbooks used, it is not useful to study these tests.

We will restrict ourselves to the final examination papers, valid for the whole country, also counting for 50%.

As the examination papers for mathematics consist of openended questions, it is not so easy to collect data from the results. Therefore CITO asked every school to send the results of five students on all questions of the examination paper. In that way we have got reliable data for mathematics A and B. Besides we have got some information about the choice of other subjects, so we are able to distinguish some subgroups in the whole examination group.

### Results and subgroups

The analysis which follows will focus on Mathematics A, since this was a new syllabus for teachers and examiners. From a sample of more than 2000 students we have got the results of the mathematics A examination each year.

First the p-values: the percentage of the mean score from the maximum score for each question:

<i>question</i>	<i>1987</i>	<i>1988</i>	<i>1989</i>	<i>1990</i>	<i>1991</i>	<i>1992</i>
1	96	71	63	91	91	96
2	91	89	74	83	66	85
3	60	68	78	96	84	58
4	53	79	43	60	34	84
5	55	35	44	73	67	47
6	79	36	46	71	91	66
7	66	26	31	40	52	68
8	61	93	77	74	27	60
9	60	67	56	64	55	71
10	68	81	25	91	73	31
11	49	69	85	89	73	94
12	54	80	71	56	77	66
13	38	27	51	54	16	95
14	29	60	78	31	77	55
15	60	78	20	42	78	38
16	60	76	27	66	78	67
17	39	71	61	47	46	61
18	51	44	26	31	18	27
19		36		67	8	
20		51				
21		81				
22		51				
23		51				
<i>mean score</i>	62	61	57	66	62	66
<i>stand.dev.</i>	17	17	16	17	14	16
<i>reliability</i>	.85	.83	.77	.83	.79	.80
-----						
<i>ceasura</i>	49/50	51/52	51/52	54/55	54/55	54/55
<i>%</i>						
<i>insufficient</i>	25	32	37	26	32	24

The maximum score one can get for this examination is 100 points. For their presence a candidate gets 10 points; the other 90 points are spread over the questions. The examiner has a strict set of correction rules, including the maximum number of points for each question.

We have also computed the mean score, the standard-deviation and the reliability of these six examinations. These figures are printed underneath the p-values.

On account of the data from these samples a decision is made what number of points will give a sufficient result, so the caesura is fixed. This will give the percentage of students having an insufficient mark.

The mark gained for this part of the examination counts for 50% of the assessment of the student, the other 50% consist of teacher-made tests. If these two figures differ too much the inspector must try to find out the cause.

The percentage of students having an insufficient final mark for mathematics A is less than the above given figures.

A student can still pass the final examination with an insufficient mark for mathematics, because in the Netherlands there is a system of compensation. Good marks in other subjects can compensate for an insufficient mark in a certain subject. (To a certain extent: a mark lower than 4 on a scale 1 - 10 can never be compensated.

Those who are involved in the preparation of the examination papers have tried to estimate the mean score, before the examination took place, by predicting the p-values of each question in one of five classes: class I with  $0 < p < 20$ , class II with  $20 < p < 40$ , etc. The prediction of the mean score in 1987 was 60; in 1988, 63; in 1989, 61; in 1990, 61; in 1991, 60; and in 1992, 59. So the difference in results in 1989 and 1990 was not predicted and in 1990 and 1992 it was not foreseen that these examinations were relative easy.

In 1987 we had some teething troubles concerning the statistical relevance of a problem and some omissions in the context, but we do not think that students were handicapped by these omissions.

Nevertheless in those years we spent a lot of time preparing good problems with correct questions. In the meantime we tried to understand the differences in the results of some subgroups of mathematics A students.

The subgroups are:

- I: Students also doing mathematics B,
- II: Students with physics, without mathematics Band
- III: Students without physics and without mathematics B.

In the graph below are drawn the mean p-values from the three subgroups for the questions of the mathematics A examination in 1992.

About 30% of the students who chose mathematics A are in group I, 10% in group II and 60% in group III.

One must keep in mind that the mathematics A curriculum was specially made for subgroup III. For university studies in social sciences mathematics is compulsory, but why bother the future students in these subjects with pure mathematics as they will use mathematics only in other

subjects? That is why mathematics A was designed.

The mean scores for the three subgroups are:

	<i>group I</i>	<i>group II</i>	<i>group III</i>
1987	73	64	53
1988	72	65	53
1989	69	63	50
1990	78	73	59
1991	72	66	56
1992	76	69	59

The percentage of students having an insufficient mark in group III was 41% in 1987, 46% in 1988, 54% in 1999, 37% in 1990, 45% in 1991 and 34% in 1992.

This increasing percentage in the first three years was very alarming; the results in 1990 and in 1992 are encouraging.

It is obvious that group I has a better result than group III, because the weekly amount of hours for mathematics is twice as much for group I as for group III. Also we think that students in group II will have advantage in applying mathematics in physics.

But we were very anxious to know what kind of questions causes the biggest difference. From 1987 to 1992 this was the case on questions of linear programming, calculus, probability, periodic functions and finding formulae.

The rather poor results in group III can be caused by the fact that this subject was new for the teachers and they had to learn how to teach these matters. Indeed during the first three years there was a large group of inexperienced teachers and it is tempting to blame them. But we think it better to look at the way the questions are formulated and study the results of the past.

According to the results in 1990 and 1992 we think we are on the right track.

In an attempt to find better reasons for the differences between the groups and between the years we have developed a threedimensional test grid. In the next section an outline of this grid is given.

#### Test grid

Up till now, for the construction of examination papers, we use a two dimensional test grid. On one side we have a list of subjectcomponents and on the other side a list of behavioral-components.

Each year, before starting the construction of the problems, we agree upon the subject-components to be tested this year and upon the percentage of original questions in the paper.

After having constructed the draft examination paper we fill in the test grid for this paper. Of course this is a rather subjective activity, but if the same group of experts does this, it is possible to compare the results over the years.

This is the way it was for constructing the final examination papers for all levels on pure mathematics. For mathematics A however it was not a suitable practice. Totally different questions came into the same cell because it was not possible to distinguish these questions on the same subject. Therefore we need another component: skills.

Solving mathematics A problems require more than mathematical skills as solving equations and inequalities. One must be able to choose a suitable mathematical model fitting in the context or to judge if a given model is appropriate. After finishing the mathematical part of the problem one must be able to transfer the mathematical results into the context in order to answer the question raised in the problem. To judge a given model is easier than to make your own model. Therefore the skill-components are needed to distinguish different questions.

We have grouped the components in main categories, as follows:

Subject-components.

I. Functions, formulae, equations and inequalities. II. Graphs, matrices and distances.  
III. Combinatorial analysis and IV. Probability and statistics. VI. Linear programming.

Skill-components.

M. To draw, make, judge, vary and explain a model.

R. To make or finish a graphical representation of a model. G. To read data.

W. To use mathematical skills.

T. To use a combination of more or less all the skills.

With this classification instrument we were able to analyse the final examination papers from 1987 to 1992.

With the help of this test grid it is possible to compare the mean scores per cell of the subgroups. As group II covers only 10% of the students we have not included the results of this group, so we shall compare the results of group I and III. Per cell and per year we have computed the mean scores of the two groups and converted these figures to percentages of the maximum score. In the next table these percentages are given.

Looking at the totals of the subject components one can see that subject II, matrices have had the best results, with an exception in 1989. In that year the two questions were original and were of a complex structure (skill component T). The subject probability and statistics (III&IV) shows the poorest results. It is a difficult subject and one can ask a variety of questions. In this subject the questions needing mathematical skills (W) show a good result; the students are able to learn the solving strategies. The questions dealing with modelling are all original.

The questions needing skill-component G (reading data) were easy for the students as one can expect.

		M		R		G		W		T		Total	
		I	III	I	III	I	III	I	III	I	III	I	III
I	'87							74	45	78	55	75	47
	'88	77	60	81	71	90	87	83	58	55	29	74	52
	'89	83	65	94	67	86	62	86	45	71	39	83	52
	'90	82	56					74	50	87	67	80	56
	'91	57	21	50	36	87	78	88	68	83	58	80	62
	'92	80	67			79	64	76	51			77	57
II	'87	86	74					76	54			79	60
	'88	82	70					78	64			81	68
	'89									51	32	51	32
	'90	91	85							81	65	86	75
	'91												
	'92	97	96			90	82	76	47	57	40	78	63
III&IV	'87			60	42					48	35	51	37
	'88	54	28	87	73			75	62	44	28	62	45
	'89	38	20							59	47	50	36
	'90	65	27					76	62	53	32	60	37
	'91	32	11					68	57	14	4	49	37
	'92	69	57					76	57	40	21	62	45
VI	'87	75	49							70	40	72	44
	'88	40	18							50	27	46	22
	'89							70	48	78	60	76	58
	'90									72	47	72	47
	'91			67	44					45	17	57	33
	'92												
Total	'87	79	59	60	42			74	46	62	42	69	46
	'88	61	41	84	72	90	87	81	59	51	28	68	47
	'89	47	29	94	67	86	62	82	46	63	45	66	45
	'90	83	64					74	53	72	51	75	55
	'91	45	16	58	40	87	78	81	64	47	26	67	51
	'92	80	70			83	70	76	52	47	29	74	55

Looking at the differences between the results of groups I and III we see a total difference of about 20%. Note that the figures at the bottom on the right hand are comparable but not the same as the mean scores on a previous page. In 1989 in cell (I, W) the difference was 41%. One can presume that it was caused by the way the questions are posed, in this problem a formula for the chain-line was given, also for the 27% difference in cell (I, R).

The difference between the results of group I and III on the subject functions is bigger than the differences on the subjects matrices and probability and statistics. This is according to the expectation, because analysis in Mathematics-B will help to increase the knowledge of functions in Mathematics-A. Matrices, probability and statistics are not subjects in Mathematics-B.

The difference in cell (III&IV, M) in 1990 is exceptional; finding a relation between mean and standard-deviation was very hard for group III, it was an original question. Looking at the behavioral components we have distinguished reproduction questions and production questions, which have some original aspects.

Again we look at the differences between the results of group I (students who are also studying Mathematics-B) and group III (students without Mathematics-B and without physics).

The next table gives the mean scores in percentages of the maximum score:

	<i>reproduction questions</i>		<i>production questions</i>	
	I	III	I	III
1987	70	49	70	45
1988	70	50	63	43
1989	74	53	53	30
1990	81	67	66	32
1991	74	55	54	38
1992	79	63	61	37

As can be expected the results of reproduction questions are higher than the results of original production questions. From 1987 to 1990 there was an increasing result of reproduction questions, while the results of production questions are decreasing, with an exception in 1990. In 1991 the results were lower than in 1990, but 1992 gives an encouraging result and we do hope that this is the beginning of a new trend.

#### Possibilities

In the Netherlands, school examinations consist of two parts:

- a central part, the same for all schools of a certain type;

- a school part, composed by every individual school.

In this school part teachers possess all freedom to pay attention to those aspects which are important in their views.

A more open assessment is quite possible in classroom practice. We have to find other forms than those where every pupil works during one of more hours at some problems.

For example we have to think of group-work: for some time a group of students will work in the indicated open way at a problem. Discussions with each other and with the teacher, research in literature and library, presentation in written as well as in oral form are extremely valuable elements. There is already some experience with this way of assessment in our country and the outcome is highly satisfying.

### Conclusions

It is useful to collect data from examinations. Interpretation of data is often very difficult; one must collect data during a long period in order to be able to give meaning to results. For the construction of examination-papers it is useful to develop a testgrid. Prediction of the difficulty of an examination is very hard, but is always worth while trying.

It is of great importance to make the mode Of assessment in mathematics so that there is a good balance between an open and a closed approach. In this way assessment is more adequate for the sort of mathematics as well as for the method of working and for the elements which the students must know and for what they are supposed to be able to do. It gives a better feedback to the mathematics teacher as well as to the student. Last but not least it is highly motivating for students as well as teachers and it contributes thereby to the solution of the immense problems of mathematics education today.

### References

Geoffrey Howson, National Curricula in Mathematics, the Mathematical Association, Leicester 1991.

Mogens Niss (ed), Cases of Assessment in Mathematics Education, Kluwer Academic Publishers, Dordrecht (to be published).

© The Author

**This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)**

**Articles available online at [www.amet.ac.uk](http://www.amet.ac.uk)**

**Original pagination of this article – pp21-31**

## An Investigative Approach to the Issue of Attitudes to Mathematics

Ian Thompson

University of Newcastle-upon-Tyne

*Two articles in the first two volumes of Mathematics Education Review caught my attention at a time when I was preparing my sessions for yet another 20-day Maths Course. Jocelyn Bell (1992) suggested a successful activity for getting students to reexperience and reflect upon their feelings about mathematics, and Mary Briggs (1993), writing in response to the marking of exam scripts for the OU course EM236, discussed the anxieties expressed by students having to engage in some mathematics of their own under examination conditions. Both articles reminded me of my first reading of the Cockcroft Report in 1982 just at the time when I had started working in teacher education. The most striking issue to emerge, and one that lingered with me long after reading the report, was that of 'confidence' or rather the lack of it shown by both children and adults in their encounters with mathematics. Other related issues in the report that were of interest to me concerned people's attitudes to the subject and their perceptions of what mathematics was actually all about. I still feel that these are important issues that have not been fully addressed ten years after Cockcroft.*

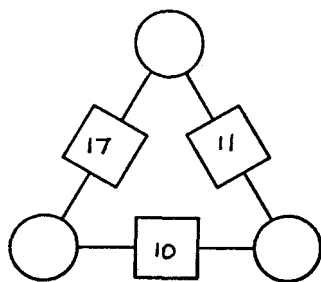
In my new job I was working with primary PGCE students, secondary 'Retrainers' and a group of primary schoolteachers seconded on a one-year Mathematics Association Diploma Course almost all of whom appeared to be totally lacking in confidence in their mathematical performance. My obsessive interest in this issue led to the enthusiastic preparation of a lecture, probably very dry and boring in retrospect, making use of multicoloured slides for the OHP and apposite quotes, not only from the Cockcroft Report itself, but from Brigid Sewell's (1981) research document and Laurie Buxton's (1981) recent book.

Over the years I have adapted and refined my 'Attitudes to Mathematics' lecture, making it much less didactic and rather more interactive and personal. A chance re-reading of an early article in Mathematics Teaching on 'Arithmogons' by Alistair McIntosh and Douglas Quadling (1975) led to my incorporating this activity into one of my practical workshops. At a later stage I decided to combine the arithmogons activity with my original lecture and this led to the development of what I feel has turned out to be one of my better workshop sessions. It is this session that I wish to describe in the next few pages.

On the very first day of any 20-day course for primary teachers I make it my policy to enter the designated room exactly on time, and, without welcoming them, give them each a copy of a doublesided worksheet. I deliberately do not smile as I ask them to work through the problems on the piece of paper in front of them.

The worksheet begins by explaining how arithmogons work: any number in a square is the sum of the numbers in the two adjacent circles. The first two examples give them numbers in the three circles and simply ask them to find the numbers in the three squares. The next two questions provide the numbers in the squares and ask for the numbers in the circles, a rather more difficult task (Fig. 1). The solutions to these two questions are deliberately arranged to be positive integers, and most people are usually successful in finding the answers by some means or another. The last two questions in the first section are similar, except that the solution to the first one involves fractions and the second one involves negative integers.





**Figure 1**

Having been presented on arrival with a daunting-looking worksheet to complete the teachers almost always choose to work in total silence, ten minutes is the longest I have been able to bear, and I usually have to intervene to inform them that they are allowed to talk and work together if they wish. This intervention usually produces a communal sigh or outbursts of laughter as the tension is released. While they are working on the problems I take on the role of the 'hovering' teacher, walking around the room looking at what they are doing. I usually ask them how they found a particular solution and jot down their responses in addition to any comments that I hear them make to their peers. While they are busy attempting the worksheet I copy their comments onto an OHP transparency to which I add further ideas that emerge in the ensuing discussion.

The activity is such that it usually leads quite naturally to the discussion of a wide range of issues, many of which have relevance either to their attitudes towards mathematics and mathematics teaching in general or to their own classroom behaviour when teaching the subject. The following are just some of the issues which tend to occur most frequently:

#### Poor self-concept

Many hate the fact that I prowl around looking over their shoulders, and some of them even cover their work so that I cannot see it, as they laughingly say things like, "Sorry, I'm no good at maths," or "I'm only a reception teacher, you know." Other comments heard between peers include: "I do feel thick" and "I'm going to ask whether I can coordinate P.E. instead next year!"

#### Problem-solving methods

Some interesting statements on solution strategies have been noted:

"I'm just using trial and error, though there must be a method." "I'm afraid I just guessed really."

"I've forgotten how to do them. I can't remember the method."

Comments like these suggest that many teachers appear to believe that for each type of problem there is a specific, unique method of solution, and that 'doing mathematics' simply involves remembering this method and executing it correctly.

#### The importance of algebra

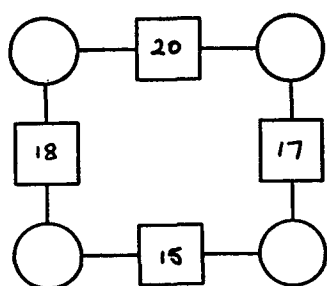
Many of those apologising for the inadequacy of their methods, despite the fact that these methods work, compare themselves unfavourably with those few teachers who are to be seen

using algebra. Comments like the following have been heard:

"Listen to that lot talking about simultaneous equations" "I never could do it. I never got over quadratic equations" "We've done it, but I'm afraid that we didn't use algebra"

An interesting aspect of the arithmogons problem is that, even though you can solve the first part of the problem by setting up an equation in one unknown or by using simultaneous equations, a much quicker solution can be found by using an algorithm based on trial and error (or trial and improvement, to use National Curriculum jargon).

It is also the case that when you attempt the next part of the problem, which involves square-shaped arithmogons (Fig. 2), your algebraic methods appear no longer to work, whereas 'guessing' or 'trial and improvement' give you an instantly correct answer because the problem is such that there is an infinite number of solutions. This is a useful example to help raise the status of 'trial and improvement' as a legitimate, and occasionally powerful, method of problem solving, whilst simultaneously shattering the illusion of the invincibility of algebraic methods in teachers' minds.



**Figure 2**

This square arithmogons section of the problem also leads to frequent comments which reveal teachers' views, impressions and feelings about other important mathematical education issues, such as the following:

### **One right answer**

When working with square-shaped arithmogons, if it is the case that the sums of each pair of opposite squares are equal (Fig 2), then there is an infinite number of solutions.

This means that if you put any number in the first circle and work your way around the diagram you will get a correct solution. When teachers attempt this section of the problem they make comments like: "Goodness, that was a lucky guess!" or, when asked how they got their answer, give self-deprecating answers like, "I got it by accident." I usually respond by saying: "That's interesting. There is someone on the other side of the room who has a six where you have a five, and yet they seem quite happy with their solution." The most common reply from the teacher addressed is to say: "Oh! I must have made a mistake." This not only reveals a lack of confidence in their own mathematical ability, but also an inability, shared by many others, to accept that there could be more than one correct solution to the problem. Some teachers who check their solutions and compare them with their neighbours' different solutions seek reassurance by asking, "Oh, you can have more than one answer can you?"

### **Mathematical problems always have a solution**

If the sums of the two pairs of opposite squares are not equal then it transpires that no solution

exists: the solution set is the empty set. The actual problem that the teachers have to tackle on the worksheet 'very nearly' works for any numbers that they try, except that they always end up being one out each time. As you might imagine this causes great consternation, and usually elicits comments along the following lines: "We think that you've made a mistake in this question", or leads to some people persuading themselves that they have done it correctly, really, but must have just made a calculation error!

### **Mind set and problem solving**

The activity also provides an opportunity to discuss the need to 'upset set' when tackling problems. In connection with the fractional and negative solutions to the triangular configuration arithmogons the following comments have been made by teachers:

"I can't do it unless they're all halves, but this might not be anowed'~

"You wouldn't put negative numbers in there would you?" "No. it can't be, you can't have negative numbers."

Responses of this type can usefully lead into a discussion of the ways in which people react when they appear to be 'stuck' when solving a problem, and of some of the possible alternative strategies that they might employ to enable them to make progress.

"Feelings of panic, anxiety and even guilt"

This quotation from the Cockcroft Report succinctly sums up the feelings of many teachers about the activity as a whole. This can often be seen in their covering up their work as I mingle amongst them; by their comments about their inadequacies; by their excuses for not using the 'proper' method or algebra, or by their discussion after the exercise of their fears that their perceived mathematical lacunae were about to be revealed for all the world to see.

### **Miscellaneous issues**

The discussion that follows the activity often brings other related issues to the fore. The fact that, despite not being told to, they usually all work quietly on their own until the ice is broken can lead naturally to a discussion of the view held by many that mathematics is something people do on their own, or the view that working with someone else might be perceived as cheating.

A review of the original problem often elicits the observation that something which seemed difficult an hour earlier now seems 'obvious', and the implications of this observation for the teaching of mathematics can be readily addressed. Focussing on the various methods they used to solve the arithmogons problem can lead to a discussion of the wide variety of possible approaches, and of the difficulties we all experience at some time in understanding the methods used by others when we have solved a problem in our own way.

At the end of the session I always apologise profusely for having put them in a potentially embarrassing situation, and promise never to resort to anything similar during the rest of the course. I feel the need to stress that the next nineteen days are going to be about broadening their understanding of mathematics and mathematics teaching, about developing their confidence in their latent ability to succeed at the subject, and about helping them develop into reflective practitioners. I usually add that one positive aspect of the activity is that it should provide them with a wealth of material to fill the first few pages of their Learning Log!

Having to confront some of their own feelings about, and attitudes towards, mathematics, and

being afforded the opportunity to discuss and reflect upon some of these issues often helps teachers to move towards ensuring that the image of mathematics that they portray in their classrooms is similar to that portrayed in the 'Non-Statutory Guidance' and that which underpins Attainment Target 1. In addition to this the session also provides a useful foundation on which the tutor can build the rest of the course.

#### References

Bell, J., Brown, L. *et al* (1992) 'Sharing practice', *Mathematics Education Review*, 1,7-8.

Briggs, M. (1993) 'Bags and baggage revisited', *Mathematics Education Review*, 2, 16-20.

Buxton, L. (1981) *Do you Panic about Mathematics?* Heinemann.

McIntosh, A & Quadling, D. (1975) 'Arithmogons', *Mathematics Teaching*, 70, 18-23.

Sewell, B. (1981) *Use of Mathematics by Adults in Daily Life*, ACACE.

**© The Author**

**This article is converted from the print version published by the  
Association of Mathematics Education Teachers (AMET)**

**Articles available online at [www.amet.ac.uk](http://www.amet.ac.uk)**

**Original pagination of this article – pp32-38**

## **The AMET Response to the Review of the National Curriculum and Assessment**

prepared on behalf of the Association of Mathematics Education Teachers by

Derek Haylock

University of East Anglia

with contributions from a number of other members of AMET

*This response to Sir Ron Dearing's review of the National Curriculum was the result of a one-day AMET workshop at South Bank University on 3 June 1993. The response addresses the four key issues which constituted Sir Ron Dearing's brief.*

AMET is the association representing those who work in mathematics education in universities and colleges of higher education. We welcome this opportunity to respond to the questions raised in Sir Ron Dearing's brief to review the National Curriculum and its assessment framework. Naturally, as a group concerned mainly with mathematics education, we have focused our responses principally on the teaching, learning and assessment of mathematics.

### *Key issue 1: What is the scope for slimming down the curriculum itself?*

1.1 The primary purpose in reviewing and modifying the National Curriculum must be to make the documents more effective in assisting teachers in the planning of their teaching and their pupils' learning, not to make national testing arrangements simpler.

The Secretary of State in his letter to Sir Ron Dearing asserts that "testing arrangements are only as complex as the underlying curriculum requirements against which children are tested." This might imply that the curriculum must be modified to make testing easier, thus subverting the primary purpose of a National Curriculum, namely to improve teaching and learning. If teachers find the present Orders complex it is to some extent because their construction is dominated by an assessment and reporting framework, rather than by consideration of planning needs. We are concerned by frequent references to 'simplifying' a curriculum which must inevitably be complex if it is to prepare pupils for the twenty-first century.

1.2 The pace and nature of the review must take into account the serious effect on teachers' morale and motivation of frequent change.

In the case of mathematics it should be recognised that the Order has already been substantially revised recently, just after many schools had revised their internal planning and recording documents and purchased new text books based on the first version. Having now modified their resources to meet the demands of the new version schools require a period of some years before further substantial change.

1.3 Changes to the national curriculum should be based on serious research and evaluation of the implementation of the present Orders.

There are huge assumptions about the nature of learning mathematics built into the present Order which, because of the contracted timetable for the development of a national curriculum, have never been properly researched. Furthermore it will be several years before we can properly evaluate the implementation of the existing curriculum for mathematics,

particularly in Key Stages 1 and 2, where teachers have so much new curriculum material to absorb into their planning.

1.4 There is no justification or demand for immediate, wholesale change to the existing content of the mathematics national curriculum in any of the Key Stages.

Generally teachers are happy that the content of the mathematics national curriculum programmes of study is reasonable. As a core subject it justifies occupying a significant proportion of the pupil's timetable. As mathematics educators with access to a wider range of schools than most others involved in education we have a feeling that there have been a number of positive improvements in schools as teachers, particularly in Key Stages 1 and 2, come to terms with the Order. It is important that we do not lose these gains. These have included a greater recognition of using and applying mathematics, the incorporation of computer-based mathematical work such as LOGO and databases, and the recognition of ways of developing algebraic thinking in younger children.

1.4.1 The Key Stage 1 and 2 mathematics national curriculum must not be reduced merely to an unjustified emphasis on number and arithmetic.

In particular, it would be disastrous for the primary mathematics curriculum to be slimmed down to little more than context-free written calculations in primary mathematics.

1.4.2 The mathematics national curriculum must be protected from attempts to shift the emphasis to only those aspects of mathematics which are easy to test. The content of the mathematics curriculum should remain as broad as it is at present.

1.5 A number of questions about the existing mathematics national curriculum should be addressed in the long-term review of the subject.

Without proper evaluation and research we are not yet in a position to answer these questions but they are important and should be addressed in due time:

1.5.1 Mathematics is the only subject for which programmes of study are organised by level rather than by Key Stage: would a reorganisation of the programmes of study by Key Stage, with due recognition of problems associated with high and low attainers, facilitate teachers in their planning, particularly in Key Stages 1 and 2?

1.5.2 Level 4 is likely to be a huge barrier for many pupils, because of the sheer extent of the content: is the current emphasis on written arithmetic operations in mathematics levels 4 and 5 justified and appropriate, particularly in an age where many written calculation skills have been made redundant by the new technology?

1.5.3 How can the fundamental and essential processes of mathematics currently appearing as a discrete attainment target (AT 1) best be incorporated into the curriculum to ensure proper recognition of their value and importance in teaching and learning?

1.5.4 Should the Orders be developed to facilitate the use of crosscurricular work as a way of developing pupils' mathematical skills through purposeful activities in meaningful contexts, particularly for Key Stages 1 and 2, and especially for lower attaining pupils in all Key Stages, and how could this be achieved?

*Key Issue 2: What is the future of the 10 level scale for graduating children's attainments'!*

2.1 The 10 level scale and the associated statements of attainment are not a good basis for teachers in their ongoing assessment of pupils' progress.

Teachers assess pupils for a number of purposes, including:

- to motivate pupils
- to monitor pupils' progress
- to provide feedback to pupils and parents on their progress
- to diagnose weaknesses and assess strengths of pupils
- to inform decisions about future teaching
- to evaluate the teacher's curriculum choices

They are unlikely to find the imprecise statements of attainment in the various levels as useful for these purposes as their own records of their pupils' success in progressing through the mathematics programmes of study, where they will be assessing pupils against more precise objectives. It would be stretching the English language to suggest that what currently appears as a statement of attainment in a particular level represents "the next step for children's learning."

2.2 The 10 level scale for assessment is preferable as a system for reporting attainment to some of the alternatives being suggested.

The 10 level scale is of some use in allocating pupils to teaching groups, in passing on information from one school to another and in collating and reporting performances of pupils, such as in league tables of schools. [Even so, we should point out that the arithmetic performed on the levels achieved by pupils, as prescribed by SEAC, is based on the unjustified assumption that this is at least an *interval scale*, in which the difference between one level and another is assumed to be the same across all levels and all subjects!] The alternative which has been suggested of having tests with GCSE-type grades at the end of each key stage has serious disadvantages, in particular the loss of the sense of progression, the emphasis on failure associated with norm-referencing, and the shift to the idea of attainment being represented by differential performance on the same material.

2.3 There are important aspects of mathematics which do not fit comfortably into the 10 level framework.

In particular, the processes embodied in Attainment Target 1 are not amenable to the 10 level structure, since many of these processes represent performances which can be demonstrated and fostered in varying degrees of complexity across the age range of schooling. [This is likely to be equally true of the comparable AT in science and important processes in other subjects.] The retention of the 10 level framework for assessment and reporting purposes must not result in the diminution of the central place of these processes in the mathematics curriculum.

2.4 The use of the ten levels as a means of public recognition of progress must be supplemented by a system of assessment which recognises the progress of all pupils, especially those whose attainment is slow.

A significant number of low attaining pupils may not achieve beyond level three during their eleven years of schooling. Hence a pupil may be at level three, for example, for most of Key Stages 3 and 4. This system of recording progress will therefore suggest an apparent lack of progression for a number of years. There is an urgent need for a national system of assessment to be designed to recognise the progress that such pupils are making.

2.5 The ten levels should be retained within the mathematics curriculum for some years.

Although the ten-levels system is clearly arbitrary and based on some unjustified assumptions about the nature of learning mathematics, this is proposed in order to ensure a period of stability for teachers of mathematics.

*Key issue 3: How can the testing arrangements themselves be simplified?*

3.1 Teacher assessment should occupy the prime position in the national scheme for assessing pupils' progress.

Teacher assessment over a period of time, drawing on a wide body of pupils' work, is by definition a more valid form of assessment than written test papers done on a particular day. The present system of disregarding teacher assessments at end-of-key-stage assessment, except in ATI for mathematics, is totally unjustified, gives too much weight to tests of dubious validity and for which there is (to our knowledge) no published data regarding reliability, and makes teachers feel that their judgements cannot be trusted. There is much important material in the mathematics national curriculum (not just AT1) which cannot be validly assessed by any means other than teachers judgements and records of pupils' work in class.

3.2 Short written tests are an inadequate method of assessing much of the mathematics national curriculum at all stages, including Key Stage 4 assessment.

It is a false assumption that mathematics is the subject most amenable to assessment by short written tests. It is this naive view of mathematics which has led to the decision for a maximum of 20% coursework for GCSE. We strongly oppose this policy and recommend that it be reviewed urgently.

3.2a External national tests should be used to moderate teacher assessments by sampling both of the curriculum and of the pupils.

This is the most sensible way to 'simplify' the testing arrangements.

Already we have teachers admitting to 'teaching to the test'. To retain the dominant role of the external national tests and to attempt to solve the problems by simplifying the tests, (for example, by reducing Key Stage 1 and 2 assessment to mainly number) would therefore have a negative effect on the teaching and learning of mathematics in schools.

A system of external tests to moderate teacher assessment could therefore simply sample the achievement of various classes in various aspects of the national curriculum. Without the demand for every class to be assessed on almost every strand of the curriculum there would be scope for the development of more valid, reliable and sophisticated means of assessment. This was actually the role of the now disbanded *Assessment of Performance Unit*; their contribution to our knowledge of pupils' understanding of mathematics and their monitoring of national standards were greatly valued and largely uncontroversial in the profession. The demise of this Unit is much regretted.

3.3 A national system of graded assessments in mathematics available for use by teachers when they judge that pupils are ready for them would be a better framework for raising standards and recording progress.

If a national system of assessment for all pupils is to be used we would support what is known as the 'Scottish' system. For mathematics, for any given level there should be a number of focused assessments available contributing to the achievement of that level. This would provide a sense of progress for even the lower attainers who could work towards 'passing' one small component of a level at a time. These component assessments would be taken at different times when the teacher judges appropriate.



3.4 National assessment of mathematics should not be simplified by resorting to multi-choice tests.

It is well established that multi-choice tests disadvantage certain categories of pupils.

*Key Issue 4: How can the central administration of the NC and testing arrangements be improved'*

4.1 National tests for summer 1994 should be abandoned to allow time for proper development of a better system.

The timetable set by the Secretary of State for production of tests for summer 1994 following this review will not make possible a serious response to the outcome of this review.

4.2 The principle of stability in the curriculum must be central to the review process.

We repeat the importance of a period of stability even where change is required. It is essential that changes are properly thought out, through genuine consultation, and are not hardened up too quickly. SCAA should be enabled to resist political pressure to produce new policies and practices in an over-hasty manner, without proper and adequate trialling and monitoring. No changes should be introduced which would affect the arrangements for pupils already started on a Key Stage. Time must be allowed for proper INSET as *preparation* for change.

4.3 It is essential that SCAA establish a mathematics subject panel including professionals in the field of mathematics education.

4.4 SCAA should seek to restore a sense of ownership of the curriculum to teachers.

This is the biggest challenge facing Sir Ron Dearing. The prescriptive nature of the curriculum, the failure to consult adequately and to respond to consultation, and the centralising of control over the curriculum and its assessment have led to teachers feeling that they no longer own the curriculum they have to teach and assess. Sir Ron Dearing's open invitation to teachers to tell him their views about the review could be an encouraging start for what we hope will be a process leading to a curriculum and assessment system in which teachers feel their voice has been heard. A less prescriptive national curriculum and a greater valuing of teacher assessment are essential if teachers are to feel once again that they can make their own professional judgements and choices about teaching and learning. This sense of ownership would restore morale and professional pride within the teaching force and form a sounder basis than we have at present for the raising of standards in our schools.

© AMET

**This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)**

**Articles available online at [www.amet.ac.uk](http://www.amet.ac.uk)**

**Original pagination of this article – pp39-45**

## **The JMC Response to the Draft Circular on Primary Teacher Training**

Prepared on behalf of the Joint Mathematical Council of the United Kingdom by

**Derek Woodrow**

Manchester Metropolitan University

*This response was prepared in consultation with a number of colleagues and representatives of various organisations on the JMC. It is made in relation to the proposals as they affect the preparation of teachers to teach mathematics in schools, and in particular to teach the statutory orders for Mathematics within the National Curriculum.*

1. JMC is strongly opposed to the dilution of professional qualifications for teachers.

In recognising in its many reports the need for teachers to be properly and appropriately qualified in mathematics - as a means of securing at least the minimal conditions for the support of good mathematics teaching - these qualifications are fundamentally dependent upon the recognition that all professions need to have clearly established and maintained standards which can only be guaranteed by through properly accredited qualifications monitored by a properly responsible body. In this connection we welcome the acceptance that H.E.I. still have a crucial role to play in this monitoring, and it is vital that any new routes into teaching should have comparability of standards, guaranteed by H.E.I. with direct reference to existing courses. Comments in the OFSTED report on the earlier Licensed Teacher route must be addressed.

2. JMC welcomes the improved specification of criteria for courses contained within the appendix.

Whilst further development is still needed these criteria do represent a move forward. In specifying output criteria JMC believes that the overemphasis on hours within the document is redundant and inappropriate.

3. JMC welcomes the increased specification of time to be spent on the teaching of mathematics ALTHOUGH it rejects the notion that a specified commitment to 50 hours on 'arithmetic' - a category of activity not currently identified within the National Curriculum - will improve the teaching of number. There is ample evidence from HMI that number is learned more effectively within a balanced curriculum, and the specified criteria for students to meet the demands of the mathematics curriculum is sufficient without further quantification. Indeed throughout the debate it is quality rather than quantity which will ensure the proper preparation of students to be teachers. It would wish to challenge most strongly any implication that this increase in time will make the initial teacher training of teachers to teach mathematics sufficient for most newly qualified teachers. Training to teach is a continuing process and JMC would regret any reduction in the commitment to the principle so strongly expressed, for example in the Alexander, Rose, Woodhead report, that initial training, induction and inservice training are all vital and necessary.

Newly qualified teachers will need to continue their study of mathematics teaching and learning through their induction period and into their further training.

4. In a report from JMC some years ago it was reported that a primary school could expect to recruit a trained specialist mathematician once in a hundred years. Recent increased recruitment to mathematical studies in the REd. degree has led to some improvement but there is no possibility of recruiting sufficient specialist primary mathematics teachers through the PGCE courses. The provision of specialist advice can only be sustained through the study of mathematics in undergraduate teaching degrees.

JMC is concerned that the suggested six-subject degrees will not adequately prepare the specialist mathematics teachers for the primary schools.

If these were embedded in a commitment to substantial (one year equivalent) retraining at a later stage in the teacher's developing career then there might well be advantage, but such a resource commitment is clearly inconceivable.

5. JMC believes that the teaching of mathematics to young children: relying as it must on intuitive responses, and critical as it is to any future learning in the subject - demands if anything more *intellectual* development and preparation than any other stage in the teaching of the subject. JMC does not believe that as proposed the new Key Stage One route can achieve sufficiently high output standards to meet the demands on teaching mathematics for key stage one.

The experience gained by adults working with children rarely includes substantial learning of mathematics. The only vehicle for success within one year PGCE courses arises from the skills in learning acquired during degree study, it is this which enables this form of training to be justified. The premise implied by the hours differential between the 'mum's army course' and the PGCE course is that adults with no recent or extended H.E. study can learn how to teach mathematics quicker than students on the PGCE course which is an astounding claim. JMC does believe that experience should receive appropriate recognition but it is the CATE Criteria which largely prevent APEL from operating properly.

JMC believes that the CATE criteria (in respect of length of subject studies) should be made more flexible to allow adults with significant experience of working with children and with alternative training backgrounds to receive APEL onto existing courses rather than creating new ostensibly poorer routes into teaching.

6. JMC generally welcomes the imposition of a science qualification on entrants to a teaching course. The length of the delay in demanding such a qualification is surprising in contrast to when a similar ruling for mathematics was imposed. It could be the year 2008 before it is fully in place. JMC is also alarmed that science is not included in the 'mum's army' course - scientific knowledge begins to be accumulated throughout key stage one and earlier. JMC would be opposed to differential criteria for either admission to or for the substance of courses of primary teacher training whether they are for key stage one or at any other stage. Alternatives should be argued against variations in appropriateness rather than convenience and would be the substance of institutional development plans.

7. JMC believes that Higher Education is only valid in the context of students assuming responsibility for learning and knowledge. The criticism of courses which 'over-teach'

students is rooted in academic arguments related to the nature of degree study.

JMC considers that the condensation of four years into three by increasing the weekly workload of students is academically unacceptable.

There may be other ways of delivering a course of teacher training within a three year period - summer schools and extended terms - but the resource implication of such developments will need to be underwritten.

**This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)**

**Articles available online at [www.amet.ac.uk](http://www.amet.ac.uk)**

**Original pagination of this article – pp1-9**