CONTENTS

Page 1 Jeannie Billington, Sandy Cowling, Derek Haylock & Helen Jenner
Mathematics in Initial Teacher Education for Primary Schools

Page 11 Rod Bramald & David Leat
PGCE Secondary Mathematics Students' Responses to an Initial Teacher Training Course

Page 16 Mary Briggs
Bags and Baggage Revisited

Page 21 Graham Newson
Classes of the Eighties

Page 28 Bob Burn
Shears

Page 30 Peter Bland
GCSE Mathematics Coursework: Some Pupils' Views

Page 33 Gillian Hatch
A Personal Approach to Teaching Mathematics to Trainee and Inservice Teachers

Page 39 Derek Haylock
Priorities for INSET: a Survey of Secondary Mathematics Heads of Department

Page 44 Ekkehard Kopp, Janet Duffin & Adrian Simpson
The Hull Mathematics Workshop: Content and Outcomes

Mathematics in Initial Teacher Education for Primary Schools

A Position Paper, November 1992

The Joint Mathematical Council of the United Kingdom

Jeannie Billington

Sandy Cowling

South Bank University

Derek Haylock

School of Education, UEA, Norwich **Helen Jenner**

Harbinger County Primary School

The above group,with Derek Haylock as convenor, were commissioned by the JMC to prepare a position paper on the subject of initial teacher education in the primary phase. Margaret Brown also helped edit the final document. At the time it was expected that this would take the form of a response to the CATE proposals for reform of primary ITE. In the event these have not yet appeared, so the exercise was seen as providing an opportunity to feed into the CATE and DFE deliberations about future developments in primary teacher training. The authors would welcome comment from AMET members on the content of the paper.

1. Introduction

This paper focuses principally on issues related to the preparation of student teachers for teaching mathematics in Key Stages 1 and 2, through both BEd and PGCE courses. Since these courses can be designed to provide only *initial* training, some consideration is given also to the implications for further professional development of primary teachers in the area of mathematics during induction and through INSET.

2. Generalist and Specialist Teaching

2.1 It is assumed in this paper that, for the foreseeable future, the training of all student teachers on primary BEd and PGCE courses will have to include substantial courses in the teaching of mathematics in Key Stages 1 and 2. The combination of the ongoing shortage of new entrants to the profession with genuine strength in this subject, the central position of mathematics in the primary curriculum and the number of teachers available to an average primary scbool, suggests tbat most matnematics will continue to be taugbt by generalist class teachers.

2.2 Increases in the numbers of graduates with strength in mathematics entering PGCE primary courses and of undergraduates studying mathematics as part of their primary BEd degree courses would clearly be welcome. It is likely however that such new entrants to the profession would be looking to use their strength in mathematics as consultants or coordinators within a school, rather than as specialist teachers. [see section 5 below]

2.3 In primary schools of average or below average size a substantial increase in specialist subject teaching is clearly impractical. In larger primary schools there may be scope for some of the mathematics of Years 5 and 6 pupils to be taught by specialists. The demanding nature of mathematics up to level 6 in the National Curriculum is a strong argument in favour of such a development.

2.4. However, a move to more specialist teaching of mathematics in primary schools would only be desirable if other core and foundation subjects were treated likewise, in order not to reinforce a view of mathematics as being a difficult or elitist subject.

2.5 It would still be undesirable for *all* the mathematical experience of pupils to be handed over to a specialist and to be separated from their other learning in school, since this would rule out the benefits of pupils developing mathematical skills through the meaningful contexts generated by a cross-curricular or subject-focussed topic.

3. Competences

Circular 9/92 (DFE 25 June 1992) *Initial Teacher Training (Secondary Phase)* specifies the competences expected of newly qualified teachers in the secondary phase within the proposed partnership model of teacher training. Although this circular refers only to secondary teacher training, it is likely that a similar categorisation of competences will be used for primary training. Specific issues related to competences for newly qualified primary teachers are raised in the areas of subject knowledge, subject application, class management and assessment. A significant question to be addressed is: what can reasonably be expected of a newly qualified generalist primary teacher emerging from a BEd or PGCE course in terms of competences in teaching mathematics?

3.1 Subject Knowledge

3.1.1 Since it is expected that some pupils in Key Stage 1 will reach level 4 in some aspects of the Mathematics National Curriculum, and some pupils in Key Stage 2 will reach level 6, it is essential that the initial training of all student teachers for these Key Stages should aim to ensure that they have a thorough grasp of the mathematics in the National Curriculum up to these respective levels.

3.1.2 This competence should be more than just knowledge at the level of skills performance required in many routine examination items. The prerequisite of a Grade C (in future a grade 6 or 7) in GCSE mathematics, whilst welcome, is certainly no guarantee of the level of competence needed by primary teachers. Those who teach the subject require a grasp of the conceptual structures of mathematics they must teach, an awareness of the ways in which elementary mathematical ideas connect together, and an understanding of mathematical processes and routines, in order that they have a sound basis of personal knowledge from which to formulate their explanations to children and to plan appropriate sequences of learning experiences.

3.1.3 In addition, primary student teachers should have substantial experience, at their own level, of the kinds of mathematical activity now explicitly included in the Mathematics National Curriculum in Attainment Target 1 - problem-solving, practical work, investigating within mathematics - to achieve a shift in their perception of the subject away from the notion of mathematics as a collection of recipes and routines. The recently-imposed 20% limit on GCSE course work, and in particular the availability of non -course work syllabuses which cannot properly assess Mal, is likely to contribute to an increase in this deficiency in the mathematical experience of those entering teacher training.

3.1.4 The limited time available in an over-crowded primary BEd or PGCE course for the kinds of mathematical input outlined in 3.1.2 and 3.1.3 especially suggests that it is unrealistic to expect this quality of competence to be achieved in initial training by many students, particularly in view of the overall curriculum demands of teaching in Key Stage 2.

3.1.5 Three approaches to remedying such deficiencies may be considered: (a) The development of new models of BEd and PGCE primary courses to give longer time and greater weight to curriculum components, in particular to mathematics. Particular problems are associated with the current arbitrary requirement for the equivalent of two years degree-level "subject studies" in BEd courses and the inadequate length of the PGCE one-year course for primary training. Any moves towards partnership models of training must take into account the crucial importance of specific mathematics curriculum input to primary initial training and ensure no loss of time for such input. [see section 4 below]

(b) The provision of an adequately-funded and properly-planned in-service entitlement for all primary teachers. [see section 6 below]

(c) A shift to more specialist teaching in Key Stage 2 mathematics. [see section 2 above]

Our view is that a combination of these approaches would be most effective.

3.2 Subject Application

Although it would not be appropriate in this paper to specify all the aspects of teaching mathematics which should be included in a mathematics curriculum course for primary student-teachers, some which are of particular importance are highlighted below.

3.2.1 Primary student-teachers should recognise the central importance of developing pupils' confidence with number and have effective strategies for tackling the extensive demands of the National Curriculum in this area (Attainment Target 2).

3.2.2 These strategies should not be restricted to drill in context-free written calculations, but should include the use of a range of purposeful activities in meaningful contexts.

3.2.3 For example, primary student-teachers should be encouraged to recognise the potential for developing pupils' mathematical skills and understanding through the contexts of their work in other areas of the curriculum, thus ensuring that pupils do not view mathematics as a text-book activity unrelated to the real world.

3.2.4 They should learn to plan appropriate activities for pupils across the range of mathematical experiences implied by Attainment Target 1 (practical work, solving real problems, investigating within mathematics) and to use these as a vehicle for the development of pupils' skills and understanding in the other Attainment Targets.

3.2.5 They should develop the abilities to explain mathematical ideas and procedures and to plan sequences of learning activities in ways which show some awareness both of the conceptual structures involved and of how children might come to understand these.

3.3 Class Management

Alongside the general teaching skills of primary class management, which apply across all subjects, there are a number of aspects which are of particular significance for student-teachers' preparation to teach mathematics.

3.3.1 Students should learn to organise their classrooms to provide opportunities for focussed talk about mathematical activities and for pupils to explain mathematical ideas to each other.

3.3.2 They should learn to organise their classes so that they themselves have substantial opportunities to explain mathematical ideas and procedures at an appropriate level to groups of pupils.

3.3.3 They should develop the organisational skills required to differentiate effectively between pupils in terms of their mathematical competence, both by outcome and by activity.

3.3.4 They should learn to organise their classrooms and lessons so that pupils can access when required the resources and materials essential to mathematical experience.

3.3.5 They should recognise the advantages and limitations of different methods of organising pupils' mathematical activities, such as individualised learning, paired work, small group activities and whole class teaching, and be able to judge when it is more appropriate to use a particular method.

3.3.6 They should learn how to use a commercially-produced mathematics scheme effectively as a resource for teaching and not as the sole detenninative of the pupils' mathematical experience.

3.4 Assessment

3.4.1 The widespread view that assessment is easier and more straightforward in mathematics than in most other subjects is misguided. It must be recognised that the level of competence in this complex area which can be achieved in an initial training course is limited, particularly since primary teachers must assess pupils in so many different subject areas.

3.4.2 The area of National Curriculum mathematics which presents the most problems for assessment for all teachers is Attainment Target 1. To be thoroughly competent in this a teacher would be able to (a) select or devise appropriate activities in the three categories of practical work, solving real problems and investigating within mathematics, to generate evidence of pupil's attainment level in each ofthe three strands of AT1; (b) implement and monitor these; (c) collect evidence from the pupil's activities; (d) interpret this evidence in tenns of statements of attainment, (e) argue the case for the evidence suggesting a level of attainment.

3.4.3 Experience suggests that this is an unreasonable expectation for newlyqualified primary teachers and must be a major focus for in-service training. Initial teacher training students should, working alongside an experienced teacher, at least have opportunities to select appropriate activities in the AT1 categories, implement and monitor them with a group of pupils, discuss the outputs in tenns of the three strands of AT1 and make an infonned judgement about the levels of attainment indicated.

3.4.4 Competence in assessment is demanding in the other attainment targets for mathematics as well. Teachers must be able to (a) provide pupils with wellfocussed activities to generate evidence of attainment; (b) make use of a range of assessment modes, such as observing, discussing, conferencing, evaluating written outcomes; (c) use these appropriately at individual, group or whole class level; (d) incorporate these into their day-to-day teaching.

3.4.5 Again this is an unreasonable expectation for newly-qualified teachers and must continue to be a major focus for in-service training. Initial training courses should at least ensure that students have opportunity to engage in (a) and (b) in section 3.4.4 with a small group of pupils across all attainment targets.

3.4.6 Newly-qualified teachers in primary schools should also be able to contribute evidence to a school's existing system for keeping records of progress and work completed. They should be able to recognise the advantages and limitations of various systems of record-keeping, and the purposes to which the data can be put. This area is particularly significant in mathematics where the inappropriate collection of marks for written tests and the invalid aggregation of these are not uncommon practices.

4. **Implications of Parnership Models of Primary Teacher Training**

4.1 Circular 9/92 established the notion of partnership schools taking joint responsibility with HEIs for the planning of initial training in the secondary sector. Whilst the development of closer partnership in initial training between schools and training institutions is supported by all concerned, the model to be developed in primary schools is likely to prove far more complex. Apart from general issues related to matters such as staffing, finance, primary teacher's workloads, parental expectations and the availability of expertise in training, there are some particular concerns in the area of mathematics.

4.2 Because primary teachers are in the main generalists rather than specialists, few primary schools are likely to be able to offer in-depth support for all ten basic subjects. In particular, in mathematics, few schools will be confident that they have the necessary experience of and expertise in, for example, the mathematical experiences required for Attainment Target 1, the development of algebraic thinking through Attainment Target 3, and the full range of experiences of data-handling and probability in Attainment Target 5, to offer the support necessary to students in initial training. It is essential, therefore, that developments in partnership, whilst welcome in many respects, do not diminish the opportunities for all primary student-teachers to receive substantial expert mathematics curriculum input.

4.3 There is evidence that many primary teachers lack confidence in their ability in mathematics and consequently resort to over-reliance on commercially-produced schemes and inflexible teaching styles. Anxiety about their own level of understanding in mathematics is similarly widespread amongst many students embarking on initial training courses. The least effective way to support such primary initial trainees would be to place upon the schools the major responsibility for developing the competences in teaching mathematics and in classroom organisation in mathematics highlighted in sections 3.2 and 3.3.

4.4 Initial training students need opportunities to reflect on practice in more than one school in order to recognise the advantages and limitations of different approaches to mathematics teaching and learning and to develop their own strategies for successful mathematics teaching.

4.5 Experience suggests that partnership will be most effective when students, working alongside teachers, are engaged in action research in their classrooms, under the guidance of the HE tutor. Participation in the process of identifying and analysing specific problems in mathematics teaching in a particular classroom context, followed by proposals for modifications in practice and evaluation of the outcomes, is the kind of school-based activity which will benefit all parties concerned.

5. **Students with aspirations for curriculum leadership in mathematics** 5.1 Students with particular strength in mathematics, either BEd students doing subject studies in mathematics, or PGCE students with substantial mathematics in their previous studies, should be encouraged to work towards curriculum leadership in mathematics in primary schools. As indicated in section 2 above it is more likely, in the foreseeable future, that they will aspire to this role rather than that of a specialist mathematics teacher.

5.2 Students having aspirations towards curriculum leadership in mathematics in primary schools should be aware of the need for their own development in three major areas: (i) their knowledge and understanding of the nature of mathematics; (ii) their understanding of how children learn mathematics and how that knowledge influences teaching methods and assessment techniques; and (iii) the development of the organisational and interpersonal skills they will need to fulfil their future consultancy roles.

5.3 Each of these three major areas should be considered in terms of what competences should be developed, when, and by whom. These competences are in addition to the competences expected of the general newly-qualified primary teacher outlined in section 3 above. It is unrealistic to expect initial training to do more than to seek to begin the development of the competences in these three areas. Access to in-service training in curriculum leadership should be an entitlement for all primary teachers with a mathematics coordinator's role.

5.4 In terms of their knowledge of mathematics, curriculum consultants in primary schools should be able to give advice and leadership to their colleagues from a basis of a greater appreciation of the power which mathematics has to help us to control, explore, improve and describe our environment. This includes an appreciation of the role which mathematics plays in our everyday lives as well as some knowledge of how pure and applied mathematicians go about their jobs in industry, commerce and research institutions. An awareness of historical and cultural aspects of mathematics is also essential so the consultant can appreciate the ways that mathematics has grown through the identification and solving of problems. The mathematics consultant should be able to set a good example to colleagues in the integration of aspects of information technology, such as LOGO, databases and spreadsheets, into primary school mathematics teaching. In addition, they should have studied the elementary mathematical ideas of the National Curriculum from an advanced standpoint and have substantial personal experience of the processes of mathematical problemsolving and investigating at their own level.

5.5 It is a shortcoming of many mathematics subject studies components of BEd courses that, because of organisational requirements for BEd students to study their specialist subjects alongside students working for different degrees, the kinds of experiences of mathematics outlined above are sidelined in favour of more traditional components of undergraduate mathematics courses. This leaves a substantial requirement for further professional development through in-service training.

5.6 Primary BEd and PGCE courses should contain components which provide those students who possess strength in mathematics and aspirations for curriculum leadership in this subject with opportunity to develop a deeper appreciation of how children come to understand mathematical ideas. They should consider how to apply this appreciation and their knowledge of mathematics in their own teaching, in planning mathematics schemes of work, in incorporating mathematics into cross-curricular work, in developing effective assessment techniques, in promoting an appropriate range of stimulating mathematical experiences for children, in planning and organising their rooms, in enhancing pupils' language and communication skills, in fostering pupils' interest and motivation, and in using a variety of teaching methods. In these respects they can seek to influence and encourage good practice amongst their future colleagues in schools.

5.7 The assessment of mathematics within the National Curriculum framework is an area where the curriculum consultant will be expected to give leadership to a school. Even if this a major focus in initial training components for mathematics specialists, there will remain a substantial continued requirement for further professional development in this area through in-service training.

5.8 Expertise in mathematics and mathematics teaching are necessary but not sufficient conditions for fulfilling successfully the role of a mathematics consultant. Also required are organisational and inter-personal skills. The curriculum consultant should aim to keep abreast of developments and literature in primary mathematics; to update their own ideas and resources; to work with colleagues in devising a school policy and mathematics record systems; to organise parents' evenings and school-based in-service in mathematics. They need skills in discussion leadership, observation and analysis, and face-to-face counselling. Their enthusiasm for mathematics must be balanced by sensitivity to other primary colleagues who may not share the same level of confidence. This aspect of professional development for the mathematics curriculum consultant should be formally recognised as an in-service entitlement.

5.9 Potential curriculum leaders in primary mathematics should be encouraged in their initial training to identify their own needs in terms of the three major areas for professional development stated in 5.2. The provision of adequately funded and substantial in-service training to meet these needs should then be an entitlement for all teachers. Professional development through INSET of primary mathematics consultants should be the key to promoting better practice in primary mathematics.

6. Induction and INSET

6.1 The central aim of Initial Teacher Education is to equip newly qualified teachers with the relevant skills, competences and understanding to teach mathematics effectively within the framework of the National Curriculum. However, further professional development, through Induction and INSET, is essential to maintain a well qualified and motivated teaching force and to provide a viable career structure.

6.2 Induction is a particularly critical period. The newly qualified teacher has the opportunity to transform teaching potential into competent, sustained classroom practice. Support and guidance from experienced colleagues is therefore vital in the successful transition from student to effective class teacher. This will be more forthcoming in schools with well defined mentoring schemes and agreed development plans for mathematics. The new teacher will be able to consolidate and expand curriculum expertise acquired during ITE and explore some of the more challenging areas of the National Curriculum. There is much evidence, for example, that ATI, representing as it does a way of working rather than a content area, presents teachers with particular difficulties. Unless newlyqualified teachers are encouraged to address the issues of mathematics application, reasoning and proof encompassed in ATI early in their professional practice, there is a danger that they will adopt too narrow an interpretation of mathematical activity and an inappropriate over-emphasis on context-free number skills.

6.3 As indicated above in section 3.4, record-keeping and assessment are particularly problematic for newly-qualified teachers. Assessment techniques in mathematics are complex, requiring good classroom management skills and skills of observation. Whilst ITE can provide students with a sound knowledge of models and procedures and some practical experience in implementing them, the induction period provides the best opportunity to support newly qualified teachers working alongside colleagues using appropriate assessment strategies in a range of contexts. The ability to provide a differentiated mathematics curriculum with progression between elements is generally not easily acquired and is dependent upon effective record-keeping and assessment. Less experienced teachers need guidance here if they are to carry out adequate teacher assessments and eventually take responsibility for administering National Curriculum tests at Key Stages 1 and 2.

6.4 The third element in the professional development of a primary teacher should be an entitlement to regular, planned INSET. As has been indicated throughout this report, there are a number of aspects of good mathematics teaching which will still require development for most primary teachers postinitial training. Through INSET teachers can use their practical experiences to reflect upon their own classroom practice, deepen their understanding of mathematics, learn new skills and competences and meet future challenges.

6.5 Because this range of activities is so varied, a "mixed economy" model of provision is the most appropriate, with contributions from both school-based inservice and that provided by HE institutions. Schools themselves may develop increasing expertise tbrough greater involvement in the school-based elements of ITE and the expansion and development of mentoring, H'there Is an expansIon of partnership models of training. Collaboration between schools and HE institutions in negotiating in-service provision can also be particularly valuable as this facilitates access to subject expertise and the mathematics education research community in a familiar, school-based context. The DFE 20-day mathematics courses for primary teachers are an excellent example of this.

6.6 As indicated in section 5 above, the role of the mathematics co-ordinator is probably best addressed through an INSET framework. Co-ordinators have an increasingly complex function in primary schools, ranging from advising on mathematics content and working with colleagues to refine teaching practice to managing formal assessment procedures and participating in moderation panels.

6.7 The challenge of devising an appropriate curriculum and strategies for teaching mathematics to low-attaining pupils, particularly in Key Stage 2, must be a major focus for INSET if the overall performance of British pupils in international comparisons is to be improved. Initial training cannot be expected to equip student-teachers adequately in this demanding aspect of primary teaching skill.

6.8 Equally important is the need to support primary teachers in developing an appropriate mathematics curriculum for the highest attaining pupils. Simple acceleration through National Curriculum levels is an inadequate model for meeting the special needs of such exceptional pupils. Again it should be accepted that initial training cannot be expected to equip student-teachers adequately for meeting this challenge and this must be a major focus for INSET.

6.9 Training for the administration of National Curriculum tests at Key Stages 1 and 2 also falls within the INSET remit, as it is inappropriate for inexperienced teachers to have sole responsibility for this aspect of assessment.

6.10 INSET, however, should be viewed as the focus for evaluating, implementing and disseminating innovations in mathematics education by responding to initiatives from practitioners, Government and other sources. Teachers need a structure within which to reflect upon their own practice and enhance their mathematical understanding and pedagogic understanding.

6.11 Induction and INSET must be funded adequately to be meaningful and central to the development of a high-quality teaching force. Changes in funding anticipated in moves both to a greater emphasis on school-based training and to an expansion of GMS for primary schools must be scrutinised to ensure sufficient resources for viable, acceptable and effective induction-support and inservice provision.

This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)

Articles available online at www.amet.ac.uk

Original pagination of this article – pp1-10

PGCE Secondary Mathematics Students' Responses to **an Initial Teacher Training Course**

Rod Bramald & David Leat

School of Education, University of Newcastle-upon-Tyne

As part of an ongoing evaluation of the secondary PGCE course, we have been looking very closely at the responses of students to an in-depth evaluation of part of their course. Initial results would suggest that our current cohort of secondary mathematics students have significantly different values from their peers in other curriculum subject areas when measured by their responses to questions about /WW they rate their satisfaction with different aspects of the PGCE course and whether or not they would recommend the course or module to next year's students. The work reported below is of a preliminary nature and whilst the claims made about 'differences' are indeed statistically significant, even for these relatively small sample sizes, they are at this stage very tentative.

The Sample

We have approximately 170 students following a secondary PGCE course. They follow one of seven different subject specialisms in the proportions shown in Table 1 The actual number who formed the sample was 138 out of the 171 (81%) and as such represented a very large opportunity sample of the whole population. However, no attempt is made to suggest the sample was in any way stratified or made representative in any other way of either this particular population or of the population ofPGCE students generally.

Subject	**No. students on course**	**No. of respondents**	**Percentage of respondents**
Mathematics	31	27	87%
English	25	20	80%
Geography	20	17	85%
History	18	12	67%
Modern Foreign Lang	30	24	80%
Science (B,C & P)	41	34	83%
Religious Education	6	4	67%
Total	**171**	**138**	**81%**

Table 1: Numbers of Respondents by Subject Specialism

It should be noted that the small size of the RE group makes their results highly reactive to outline data. Readers should therefore read any results related to the RE group with extreme caution.

The Data

The students were asked to rate their *.ati.faction* with various aspects of the course: their *Curriculum* specialism course and four generic modules relating to *Assessment, How Children Learn, Information Technology*, and *Classroom Management.* They were also asked if they would *recommend* that particular part of the course to next year's students. The relevant results of their responses are reproduced in Tables 2 and 3 below. The data was collected using a Likert 5 point scale where 1 represented 'Definitely Yes', 2 - 'Yes', 3 - 'Uncertain', 4 - 'No', and 5 -

'Definitely No'. Thus it can be seen that low scores represent 'high' or 'positive' *recommendation* or *.atiBfaction* ratings.

How satisfied are you with this component?

	Geog	Hist	RE	Sci	Math	MLan	Eng	Mean
Curriculum	1.60	1.54	1.00	1.61	2.19	2.04	1.70	1.79
Classroom manage't	1.65	1.41	3.00	1.94	1.70	1.66	1.95	1.79
Assessment	2.53	2.08	3.00	2.56	3.26	2.50	2.75	2.68
How children learn	2.64	2.75	2.50	2.47	3.15	2.91	3.40	2.86
Information Tech'gy	2.60	2.80	3.50	2.90	2.20	3.20	3.90	2.93

Table 2: OVERALL SATISFACTION (Means)

Would you recommend this component to next year's students?

	Geog	Hist	RE	Sci	Math	MLan	Eng	Mean
Curriculum	1.25	1.09	1.00	1.11	1.77	1.39	1.15	1.25
Classroom manage't	1.35	1.08	2.75	1.47	1.26	1.22	1.21	1.34
Assessment	2.29	1.50	2.75	2.47	3.00	2.17	2.35	2.41
How children learn	2.58	2.41	2.25	2.44	3.08	2.79	3.00	2.72
Information Tech'gy	1.80	2.10	2.50	2.10	1.80	2.60	3.45	2.16

Table 3: RECOMMENDATION RATINGS (Means)

Analysis of Data

Informal inspection of this data suggests that there appear to be particular differences between the different curriculum group responses. For example, humanities students (Hist and Geog) appear to be the most satisfied across the five components. Modern Languages, Science and RE form the middle ranks whilst Maths students (especially if one removes the IT component) are the least satisfied and give the lowest ratings to their particular curriculum area. Having said this, it is important to note that even so, their ratings are still positive!

A one-way analysis of variance was done upon the responses to the two basic questions of, 'Were you satisfied with the component?' and 'Would you recommend it to next year's students?' for the *Curriculum* component of the course. It is accepted that this is not necessarily the best form of analysis for data of this nature (single score Likert scale) but it is emphasised again that this is intended to be initial, base-line data. However, accepting this, the results were quite illuminating. (The details of the statistical analysis can be obtained from the authors.)

This analysis established that there was an apparent difference between the mathematicians and at least most of the other subject area groups (with the mathematicians being the least satisfied with the curriculum component and least likely to recommend it to next year's students). The magnitude of differences at the 95% confidence interval between the respective groups was then explored using the Least Significant Differences Range test. The results showed that the group of mathematicians was indeed significantly different from each of the other groups beyond the 0.05 level and even beyond the 0.01 level for all groups except Religious Studies group. (It should be remembered here that the RE group was a very small sample.)

The results of these analyses were of sufficient significance to warrant further investigation beyond the 'Curriculum' part of the course and so each of the modules was analysed in a similar way. What became apparent was that 'Satisfaction Ratings' were producing more interesting results than 'Recommendation Ratings' and so the authors decided to concentrate upon the former. The results are summarised below.

Summary of Results for Satisfaction Ratings The CLASSROOM MANAGEMENT module

The RE group's low score, indicating a high level of satisfaction, was significantly different from all the other groups. No other group differed 5ignificantly from any of the others. In light of the extremely small sample (four) in the RE group (see above) this result needed to be treated with extreme caution.

The ASSESSMENT module

No single group stood out as being different, but the mathematicians, with a high score indicating a relatively low level of satisfaction, did have significantly different results from the historians and modern linguists.

The HOW CHILDREN LEARN module

The English and mathematics groups (relatively low levels of satisfaction) differed significantly from the scientists, but no single group differed from all the others.

The INFORMATION TECHNOLOGY module

Again, no single group stood out, but the mathematicians' relatively high level of satisfaction did differ significantly from the English and Modern Languages groups. Also significant was the difference between the ratings of the English group and the scientists.

Conclusions and Discussion

From this initial look at our results, it would appear that this particular group of secondary mathematics PGCE students were indeed different from all their peers in their collective expressions of satisfaction, being significantly less satisfied than some other groups with the curriculum component and the components on how children learn and assessment, whilst being significantly more satisfied with the Information Technology component. It is accepted that there are a great number of cautionary notes that must be added to this conclusion. This is, of course, only one particular group in one particular University with these particular tutors. But it is still very interesting that with such relatively small numbers such high levels of significance have been found.

These apparent differences then raise several questions. In general terms, do students from different curriculum disciplines have differences in their learning styles and, if so, are these satisfied to varying degrees by the course? One can speculate for example that the mathematics students have arrived at their level of success (graduates) through a system which values 'right' answers and which has a low tolerance for uncertainty. Initial Teacher Training is not renowned for providing 'golden rules' which can be applied with confidence to achieve predicted outcomes. This argument would appear to be supported by the fact that the mathematicians are the most positive group about the IT module where uncertainty is limited and there are more 'right' answers - "If you do 'this' then 'that' will always happen" etc. However, the possibility cannot be ignored that, as a group, the mathematicians are also those most familiar with, and therefore less inhibited by, computers which presumably engenders confidence and interest. Certainly, Kolb (1984) found in MIT that students from different faculties had different learning styles. Furthermore, students with atypical learning styles for their faculty were frequently at odds with the environment and culture of that faculty. Can we explain the apparent stronger identification with their particular course of the humanities' students or the relative degree of discordance of the English students? Are the methods of teaching used in education courses or the ways of approaching issues more familiar to the geographers and historians?

The authors invite colleagues around the country to repeat the experiment and explore their circumstances to see if this is a one-off'result or whether there is indeed a genuine, repeatable difference to be found between groups of PGCE students as defined by their subject specialism. Ideally, more than one question about satisfaction can be asked so that some form of satisfaction scale can be constructed rather than the single score available in this first run. Such a move would certainly add power to the analysis and weight to the conclusions.

The work of these two writers will continue with investigations into learning styles (as defined by Kolb's learning inventory) and, of course, into next year's cohort to see if the result is unique or repeatable. Other questions which will be pursued are those concerning selection criteria, teaching and learning styles within the specific units of the course and, ideally, direct comparison with at least one other ITT institution. (But clearly this last idea will depend upon someone else being sufficiently interested in the idea to want to collaborate!)

Reference

Kolb, D. A. (1984) Experiential Learning. Prentice-Hall, Englewood Cliffs, New Jersey.

Original pagination of this article – pp11-15

CLASSES OF THE EIGHTIES

York PGCE Mathematics Students· where are they now?

Graham Newson

Department of Educational Studies, University of York

I arrived at York in September 1979 *and over the next nine academic years supervised* a *total of* 89 *Mathematics PGCE students through their one-year course. My quota for most of those years was twelve - luxurious times as, since then, it has risen to fifteen and now twenty (albeit at leaBt with some part-time help). In the summer of* 1989 I *sent* a *questionnaire to all the* 89 *students containing* a *variety of questions about their present teaching job, attitudes to teaching, etc. Over the next six months I received* a *total of* 52 *replies.*

Where Are They Now?

The breakdown of the 52 replies was as follows:

39 still teaching

9 left teaching for other employment

4 left teaching to raise a family

Of the 39 still teaching, their type of institutions broke down as follows: 31 in secondary comprehensive

4 in independent schools

3 in Sixth Form Colleges

1 in College of Technology

One of the questions asked what salary scale they were currently on. The results for the 34 in the state school system were:

Allowance D -	1
Allowance C	0
Allowance B	2
Allowance A	3
MPG	28

Omitting teachers from the last three cohorts, the 15 who left between 1980 and 1985 answered as follows:

D	1
C	0
B	2
A	3
MPG	9

Thus even for the teachers with at least three years teaching experience, nearly two-thirds were still on MPG.

The nine who had left teaching for other employment were spread evenly over the nine cohorts.

Cohort	Responses	Left Teaching
79-80	1	0
80-81	5	1
81-82	8	1
82-83	4	1
83-84	7	1
84-85	6	2
85-86	5	1
86-87	9	2
87-88	8	0

The new occupations of seven who had left were:

Senior Field Officer (Conservation Volunteers) H.M.Inspector of Taxes

Chartered Accountant

Social Worker

Computer support for an R&D company Calculations Assistant (Assurance) 'Travelling'

Of the four teachers who had left to raise a family, two expected to return to teaching, one definitely did not anticipate going back and the fourth did not complete the relevant question.

Opinions on Mathematics Teaching as a Career

The table which follows shows the responses of the ex-students to the question: if asked by a young Maths honours graduate, interested in becoming a Maths teacher, whether you would advise them to proceed, would you:

	Teaching or raising family	Left Teaching
a) strongly encourage them	1	0
b) encourage them with reservations	24	4
c) be unsure what to advise	10	0
d) discourage them with reservations	3	2
e) strongly discourage them	2	2
	(not all completed)	

As can be seen, the eight who had left teaching were exactly split on encouraging potential teachers. The four who gave b) as an answer seemed to be, by implication, accepting that, though teaching might not have been for them, they could see that some people would find it fulfilling though they should be aware of conditions etc.

The five teaching or raising a family who answered d) and e) gave invited comments as follows:

- *Career prospects very poor*
- *Standards in school have dropped dramatically. Facilities for pupils and staff are severely restricted. Very few textbooks or resources were available for use. Pupils were never given*

textbooks for sole use.
- *With less motivated children I would be very unhappy*
- *Lack of promotion prospects, lack of pay, lack of parental support, inefficient senior management*
- *Lack of status of the profession and lack ofsupport from parents and government*
- *They would have to be very committed, not just interested to withstand the increasing pressures.*
- *Ever increasing workload. Decrease in resources available. Too*

many initiatives.

Though in some senses it is encouraging to see over half ofrespondents willing to encourage such a person (even with reservations), 19 out of the 48 answering this question (almost 40%) were unable to be positive.

In another question, eight disadvantages of Mathematics teaching as a career were listed. Respondents were asked to rank them in order of significance; they did not have to rank them all and there was an opportunity to name and rank any other disadvantage(s) not listed. Tallying the rankings, I added together the total number of times each disadvantage had been ranked either 1,2, or 3 in order.

The results were as follows:

Disadvantage	**Times ranked 1, 2 or 3**
Poor career opportunities	29
Poor salary scale	26
Increase in Administration	23
Disruptive children	17
PoorsUrrtingpay	16
Directed changes in curriculum	14
Others	12
Lack of Inset	9
Teaching to Exams	5

The 12 *others* cited came into the following categories:

Lack of status	5
Time pressure	2
Work load and conditions	2
Marking	1
Stress	1
Other staff	1

As can be seen, poor starting pay was not seen as significant as the related perceived problems of salary scale and career opportunities. Again, problems of children and the curriculum were seen as less important than increases in administration (likely to receive even more prominence if the exercise was repeated today).

A similar question was asked inviting the respondents to rank a list of 'positive aspects' ofMaths teaching. The results were as follows:

Positive aspect	**Times ranked 1, 2 or 3**

Ability to use own initiative in classroom	38
Working with children	36
Variety of work pattern	25
Working with Mathematics	23
Working with other teachers	9
Oth&s	6
Job security	5
Good pay	1

The six others came into the following categories:

Holidays	4
Mobility (of job location)	1
Immediate rewards (instead of very long term planning as in most jobs)	1

It is interesting to reflect that the mounting pressures in recent years have come in the area of 'directed' teaching and testing (National Curriculum, SATs etc) possibly at the expense of being able to use individual initiative - and much more record-keeping/administration/accountability - possibly at the expense of priority to actually working with children.

In-Service Training

Types of INSET courses attended since finishing the PGCE course (not including what were then known as Baker days).

Type of course	Teachers attending one or more
1 year full-time	0
1 term full-time	1
1 year part-time	3
1 term part-time	2
2 days/week full-time	17
Evening/afternoon or Day	31
None	6

A total of 41 teachers replied to this question. Of these, 35 claimed to have attended at lest one course. The predominant pattern was for one-off evening, afternoon or day courses (some had attended several). Longer courses were much less prevalent.

Of 32 teachers answering a question about LEA Teachers' Centres, 27 had one with Mathematics resources available and 5 did not. Asked how much use they made of it the answers could be broadly categorised as:

Regular use	2

Occasional	5
Very little	6
None	14

A related question about Mathematics teaching organisations was asked. Of the 39 still in teaching, 13 belonged to the Mathematics Association and two to the ATM. Six of their schools belonged to the MA, nine to the ATM.

Use of Micros in Maths Lessons

The respondents were asked to give brief comments about the nature and frequency of their use of micros in Maths lessons in their current PQlSt. The 37 responses could be categorised thus:

Very frequent use	1
Regular	5
Occasional	6
Rarely	10
None	15

Five of the responses could not be easily categorised. It is interesting that only six teachers claimed anything greater than occasional use. (Would the picture be different today?)

Miscellaneous Data

i) The ex-students were asked which main published scheme was being used in their school at 11-16 ages. From replies received the picture emerged as:

Scheme	No. of schools
SMP	13 (2, part of 11-16 range only)
IMS	9 (3, part only)
Understanding Mathematics	4 (3, part only)
Rayner	3 (all, part only)

Five other schemes were in two schools or less.

ii) Teachers were asked whether they had their own classroom base: 27 replied Yes; 11 replied No.

iii) Teachers were asked to name any non-text books in Maths which they had found especially valuable. Several books got one mention (including The Bible 'to help get you through the first term') but the following got two or more: The Bolt series, Mottershead Investigations in Mathematics, Starting Points, Head of Department (MA), Away with Maths, These Have Worked for us at 'A' Level (ATM).

General Comments Made

At the end of the questionnaire, the ex-students were invited to make any other comments on their experience as a Mathematics teacher since completing their PGCE. Below is a representative cross-section of replies:

• *I've enjoyed teaching - it makes a big difference if you have enthusiastic colleagues.*

• *The teaching has been fun. The teachers have been consistently disappointing.*

• *I'm glad I chose teaching. However, I* do *feel undervalued and we lack the proper resources.*

• *I have found .... very few Maths teachers who are really interested and enthusiastic about the subject.*

• *Two out of* 11 *Maths graduates in our Department has meant that the majority of my time is taken up teaching 'A'level.*

• *Schools in a sorry state of repair .....• lack of space. lack of books and resources . • Morale has gone down ..... only the children remain the same (thank goodness).*

- *Teachers are generally too insular.*

• *I am increasingly disheartened to be in a profession where to 'get on' you must get out of the classroom.*

• *Class sizes are far too large in some schools .... .individual work schemes used with the less able are particularly difficult .*

• *Very rewarding and worthwhile job. Very hard work. It doesn't get any easier - you keep challenging yourself.*

- *I'm enjoying myself immensely.*
- *The workload is getting worse. Morale is very low.*

• *There are a lot of very poor Maths teachers out there ..... need to encourage better graduates into teaching. not just the ones who have nothing better to* do.

• *Really good fun most of the time. I can't stress too highly the importance of taking an interest in the children.*

• *Good points: still excitement in many lessons for me. Bad points: low morale of colleagues. Lack of status (outside education and within it) for the basic grade teacher.*

Summary

Though this data is some three years old, it seems likely that the general feelings will have remained similar if not intensified. One could argue that much of what has happened in the last three years will only have served to accentuate worries and frustration voiced. We at York are lucky enough to attract generally well· qualified and highly motivated PGCE students; given this, it is disturbing that nearly a fifth of those who responded had left teaching already. It is noticeable too that broadly speaking initial pay is not seen as a critical negative factor (though the career structure may be) but that lack of resources and conditions and status are. Several replies specifically mentioned lack of status and a consistent undertone throughout is the feeling of being undervalued. It is not hard to speculate that the media and the government have only served to strengthen this feeling in the last three years.

This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)

Articles available online at www.amet.ac.uk

Original pagination of this article – pp21-27

Bags and Baggage Revisited

(or another suitcase for the rack) Mary Briggs

School of Education, Open University

This article was written in response to marking the examination scripts for the Open University course EM236 Learning and Teaching Mathematics. This year (1992) was the first year of presentation for this course. This is a second level Education course available to both undergraduate and associate students, aimed at teachers teaching mathematics to children aged 5-16. *Although the course is available to this wide range of teachers those quoted in this article are mainly from within the primary sector. In writing this article I wish to draw attention to the anxieties about mathematics in those who teach the subject, how inservice courses can help teachers to recognise these anxieties and those of their pupils, and how possibly to begin to alleviate the problems.*

Some Extracts from Examination Scripts

• *As with most practical mathematical tasks on this course which are out of the sphere of my limited mathematical knowledge, my heart sank when I read question* 3 *and my brain said* "I *can't do that"* .

• *At the start I had a block, I hate mathematics and I hate being asked questions on it. However I eventually re-read the question--- I couldn't remember what sum of consecutive numbers means! It makes me feel sick thinking about it. I checked the example and thank goodness I understood!.*

• *My initial reaction when presented with a task like this is to panic. In fact I panicked so badly that I began to tackle the task quite incorrectly. I began by trying to make a pleasing pattern of sums that added up to all whole numbers between 0 and 10. After realising that I hadn't been working on consecutive numbers I started the task again-now leaving very little time for the task.*

These extracts were taken from a number of recent examination scripts written by teachers on a mathematics education course. The question they were asked to answer is the following:

In this question you are asked to describe and analyse your work on a numerical/algebraic task. First you need to work on the task given below.

Consecutive numbers.

Investigate the ways in which whole numbers can be expressed as the sum of consecutive whole numbers., e.g.

7=4+3

9=4+5=2+3+4

a) Give a brief account of your work on the task. b) Analyse what you have done in terms of:

- *stressing and ignoring;*
- *expressing generality;*
- *the following aspects of mathematical knowledge: imagery, experience, techniques.*

This was the third of three questions on the paper. The other two focused on planning and teaching issues, and observing and evaluating learners' experiences. Question three was an obvious attempt to engage the teachers in some mathematics of their own and to put them in the place of the learner. The reader may feel this question is relatively simple, however, what none of us can afford to forget is that for those people who feel real anxiety about mathematics this would not be a straightforward exercise (even if you do not need to resolve the mathematical question to answer the examination question). Whatever age you teach there will be pupils/students/teachers who feel this anxiety about mathematics.

Carrying Baggage

In the original article "Bags and baggage"(l), John Crook and I explored both student-teachers' and teachers' experiences oflearning mathematics. Here I am not only looking at a stressful situation - 'sitting an examination' - but at being asked to do some mathematics during that experience. These teachers carry their baggage into the examination room with them and open the bags to reveal insights in their answers to analysing the problem.

• *I do feel that an important factor of the analysis is the panic at the start. This must reveal something very important about my own mathematics education which I am hoping I never pass on to my pupils: the fear of getting it wrong. When I was educated in the 60s, it was by mathematics teacher and textbook.*

This raises tbe question of transmission of anxiety. Most of these teacners were concerned about not passing on their fears to the learner, for example:

• *I hope that even my little learners never suffer from* a *similar fear and I 1wpe that my teaching encourages them* to *express themselves freely and never be afraid of getting it wrong.*

• *I know what the lack of experience can do to one's self-esteem. It creates* a *massive mental block which children need to be freed from in order to progress---myself included.*

The other area mentioned was the lack of access to any practical apparatus to help these people engage in mathematics.

• *My experience as* a *learner of mathematics has influenced my work greatly. I am afraid of numbers because I had bad experiences of arithmetic teaching in primary school. The emphasis then was heavily on mental calculation. I badly needed the physical objects to act on. But these were taken away. I did not understand the procedures or algorithms that were taught. I gained no fluency in using numbers. I had no understanding therefore I could not practice to gain any fluency. I feel this personal framework of understanding-practicefluency is important for me and it has not happened in personal experience.*

Students are allowed to take calculators into mathematics examinations. But what about additional apparatus?

• *I would find it hard to help children solve this sort of problem but I would probably use something like 'Cuisenaire' or 'colour factor' rods so that the children could see the patterns emerging. Equipment such as this would have helped* me *too!.*

If Cuisenaire rods would alleviate anxiety in an examination then why not make them available? (There are practicalities to be considered here: since this question is not a seen one, who can anticipate what each student might require to help answer it?)

The anxieties expressed about using the numbers with confidence were related to their size and the potential complexity of the operations which might be required.

**I thought if I kept to small numbers where I have more confidence, maybe I could* see *the pattern.'*

• *My mental ability is very limited when it comes to number bonds, and mathematics in general.*

• *I feel uncomfortable with numbers. I am nervous about getti11lJ them wrong especially if it means exposing my ignorance publicly.*

My own work, looking at people's automathsbiographies, included a question for teachers about whether their experiences of mathematics influenced their choice of age range to teach. A number of teachers linked their choice of teaching younger children with their lack of confidence in mathematics. There were echoes of this in some of the comments in these examination papers. The most striking was:

• *I immediately chose to work with numbers below 10 because at present my experience is working with very young children and although they have experience and knowledge of much larger numbers (with use of calculators) it is mainly with lower numbers that I work, I think I feel rather out of depth with large numbers.*

However hard it seems we try to stow it, the baggage stays firmly attached to us as we travel through our mathematical experiences. (I have deliberately changed to plural as I am aware of my own and others' baggage all the time.)

• *We all bring our experiences to bear, whether they be good, bad or nonexistent. I know my own mathematics education involved a constant turnover of mathematics teachers, I know I feel threatened.*

All this could be leaving the reader with the feeling that, whatever anyone does, mathematics anxiety is here to stay. I would like to take a more positive view. These teachers clearly express their feelings and they show awareness of the potential for transmitting their fears to their learners. I feel this makes them less likely to put their learners in difficult situations without support than those who have no awareness of the overwhelming anxiety which "doing mathematics" can bring to some. In completing the course, including the examination, these teachers have raised their own levels of confidence: this is evident in the last two extracts I have chosen.

• *However I must admit I feel much better having tackled this exercise now than I did to start with. I find I have used imagery but I have difficulty trusting in it to begin with for fear of being wrong.*

• *I ignored my experience which told me I couldn't and relied on a technique to try to do it.I can now try to think round problems and try to see it from the pupils perspective and at least give it a shot which I would not have done prior to this course.*

This is not intended to be an advertisement for this or, indeed, any other mathematics education course but it shows the potential for raising awareness that mathematics anxiety exists and how these teachers are learning to cope with the issue. I feel they have come a long way despite no marks being awarded in this examination for comments in the affective domain!

Perhaps it is not surprising that comments like those above appeared in the examination answers since the students were directed to consider the issue of the influence of prior experiences upon the learner throughout the course. Early in the course material there was reference to Dick Tahta's article "Understanding and desire". This was followed by reference to the article mentioned above, "Bags and baggage", and later to Robert Nicodemus's "Psychoanalytic perspectives". In reviewing the answers to the examination questions and this course in its first year it has become clear that there should be consideration of the inclusion of marks for comments in the affective domain in next year's examination paper.

References

Briggs M, Crook J, "Bags and baggage" in Love E and Pimm D (eds) 1991, Teaching and

Learning Mathematics, Hodder and Stoughton.

Tahta D, "Understanding and desire" in Love E and Pimm D (eds) 1991, Teaching and Learning Mathematics, Hodder and Stoughton.

Nicodemus R "Psychoanalytic perspectives" EM236 Course material. Open University.

Original pagination of this article – pp16-20

Shears

Bob Burn

School of Education, Exeter University

Shears are easy to demonstrate with a *pack of cards and have become* a *part of school maths and teacher training. They add significant variety to the discussion of geometric transformations. In the early days of geometric transformations in school, courses about shears, e.g.* [4], *dealt with readily generalisabk properties. More recently, e.g.* [1, *p* 377] *and* [3], *properties of shears peculiar to the geometry of real numbers, or peculiar to two dimensions, have been added, often without acknowledging that* a *change in the basis of the discussion has taken place. This has happened in BEd courses.*

Test your own awareness on the following list of shear properties. Which properties hold for a plane shear, and which *also* hold for a shear in 3 dimensions, with a *plane axis* of fixed points? Which properties hold when the number field is arbitrary, and which hold only when the field is real?

1. Shears preserve area.
2. A shear is uniquely determined by its axis and shear factor.
3. A shear is uniquely determined by its axis and the image of one further point.
4. Under a shear, the line joining a point to its image (if distinct) is parallel to a

line of fixed points.

5. The matrix of a shear has determinant 1.
6. A matrix is a shear matrix ifits determinant is 1 and its trace is 2.
7. The group of shears wi th a given axis is isomorphic to the field of numbers under addition.
8. Any matrix with determinant 1 is either a shear matrix or a product of shear matrices.
9. A triangle and its image under a shear illustrate Desargues' theorem.

The properties which only hold in 2 dimensions are 1, 2, 6 and 7. The properties which only hold over the real field are 1 and 2.

Properties 1 and 2 are the most restricted. But the area-preserving property not only recalls parallelograms on the same base and between the same parallels, and invites a generalisation to volume in 3 dimensions, but also provides motivation for property 5 with all its generality.

The notion of shear factor, by contrast, is limited. Increase the dimension and shear factor no longer specifies a particular shear. In real 3-dimensional space, there is an infinity of shears with a given plane axis and a given shear factor.

Furthermore, even in two dimensions, the possibility of defining a shear by axis and shear factor depends on special properties of the real numbers. Over the field of rational numbers, for example, there is no shear with axis $y = x$ and shear factor 1.

The treatment of shears in [3] shows how the notion of shear factor is based on the supposition that there are rotations taking any line through the origin to any other line through the origin.

A further practical discomfort with shear factor is that the shear $(x, y) \to (x + y, y)$ has shear factor +1 but $(x, y) \to (x, x + y)$ has shear factor -1.

This article has been written because of the fondness for shear factors which the author has found amongst BEd examiners. There are many other interesting and more significant aspects of shears to investigate.

The general properties of shears are not metric. Geometrically a shear maps lines to lines, and preserves one family of parallel lines. But a shear alters angles

and lengths. It is just field axioms that make the transformation (x, y) -+ $(x + ay, y)$ carry lines to lines and make matrices with determinant 1 form a group. To see the idea of a shear working in another context, it is quite fun trying to write down the matrices for plane shears, fixing the origin, over the field Z_a. Four little groups emerge.

For a school level treatment which could lead on to this kind of generalisation see [4]. For a more advanced treatment, but still one which has been used in a BEd course, see [2].

References

[1] Anton, H., 1991, *Elementary Linear Algebra*, John Wiley. [2] Burn, R.P., 1985, *Groups: a path to geometry*, Cambridge. [3] Smart, D., 1988, *Linear Algebra and Geometry*, Cambridge. [4] S.M.P., 1974, *Book Z,* Cambridge.

Original pagination of this article – pp28-29

GCSE Mathematics Coursework: Some Pupils' Views

Peter Bland Huntington School, York

& Department of Educational Studies, University of York

Having listened to teachers, educationalists, the examination boards and parents all expound on the various merits and demerits of coursework I felt I would like to consider the pupils' point of view - especially with the new GCSE examinations and their various options on the horizon. This article is an account of the results obtained from a *questionnaire given to* 173 *Year* 11 *pupils, studying at my school, (an* 11-18 *mixed comprehensive) just before they left to take their GCSE examinations. The students were entered for the NEA Mathematics GCSE, Syllabus A, and ranged from level R to P.*

Percentage of Coursework Preferred

Of the 173 pupils, 81 were boys and 92 girls. The table below records the style of assessment (i.e. ratio of course work to examination) they would prefer:

% Course work	% Examination	No. pupils	% pupils	No.boys	No.girls
0	100	17	10%	16	1
20	80	64	37%	22	42
25	75	16	9%	7	9
30	70	16	9%	6	10
50	50	26	15%	14	12
100	0	2	1%	1	1
other combinations		32	19%	16	16

Although there were 11 more girls than boys in the sample, it was interesting to note that mostly boys (16:1) would prefer an examination-only GCSE. These 16 were spread over all the ability levels.

The 19% of pupils which represented other combinations chose various ratios which ranged from 10% course work and 90% examination, to 90% course work and 10% examination.

The results indicated that 10% would prefer an examination-only assessment, 37% would prefer the 20:80 combination and 53% would prefer other combinations.

Do you like course work?

The pupils were asked whether they liked course work. The replies indicated that 14% of the students did not like course work. Of the 86% who did, 37% were *yes* and 49% *sometimes.* They were also invited to give the reasons for their responses.

Among the reasons given by those who said 'yes' were the following:

- *I enjoy solving mathematical problems. However I do think that on some of the earlier pieces of coursework we had to spend far too much time writing up the coursework neatly and sticking diagrams in the right places.*
- *It is challenging and allows you to reach your full potential whilst working in the absence of examination pressure.*
- *It's different from just working on text books and it gives you* a *better chance of getting* a *good grade.*
- *Change from normal classwork and spreads* a *little of the work to the final grade across the year.*
- *You can give as much effort as you want to get* a *good mark. Coursework is an opportunity to help boost your examination grade.*
- *It helps your examination grade (or it can if you work). There is not as much pressure as there is in examinations.*
- *Because there is no examination pressure.*
- *It gives the pupil more time to do the work, under less pressure, so they are therefore more likely to do better.*
- *I like coursework* as *it helps you solve problems and it helps the grade in your examination.*

Among the reasons given by those who said 'no' were the following:

- *It takes away time when you could be learning something.*
- *Usually involves investigating theories which means doing the same thing repeatedly.*
- *It's too boring especially investigations, they seem to go on forever and get you nowhere.*
- *Boring, totally pointless.*
- *Unfair to certain pupils and therefore pupils can fail by bad coursework but do well in the examination.*
- *It's tedious and boring and sometimes difficult.*
- *I know* a *lot of people who have* a *lot of help on coursework by their parents. With 100% examination it is* a *fair test of that one pupil. The system at the moment is incredibly unfair.*
- *I am not very good and would prefer an examination. I find examinations quite easy but don't like coursework*

Because we have too much coursework in other subjects and so we are weighed down with work, by the time of our examinations we can't be bothered.

- *I do not believe that coursework is worthwhile in maths, pupils tend not to learn as much as they would taking a topic area head on in class.*

The comments made by those who said 'sometimes' included the following: • *It's OK but it does get a bit boring at times.*

- *Some coursework takes too long - the shorter pieces are better to concentrate on.*
- *Some topics are boring and monotonous.*
- *Sometimes course work becomes tedious, although it does give the opportunity to use your full capability to do your best without the pressure of examination conditions.*
- *I find it totally uninteresting, but some of the courseworks are OK*
- *It depends what the topic is as sometimes it is a topic that is useless and so it tends to be boring.*
- *Some courseworks aren't as enjoyable as others.*
- *Sometimes the course work is not explained and the amount of time allocated is not*

always long enough.
- *Many pieces are boring, although others are not quite as easy. Most are too time consuming.*

Conclusions

It was a pleasure to find that the majority of students do not dislike some coursework element contributing to their final examination result, for the new criteria stipulates a 20% coursework 80% examination combination. However, as one who was involved in setting up course work for the GCSE examinations in June of 1990 and 1991 with the advice of the examining boards, advisers and resource books on the subject, I am struck by the implication from the comments made by the pupils that the messages that this is an interesting way of studying mathematics and that it is relevant to the understanding of the subject are not getting across to a substantial group of the candidates. Hence, for us, we need to review our coursework, particularly in the light of the new criteria set out in the 1994 syllabus, both in terms of the areas in which we set coursework and the approach by which it is implemented.

I would be interested to know of the results of similar surveys undertaken in other schools. A more detailed study could be taken to identify which pieces of course work the students liked and which pieces they did not like. A similar survey could be conducted on other subjects: first to discover what the students felt about the course work in that discipline and secondly to find if students perceive some subjects being more appropriate to coursework than others.

This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)

Articles available online at www.amet.ac.uk

Original pagination of this article – pp30-32

A Personal Approach to Teaching Mathematics to Trainee and Inservice Teachers.

Gillian Hatch

Centre for Mathematical Education

The Manchester Metropolitan University

This article arose from the special issue of Mathematics Teaching entitled How I Teach *[1J, in which* a *small quotation from this is included. When faced with the question of how I personally teach mathematics I was unsure about my willingness to try to answer it, despite my regret that little is written about teaching academic mathematics to adults. I was helped by* a *phrase in an unpublished paper by* a *colleague of mine, Andy Pickard. It concerned teachers on the reflective MA-in-teaching course which he runs; but it focussed the task I was considering:* " *... the sense of their own distinctiveness as* a *teacher becomes more marked ...* " *Now the task seemed to be one I could see value in. But it felt vital that somehow I should find out what truth my own impressions of my teaching contained. Then I realised the advantage of teaching adults. I decided to ask all the students I taught during one week to write* a *few sentences about me as* a *teacher. I guarantied no reprisals and that they would be anonymous except where I could identify handwriting! I have also made use of module summary writings done by inservice students as part of their course.*

The Context

I work in the Centre for Mathematics Education at Didsbury School of Education The Manchester Metropolitan University. Currently all my teaching is mathematics rather than overtly professional work. The learners in my groups are all teachers or intending teachers and range from those in the first year of a BEd course to those doing an MSc in Mathematical Education. The groups are small, usually about twelve. There are two further facts which make the context distinctive in ways which are significant to me: the syllabus I teach is usually one which I designed, and if there are examinations then I set them. All in all, it amounts to a privileged teaching situation and one which allows me to experiment.

The fact that they are all involved in teaching permits me to use the learning process to help them to learn the mathematics, to become aware of the nature of that learning and to draw from it implications for teaching in school. Clearly there are differences between the teaching of children and adults but it appears that this approach does create valuable insights. This two-level approach is made absolutely explicit within the inservice courses and is an essential part of the work the teachers undertake. Within initial training it is present but not so clarly emphasised, however from time to time I stop and discuss the learning process.

A Group Teaching Method

I work with the group sitting round a table. We write on a pad of A3 paper with large felt tip pens. This means that we can work on several sheets at once, go back to earlier sheets and that any member of the group can do the writing. Some people have to read upside down and so the group have to sit close together. It is certainly one of the things that is distinctive about my teaching. At times it raises violent comment, but it changes the whole dynamic of the learning process. It seems to decrease the authority role of the teacher and therefore it makes it easier for people to break in. It also helps to convey the idea that it is the *group* who are working *together* on the mathematics. Obviously one does not have to be stupid about such a recording method.

There are times when the algebra is so complicated that we do return to the blackboard, but in this case I often ask a student to record the group's work. If we work on paper I write up a set of notes after the lecture which describe what was done.

Students' Perceptions

So now how do the students perceive what I am doing? First they recognise the diminution in the domination of the teacher. The physical change achieved by sitting at a table seems to me to be significant in relationships and this is recognised by many of the students.

- *Gill likes to become one of the group rather than have a teacher pupil barrier.*
- *.... a leader working from within a group*

I find it easier to be aware of individual's reactions when I sit round a table with them. I realised recently that they are no longer a 'class' but very much a set of individuals, and that I am aware of each one of them separately.

- *Gill tries to get to know her class very well, to create, I assume, an ease between leader and pupil.*
- *I feel that you also know exactly when somebody understands and when they don't - stopping for discussion to explain to those who don't.*

I found it surprising that one student at least had a sense, even in a group of 12, of being taught as an individual.

- *You can feel as if you are being taught on a one-to-one basis.*

I now find it very uncomfortable and very distancing to use a blackboard - a sense of being out of communication. The use of a pad seems to have a two-fold effect. First it allows the students to feel, in some sense, in control of the work, as the pace of the lesson seems to be set by the class; secondly, it creates a strong sense of group-creation of the mathematics:

- *Gill uses the group as resource rather than telling us how to do this and that.*
- *The large piece of paper and several felt tips approach is one which, once you learn to read upside down, is both communicative (ie any / all of the group members can interact with it at once) and informal (in that it transforms the learning situation from a teacher / student situation to a group discussion).*

One issue is whether the students feel that the whole process is a fraud, ie. that I am really still in control of the outcomes. This has to be a concern. There is an unavoidable tension between the investigational commitment and the need to avoid spending hours trying to get students to re-invent the mathematical wheel in an unrealistic fashion. So there are times when the situation has to be prestructured. The most extreme example is when students are trying to construct proofs as a group. I was pleased to get comments that showed both an awareness of the agenda that I bring into a lecture but also a freedom within that agenda to control pace and detail of content:

- *There is an element of flexibility inherent in the investigational approach which makes the learner feel more responsibility for an involvement in the learning process.*
- *Doesn't necessarily stick to plan if something more interesting comes up (although usually makes us go back)*
- *Everybody's suggestions followed even if way off track*

- *Firm control over the situation, does not let things go off at a tangent*

unless sees some profit in this.

Groups working this way repeatedly astonish me by the mathematics they can achieve. I suppose I do ask pertinent questions, I do sometimes "hear" the more helpful suggestion but it does not seem to destroy their feeling of corporate endeavour. People react to this in varying ways:

- *Group involvement all the time make the concentration span longer.*
- *Makes every student feel valid. a part to play in the group.*
- *We work by group investigation which can be very lively and stimulating but potentially also rather stressful*
- *Working together in a. group situation ha.s been beneficia.l in that it has created a relaxed satmosphere which has allowed all members of the e group to contribute.*
- *Feel more at ease discussing in this situation than facing board.*

My emphasis on understanding is widely understood and the situation seems to enable most people to admit problems.

- *In terms of the understanding of the student Gill seems to hold this as of paramount importance often if there is* a *loss of time.*
- *Her informality creates an atmosphere that puts me at my ease and I am now able to say when I do not understand something rather than being afraid to interrupt.*
- *Encourages students to explain to each other or question each other.*
- *It is also good the way in which you get others to explain to someone who doesn't understand as the explainer gets an even clearer picture.*

There is widespread awareness of my intentions in supplying post-lecture notes. However I have perhaps not always empowered people to take their own notes ifit helps, although some people do do this and find it helpful.

- *Allows us to "think" rather than having to write notes by writing up our notes for us.*
- *I prefer to take my own notes. I think personal notes should be taken during the lecture to back-up class notes.*

The inservice students have to take their own notes because we only meet weekly. This does not seem to be a problem and even for them I often write up my version of some of what happened. I have not, however, solved all the problems. At best I write up the notes the same evening, duplicate them and get them back to the students the next day. Often the exigencies of school visits, or days without lectures make this impossible. Perhaps I should encourage a little more note-taking during lectures .... ?

Two of the fourth year group have recently taken over the writing-up task, because they like to review the lecture the same evening. The other students seem to read their notes more critically than mine, which is useful! I am considering whether or when I might actively attempt to extend this to other groups. In the case of the fourth year the impetus came from them. This I found interesting because I always felt it was the logical extension of what I was doing, but hesitated to impose it.

For me there are important issues about the students who still hide their lack of understanding in this less-formal situation. Most students would echo this uncompromising comment: I *feel I can* *stop the lesson if I don't understand.*

However, there are a few acknowledgements of ongoing reluctance, such as: I *feel I would not want to do this too often for fear of appearing foolish.*

I am sometimes aware of people behaving like this, but strategies to deal with it without making them feel "foolish" are hard to find. I look for oblique excuses to get them to join in. I believe that the continual informal group interaction means that this happens less often than in a more formal situation. Indeed one inservice student commented on what happened when a group was really responding well in this situation.

- *If the teacher has the ability to nurture even the slowest learner without showing any signs of impatience* or *exasperation then it is possible to create a working atmosphere within the group so that your peers also have this ability.*

The Teacher's Experience

What is my experience of working in this way? Well, it is often an alarming feeling of swimming in a sea of mathematical ideas and only just keeping my head sufficiently above water to have any idea of the profitability of the current line of endeavour. But working with a group to build up the solution to a particular standard problem (eg second order linear differential equations) for the first time and seeing the solution gradually emerge is exciting in the extreme. It is also stressful because of the unpredictability of what happens and the need to choose exactly the right nudge which keeps the momentum going but leaves the ownership of the problem with the group. One needs to be very confident with the material that is under discussion or very committed to the method to be willing to risk it. This year I am enjoying the stress of applying the process to a new content area.

So where do the problems lie for me? The most obvious problem is ensuring that students know that I know that reading upside down can be irritating. I try to meet them halfway by learning to write upside down but I do have to be very careful, for as someone said, "Life on the outskirts can be frustrating". Groups need to be prepared to arrange tables and themselves sensibly to facilitate the work. They are sometimes surprisingly reluctant to do this and I do not know why. As long as I do go back to the board if there is very technical work to be done or hand the pad round, then these problems can be overcome. The research I conducted for this article at least confirmed that the majority saw enough advantage in the pad to tolerate the method.:

- *Initially I found it mildly irritating because it is not always easy to see, but now I would not welcome a change to a more formal classroom situation.*

I have to remember to allow enough time for appropriate review and consolidation. It is easy to rush onwards on the back of the group's enthusiasm and leave them only with a sense of the process, not the content:

- *.... it leaves me without a clear view of what conclusions we come to or by what structure they are interrelated but with a clear memory of how we got there ....*

In preparing students to teach I believe there is advantage in their meeting a teaching method sufficiently different to force them to reconsider what is possible. Their surprise is reiterated by many:

- *something I have never experienced before*
- *Gill's teaching method is a completely new one to me*

- *a totally new approach*
- *very individual way of teaching*
- *a revelation to me*

• *at first I could not adjust to the method but now I am fully confident in it.*

So, although what I have described is distinctive to me and my personal teaching context, I hope I may have influenced a few people to try a less conventional, possibly more risky approach to teaching mathematics in their own situation.

Reference [1] Mathematics Teaching 139, June 1992

Original pagination of this article – pp33-36

Priorities for INSET

A Survey of Secondary Mathematics Heads of Department

Derek Haylock

School of Education, UEA Norwich

A survey of heads of mathematics departments in 200 secondary schools in England provided valuable information about the perceived priorities for professional development for secondary mathematics teachers. The areas where INSET is perceived to be the greatest priority are ,he assessment of Attainment Target 1 *in the National Curriculum, and the problems of meeting the needs of both high and low attainers in mathematics.*

Background

As Chair of AMET, I was invited to join an *ad hoc* group at the Department for Education, meeting with DFE officers, NCC and SEAC officers, representatives from other associations concerned with mathematics education and OFSTED, and a number of educational broadcasters. The first meeting in October 1992 focussed on training needs for secondary mathematics teachers.

The DFE officers presented a summary of what were perceived to be major priorities for in-service training for teachers of National Curriculum mathematics. These were: (i) subject knowledge of non-specialists, with priority accorded to primary teachers; (ii) management and curriculum leadership skills amongst coordinators and heads of department; (iii) curriculum development; (iv) assessment; (v) pedagogical issues such as strategies for continuity and progression, integrating ATs 2-5 into AT1, classroom management, and developing a range of teaching styles and assessment; (vi) the needs of pupils with SEN.

On behalf of AMET I outlined the priorities for secondary mathematics INSET which were identified by AMET members through discussions at the annual conference in September 1992. (See AMET Newsletter number 5 December 1992 and the paper presented to the Joint Mathematical Council in November 1992.) These focussed on five major areas: (i) teaching styles, (ii) assessment and recording, (iii) pupils with special needs, (iv) mathematics content, and (v) heads of department.

Survey of Secondary Schools

Following this meeting questionnaires were sent via members of the AMET committee to heads of mathematics departments in 200 secondary schools in East Anglia, the Sheffield area and around Newcastle-upon-Tyne. The letter with the questionnaire explained thsat as Chair of AMET I had then opportunity to advise DFE officials about training needs for secondary mathematics and encouraged the heads of mathematics departments to use this survey as a way of making their input to the discussion. Replies were received from 137 schools.

The questionnaire suggested 14 areas where further professional development amongst those teaching mathematics in secondary schools might be welcomed. Heads of department were asked to indicate the extent to which they would agree with these, by scoring each from 0 to 3, where

00 means: *not an area where professional development is needed by their mathematics staff;*

3 means: a *major priority for professional development for their mathematics staff.*

The survey thus provides valuable information about the perceived priorities for professional development in secondary mathematics from those in the schools with the major responsibility for the subject.

Results of Survey

The 14 areas proposed are listed below, using exactly the wording which appeared in the questionnaire, with the mean score attributed by the respondents to that area in brackets. The statements have been ordered according to their priority rating from highest to lowest.

1. Learning to select or devise appropriate activities in the three categories of investigating, practical work and solving real problems, to generate evidence of pupil attainment in AT1; to administer and monitor these, to collect evidence from the pupils' activities and to interpret it in terms of the three strands of AT1. (Mean score: 2.27)

2. The use of the range of activities implied by AT1 to promote learning in the other ATs. (Mean score: 2.22)

3. The assessment of AT1 - working with colleagues from other schools in INSET activities to establish a shared understanding of what is to count as evidence of a pupil performing at a particular level in the AT. (Mean score: 2.18)

4. How to challenge the more able pupils, in ways other than simply resorting to the acceleration model which might be perceived as the implication of the structure of the mathematics national curriculum. (Mean score: 2.06)

5. The development of strategies for motivating low-attaining pupils, for diagnosing their difficulties and for determining an appropriate curriculum. (Mean score: 1.96)

6. Developing different modes of assessment, including observation, discussion, the interpretation of written outcomes, individual and group assessments. Using these appropriately in teaching to generate evidence of attainment, to diagnose difficulties and to plan future teaching. (Mean score: 1.91)

7. The use of resources in mathematics teaching, especially computers, calculators and graphic calculators. (Mean Beore: 1.87)

8. Heads of Departments: In-service for heads of department might focus particularly on the development of leadership and management skills, the running of school-based INSET, and the development of school mathematics policies and schemes of work. (Mean score: 1.77)

9. The organisational skills required to differentiate effectively between pupils in a mathematics class in terms of their competences, both by outcome and by activity. (Mean Beore: 1.74)

10. Structuring lessons to provide activities which generate useful, diagnostic evidence of pupils' learning. (Mean score: 1.69)

11. Interactive teaching, including the skills oflistening, questioning, motivating, observing pupils; in particular developing effective ways of engaging pupils in mathematical activity.

(Mean score: 1.56)

12. The advantages and limitations of different systems for recording pupil progress in mathematics. (Mean score: 1.49)

13. Mathematics Content: it is essential that all mathematics teachers have a secure grasp of the content ofthe mathematics National Curriculum. This would include an understanding of the nature of mathematical activity implied by AT1 and a grasp of the conceptual structures of the concepts and principles embodied in the other attainment targets. (Mean score: 1.42)

14. The use of problem-solving as an essential context for mathematical learning. (Mean score: 1.39)

All 14 areas scored a mean rating of 1.39 or more. Clearly, each of them is recognised as an important INSET need by a significant number of the heads of department who responded in the survey. Table 1 below shows the actual percentages of respondents rating the various areas as zero (not an area where professional development is needed), one, two, or three (a major priority for professional development). Even the area at the bottom of the table was rated as priority level 2 or 3 by 43% of the respondents.

The three items at the top of the list make clear that what concerns secondary mathematics teachers most are the problems associated with the assessment of Attainment Target 1 in the mathematics National Curriculum. The heads of department recognise as a major priority the need for training in devising activities in the various categories implied by this Attainment Target (investigating within mathematics, practical work and solving real problems), in order to generate evidence of attainment, the interpretation of this evidence, the development of a shared understanding of what is to count as evidence of a pupil performing at a particular level on AT1, and the use of these kinds of activities to promote learning in the ATs 2-5.

The second major concern indicated by the survey is the provision of an appropriate mathematics curriculum for pupils with special needs. The problem of challenging the more able pupils in ways other than simply accelerating them through NC levels was rated as priority level 2 or 3 by 74% of the respondents. Similarly, the problem posed for teachers by low attaining pupils in mathematics was rated as priority level 2 or 3 by 68% of the respondents .

Area for Secondary maths INSET	**zero**	**one**	**two**	**three**	**mean rating**
Activities for evidence of AT1 attainment	2.2	22.6	21.2	54.0	2.27
Use of AT1 activities to promote learning	0.7	21.9	32.1	45.3	2.22
Understanding ATl levels of assessment	8.8	17.5	21.2	52.6	2.18
Challenging more able pupils	2.9	23.4	38.7	35.0	2.06
Strategies for low attainers	6.6	25.5	32.8	35.0	1.96
Developing modes of assessment	5.1	27.7	38.0	29.2	1.91

Use of resources, computers, calculators	8.0	28.5	32.1	31.4	1.87
Heads of department: management ete	14.6	26.3	26.3	32.8	1. 77
Classroom organisation for differentiation	8.0	36.5	29.2	26.3	1.74
Generating diagnostic evidence oflearning	6.6	38.7	33.6	21.2	1.69
Interactive teaching, engaging pupils	11.7	40.9	27.0	20.4	1.56
Systems for recording pupils progress	19.0	29.2	35.8	16.1	1.49
Mathematics content of National Curr'm	18.2	36.5	29.9	15.3	1.42
Problem solving as a context for learning	10.9	46.0	35.8	7.3	1.39

Table 1

Percentages of respondents rating the priority of 14 INSET areas as 0,1,2, or 3

The questionnaire also gave respondents the opportunity to indicate any other areas which they considered important focuses for professional development for secondary mathematics teachers. Not surprisingly, only a small number wrote anything in this section; the responses are listed verbatim in the Appendix. The only common theme which emerges from these is the sense of frustration and stress associated with the rapid changes in the demands being made upon teachers at present.

APPENDIX

Suggestions made by respondents for other focuses for professional development for secondary mathematics teachers

- Is it not ludicrous for each school to be asked under a 'national' curriculum to devise its own recording scheme when what we need is a system which is universal? You seem to be being asked to shut the stable door at the usual inappropriate time.
- How to get an infinite amount of work into a finite amount of time.
- Surely we ought to have been provided with INSET for the National Curriculum <u>before</u> it was introduced into our schools!
- The further development of reporting procedures to parents, during key stages, which give detailed information in a user-friendly context. The further development of encouraging pupils to be aware of ATs and levels and to claim mastery when they feel they have achieved. The assessment statements overleaf appear to leave the pupils out of the equation. We have involved pupils and benefited from the dialogue generated. Pupil involvement is a vital element in assessment in our view.
- How to teach children with dyslexic problems related specifically to mathematics. There is very little done in this respect at the moment.
- Maths in cross-curricular themes and topics. Equal opportunities in mathematics.
- At present there is hardly any time to stand still and assimilate what is happening, let alone be objective about the relative needs of INSET v other activities, all of which can be said to be vital to the coherent development and working of the department. Time is being squeezed from

several directions and the thing that will 'give' is the teachers themselves.
- Enough overleaf.
- Developing resources for less able, related to National Curriculum in the secondary school.
- How to organise things so as to end up with a reasonable and realistic length of working day.
- A-level coursework.
- Many of the concerns about assessment overleaf would be solved if the head of department had further training, which could then be communicated to others.
- AT! might not be so much of a problem if SEAC knew what they meant!

Original pagination of this article – pp39-43

The Hull Mathematics Workshop Content and Outoomes

Ekkehard Kopp, Janet Duffin and Adrian Simpson

University of Hull

This article details the experience and understanding we have gained over the past three years in providing a *Saturday morning Mathematics workshop programme for school pupils. We have used the workshops as an opportunity to select anecdotes which highlight aspects both of the pupils' and of our own experience of mathematics during the sessions. This has led us, through discussions about the incidents, to develop our own understanding of teaching and learning.*

The Layout and Contents

The pupils are from 12 to 14 years old, come from a wide range of schools and attainment levels, and are drawn from the whole of the county of Humberside. Hull University is not unique in providing such a service in its locality: following Prof. Sir Christopher Zeeman's inspiring televised Christmas Lectures in 1978, the Royal Institution created a network of "Mathematics Masterclasses" which has spread from London to a large number of provincial universities.

While these developments provided the impetus for the establishment of our workshop programme in January 1989, that programme quickly developed a distinctive focus, leading to our current efforts to provide mathematical stimulation for a wide range of pupils: interest, rather than proven ability, remaining the chief criterion for participation.

This principle has led to a markedly different format and to a rather less didactic style than those of "Masterclass" programmes. We do not provide an overview (or even a "golden thread") linking a coherent sequence of topics. The work is done in small, essentially autonomous groups with short plenary discussion sessions at the beginning and end of each Workshop. While many "Masterclasses" require significant time commitment from the pupil (indeed, one of the conditions laid down by the Royal Institution for recognition - and thus start-up funding! - of any proposed scheme, is that each pupil is expected to attend for at least 5-6 sessions), we decided early on that such expectations would be unrealistic for our principal target group. Despite our concern for the needs ofthe mathematically gifted and committed youngster, we felt that a more urgent need lay in the provision of stimulation of a wider interest in the subject and that the effort should be aimed at an age group where such interest has not yet been swamped by the twin demands of GCSE and adolescence.

For these reasons the Hull Mathematics Workshops provide a set pattern of sessions on two consecutive Saturday mornings for each pupil in the second year of secondary school, with a pair of follow-up workshops a year later. In 1990/1 this enabled us to cater for well over 300 pupils from 33 schools.

A programme on this scale poses logistical problems. Should the whole programme be presented at a single (preferably central) venue, or do we embark on a "roadshow", bringing our presenters to various schools? We have (so far) opted for the former (Hull University) but are exploring the latter. The question of the optimal size of the workshop sessions poses fewer problems once the venue has been decided: for reasons of space, our 'ideal' workshop size comprises 60-70 pupils. The implementation of the 'self selection' principle for participation we leave to the teachers. Undoubtedly some concentrate on their brightest pupils, but we prefer to avoid formal selection criteria or competitions, which are contrary to the co-operative ethos we wish to encourage.

The format of our workshops suggests itself from the above. A brief introduction to the day's material is followed by dividing over 60+ pupils into 20+ groups of three (chosen randomly and without regard for friends who seek shelter in familiarity) who tackle the day's worksheet co-operatively. The sheet contains instructions and hints. These groups work together, largely at their own pace, for about two hours, broken in the middle for a short break for biscuits and orange juice, and finally all groups are gathered together for some 15 minutes to discuss what they have found and to draw out some tentative conclusions. Pupils are encouraged to pursue unsolved questions further with their teachers (though no formal mechanisms monitor this at present).

At a workshop of 60 participants there would typically be about a dozen helpers, who are available to provide guidance and encouragement, but who are enjoined not to interfere unnecessarily in a group's discussions. We have found it very easy to recruit student volunteers, who have found the experience both rewarding and stimulating. The students, like the pupils, are self-selecting, and their mathematical confidence varies widely. Most of them have teaching in mind as a future possibility and attend to experience some of the rewards and challenges of the profession through the workshops. The presenter, members of staff and undergraduates are sometimes joined by interested teachers and this has proved to be an effective formula for interaction among the groups.

We find it important to produce worksheets that continue to evolve as our experience grows. The focus with second-year pupils has been to provide one workshop on shape (polyhedra) and one on number (factors and primes):

The first involves the pupils in building up various polyhedra, notably the icosahedron, by sticking together various numbers of just four basic shapes. Whilst this holds a fascination for the students on the level of a three-dimensional jigsaw puzzle, the pieces are designed to allow much quite deep mathematics to be explored by these relatively young mathematicians. For example, they canargue about the volumes of the shapes they start with – how can they prove that some of them have the same volume? - and from there to the volumes of the polyhedra they are aiming at. Working in groups of three, they require quite sophisticated visual reasoning to put the pieces together and often this can act as a leveller of ability: one pupil who has contributed little often suddenly sees how the pieces fit and, to the fascination of the others, explains things that they had not seen.

The second workshop focuses on number. Pupils are encouraged to explore how many factors a number possesses by relating this to its prime factorisation. They invent methods for finding prime numbers (they use a computer to test larger numbers), move on to looking at factors and splittings (prime factorisations) and try to discover and justify a rule for finding how many factors a number has, given its splitting.

The Outcomes

There are more outcomes of, and issues raised by, the Hull Mathematics Workshop than the mathematical content. The workshops have a significant effect on all involved: pupils, teachers, lecturers, student helpers and occasional parents who stay on to observe or even participate. As we began to write this article, we became aware that each person gained something different: whether this was a new perspective on a piece of mathematics, a better understanding of the way in which groups work or fail to work together, or just help in expressing a question or issue. At the end of each workshop, as we clear up, the group of helpers discuss how they feel the morning has gone. One of the motivations for this article was the variety of anecdotes this 'debriefing' throws up, each providing a poignant perspective on the skills and uncertainties of the individual, as well as highlighting the viewpoint from which they approach the session.

We have collected a few of the anecdotes related to us after two of the most recent workshops and we present them here wi th a discussion of some of the issues they raised for us. It should be noted that, whilst we have presented each as a separate anecdote and discussion pair, the web of issues they raised allowed our discussion to move in various ways, and in reading them you may well find yourself saying "but that also raises the issue of ... " . We hope so.

"We're sticking to this 'cos we know what we're doing"

One of the most striking incidents in one workshop concerned a pair of groups, attacking the problem offinding all the primes between 1 *and 100. The first had written out the numbers and were crossing them out by testing with a calculator before one of them said to another, "Why don't I cross out all the ones divisible by 2, you do the ones by* 3 *and Laura, you do the ones by* 4 *... no,* 5?" *and they had soon devised a sieve method for themselves. The second started by trying to* do *some sort of sieve algorithm, but dropped it in favour of a homespun method based on known facts and the calculator, saying, "Well, we were taught it [the sieve method] at school, but we can't quite remember it, so we're sticking to this 'cos we know what we're doing this way."*

This highlights some of the value there is to be found in discovering parts of mathematics for oneself. The first group who discovered the sieve method found out why it worked as a by-product of inventing it for themselves, and with it got quite a feeling of excitement and achievement. The second group could only partially remember what they had been shown as a prime finding method, but had no idea why the algorithm worked. Even though they discovered and used a less efficient method, they gained a sense of satisfaction and mastery over the mathematics, because it was their algorithm. In this sense, their reactions were not dissimilar to those of the first group.

Observation of other groups demonstrated a wide variety of methods used by pupils tackling this task, countering the impression gained by helpers that many pupils have a poor level of number competence. It seemed possible that, when invited to proceed in any way they chose in undertaking a task such as this, pupils could employ skills and competence not revealed when only simple number facts were asked for.

Many of the groups became quite at ease with swapping strategies halfway through, discussing the merits of different methods and evaluating the progress of the algorithms they had chosen. The confidence to do all that is perhaps as important an attribute for a mathematician, or indeed anybody, as a strong database of facts.

"Groups let you know when they want you"

A student helper, Alison, reported, "They really let you know when they want you to help them. I was with one group and, instead of telling them what to do, I *just asked them what they had done so far and then we talked about it. A bit later, I left them and went into another room. I was amazed when a few minutes later they came to find me and wanted me to go back to see where they had got to* ***now."***

Several helpers made comments about the attitudes of pupils in the various groups. Some groups getting on nicely on their own were able to indicate that they did not want to be interrupted. Some appeared to be grateful for the help given though not actively seeking it, others conveyed the impression of needing help by just sitting contemplating the material in front of them, while others would actively seek it by capturing a nearby helper and putting a problem to them.

But most rewarding for the helpers was a group seeking out the same person aain for more guidance or to show what they had managed to achieve on their own. It did seem that, for the most part, showing an interest in what pupils had achieved for themselves was a powerful incentive to further consultation.

This opens up the question of how best to give and offer help in order that pupils will get the most out of the workshop experience; how to judge the right moment to intervene and what form that intervention should take.

"I can 't help but want to put them right"

In a discussion with one of the student helpers, Paul, he told me *that he felt quite confused about how much help to give. I asked him to explain what he meant.*

'Well, there's a real problem here: I know that if Ijust sit down and show them how it's done, they're not really going to learn much, and anyway, half the fun for them is when they suddenly see how it fits together, but I can sit and watch some of the groups and I know that the way they're going isn't going to get them there - they're going to build some horrific monster, waste their time and get frustrated. I can't help but want to put them right. "

This statement poignantly highlights a dilemma that many helpers feel about their role in the workshops. If they are prescriptive they denude the activity of its excitement for the pupils and probably detract from the learning process, yet if they are acting as mere listeners without any intervention, some groups will not only get stuck, but will waste a lot of time exploring quite fruitless avenues and may leave the workshop with nothing more than a sense offrustration. This is the eternal dilemma of teaching.

Of course, Paul's statement only highlights the extreme positions in the role the helpers can take and perhaps the answer to the dilemma lies in the roles that lie between these. The student's comments made us look more carefully at the levels of prescription we saw being used by helpers when groups were stuck in subsequent workshops. We identified what seemed to be four coherent levels:

1. Giving the answer if the group says it is stuck.

2. Giving a directive strategy to the group that will lead them to an answer ('Write down this, then do that")

3. Giving a hint of a strategy or a question towards it ("What would happen if you went a different way at this point?")

4. Asking the group what their strategy is ("How are you going to go about this?")

In rare cases helpers maintained just one role throughout the workshop, most commonly position 1. We felt that this could be because of insecurity about the mathematics involved (a reaction that is, perhaps, surprisingly common considering that the helpers are all at least at the l~vel of second year undergraduate mathematics) or alternatively because such a helper was very excited by the mathematics and was trying to get their enthusiasm over by showing hislher discoveries to the pupils. More commonly, however, helpers varied their role between situations as their instinct led them.

Paul is one of a number of the student helpers who are at the stage of thinking about teaching as a possible career, and the workshops offer them all an opportunity of experiencing some of the rewards that teaching can offer them as well as highlighting the dilemmas that a teacher can face. The solution to Paul's dilemma lies in the practice of good teachers who can vary their role between situations. Their experience and perception enable them to choose which position to adopt so that they can allow pupils to be excited by discovery, without getting lost or frustrated, and where they, the teachers, can still be enthusiastic about their mathematics, but won't overwhelm the pupils own feelings for it. It is refreshing to see that, even before starting his training, Paul is aware of the dilemma and is thirsting for an answer.

"Isn't this kind of maths marvellous?"

I was discussing part of the current activity with two boys in a group of three. I noticed that the third did not seem to be taking an active part in what was going on. I asked him how he was getting along, to which, to my dismay, he replied, "J'm completely lost; I've been lost for quite a while." Anxious that he should catch up as quickly as possible, I went through the work in a way that I felt was too quick and too prescriptive, but I was amazed to see the boy's face light up suddenly as he said, "I see it all now. Isn't this kind ofmaths marvellous? "

The helper concerned felt strongly that she had been too prescriptive in her attempts to help the boy catch up the others but this was overshadowed by delight at the boy's comment. Other helpers, told of the incident, shared the delight but expressed some concern that we might not be being vigilant enough about how individuals were working within their groups. Our practice of random selection of groups, designed to maximise interaction between pupils from different schools, has potential for both positive and occasional negative outcomes, as our next anecdote shows.

You can't communicate when you've nothing to say

One group seemed not to be communicating at all. Each member was somewhat vaguely moving different shapes around on the table in front of them with no communication taking place at all. Suddenly somebody said, "I see how it goes" and fitted his shapes together to make what was required. the others immediately brightened up, the idea was shared and the group worked well together *for the* rest of *the* session. At *the* end *they were* seen enthusiastically exchanging telephone numbers.

Groups were observed to work in a variety of ways, sometimes, at least initially, appearing to work entirely on their own. Sometimes one member appeared to be dominant while others merely looked on. Sometimes, when there was an activity which could be shared, two would do this while the other appeared to be doing nothing, though, on occasion, a group became aware of this in retrospect and shared the work out better later. Sometimes healthy arguments ensued. Clearly some of these variations arose from the composition of a particular group but in other cases there seemed to be a specific occurrence which sparked off communication and genuinely collaborative working, as in the example above.

Although much mathematics can be usefully and fruitfully done alone, and there will remain some who prefer always to work in this way, for many the workshops offer what is perhaps a unique opportunity for learning to work collaboratively in an atmosphere which is more relaxed than some schools can be. But it is unrealistic to expect this to happen immediately amongst pupils who have only just met each other. Something which causes one member of the group to want to communicate what they have done to others in the group can form a spark which lights the fire of a group's enthusiasm.

Thus this anecdote draws attention again to the importance of communication and the fact that for it to occur there must be something to communicate about. This may take time to emerge and, for some groups, there may be a variety of reasons why it does not seem to happen. Skill is required both in recognising why a group appears not to be communicating and what can be done to help it along. Since helpers are encouraged to move around from group to group rather than concentrating all their attention on a single group they have to be sensitive to the possibilities and to be able to sum up a situation quickly in order to take effective action. This could be one of the most important skills being developed by the helpers in the workshops.

"It gets me that way too"

On one occasion,] *was talking with one of the student helpers, who said,* "] *feel so inadequate with some of this because] don't know the notation and it seems to make me paralysed and unable to think."*] *replied, "Yes it gets to me that way too, it used to get me that way when] was* a *student as well. "*

We think it would be true to say that it was unusual for a student to be quite as open as this about feelings ofinsecurity relating to mathematics and perhaps, in itself, the episode is a testimony to the sense of mutual support being generated amongst helpers who interact as peers in the workshops, quite independently of their status in their usual teaching and learning setting. From A-level and beyond, students seem to become more reluctant to admit difficulties; an exam-orientated curriculum breeds an attitude amongst both students and teachers that it is what one can do that is paramount and admitting to difficulties is hard to do in a success-based system. One of the notable achievements of the workshops, as illustrated by this anecdote, is that students do feel able to admit to misconceptions and general areas of difficulty. Staff accept these and students see that staff, with their own insecurities and misconceptions, are human too. This accord, where helpers could be total strangers to each other in some cases, while in others they had been used to meeting each other in quite different circumstances, was something that gradually developed over time and this is one of the aspects which contribute in a special way to making our workshops so worthwhile. Very few helpers drop out once they start to attend.

This, though, goes only part of the way to overcoming the problem of student insecurity. One of the long term things which we might be looking for from the workshops could be whether they have any effect on future classrooms and future generations of students. As pupil participants move on to become students, will they take what the workshops have done for them forward to their undergraduate experience? Will the student participants, as they move into classrooms as teachers, encourage their pupils to be more articulate, more able to talk to and question each other and their teacher? Will the workshops help to generate a view of mathematics in which learners actively seek to develop their own knowledge and skills rather than being the passive receivers of their teachers'? Will this then result in an environment in which the fear of expressing insecurities and inadequacies has lessened?

'This shape goes here"

Groups were asked to explain why one shape they had made was eight times the volume of another. A variety of strategies emerged: taking off a *face and trying to fill the interior, trying to recreate it by using shapes whose volumes were already known, using* a *known 'rule' about volumes etc. Suddenly one girl in* a *group took up the shape and, with gestures pointing to her perceived spaces within it, said, "This one goes here and this one goes in the spaces in between leaving* 4 *to fill the space in the middle." Everyone else in the group, including me, was mesmensed by this confident performance and I don't think that any of us exactly followed what she was showing us, but her own confidence convinced us that she was 'seeing' what we, as yet, could not see.*

Such insights, with or without the ability to persuade others of the truth of what one can see, are little gems of experience to be shared and for which the building up of the ability to articulate the insight in a way which others can understand must be one of the most exciting outcomes of the workshops.

Another episode showing how pupils can develop the skill of communicating mathematics to each other was also interesting:

Another question asked the pupils to explain why four of a shape TTUlde up another shape they had made. A group indicated they were having difficulty in explaining this though they felt they 'knew it'. I *talked with them for a while until it seemed as if a satisfactory explanation was emerging. I left them trying to make the explanation more convincing. They* *sought me out again to tell me about it and by this time both they and I were satisfied.*

The ability to communicate mathematics through the spoken word has always been a feature of the interchange between professional mathematicians who can often be found absorbed in discussion about one issue, global or minute, which has arisen in conversation. *As* a medium for learning mathematics it has not, until fairly recently, been highly regarded.

Yet if one were to ask the professional or research mathematician about this, they would speak about the way in which they can use it to clarify their own thinking; they will say it 'takes them on a step' in their thinking and that it serves to highlight conflicting perceptions which need to be resolved for progress to be made. Sometimes a scarcely recognised conflict will be brought to light from interchange with colleagues which will help them to resolve such a conflict and take them on that further step in their thinking. The workshops are offering the pupils and students an opportunity to use interchange in the way that their teachers and lecturers use it. But many of them, because mathematics in the class or lecture room can be seen as having more to do with communicating through the symbols of mathematics, have not had much practice at it before. And like everything else, the ability to describe, explain and justify is something which improves with practice, so ought to be seen as an accepted and to be expected part of the learning of mathematics.

Pupils attending the workshops often display a lack of such practice but, given the opportunity, do respond and do become better at it. By improving skills and clarifying ideas, opportunities for discussion also help learners to be more confident because they increase the store of internal images of mathematics that their learning is helping them to develop. The importance of the notion of discussing mathematics led us to analyse the values associated with communicating mathematics a little more deeply.

In *talking* about our mathematics we find a need to clarify what it is we are going to say in our own minds, we need to pick up the quite subtle skill of reading both our own mathematics and that of other people, and we pick up the valuable skills of communicating, presenting ideas to others and constructing viable arguments. Talking about mathematics is traditionally the teacher's role rather than the pupil's. In *listening* to mathematical talk, we pick up the skills of visualising a situation and translating it into our own framework, and we learn to follow an argument. The listening role is traditionally that, of the pupil in the classroom. In *discussing* mathematics, Wt! are not only provided with a situation in which everyone, teachers and pupils, can enhance the skills of both listening and talking about mathematics, but the to and fro of a mathematical discussion can bring conflicts and misconceptions to the fore, can help resolve them and allow us to 'bootstrap', let us grow from each other's knowledge and reach heights that none of us could have reached alone.

'They just wouldn't try to build it in 3-D"

One of the student helpers, Ari, was discussing the morning's workshop and pointed to some related incidents he saw as *he watched groups building shapes for instructions like: make* a *polyhedron out of* 3 *pentagons and* 5 *equilateral triangles; make* a *polyhedron out of* 1 *pentagon and* 5 *equilateral triangles; and* so *on. "Some of the kids would first try and stick all the polygons together - flat* as *some kind of net and then they'd try to fold it together and of course it didn't fit. They just wouldn't try to build it in 3D or by trial and error."*

This was quite a common observation and, with other anecdotes, spoke to us of the way in which our experience constrains our thinking and colours our attitude to others. As mathematics becomes more formal and work becomes book based, there is an inevitable concentration on the two-dimensional. If, in addition, the visual is sacrificed to the written page, even 2D perception can be lost.

As we discussed Ari's comments amongst the helpers, we began to realise that many of us suffered from this same constraint and it was pointed out that it is so prevalent that 'cornflake packet' puzzles such as "make four equilateral triangles out of six matches" are based on our pre-occupation with plane thinking.

Another anecdote told of the way that experience constrains us in our expectations of others.

One student was working with a *group which had been calculating the number offactors of2,4,S,16 and* so *on. She commented afterwards: "They were quite happy writing* 2 as 2 , 4 *and* 2 , *S* as 2 *etc, but I couldn't get them to grasp that* 1 *is* 2 *which seems such* a *natural progression"*

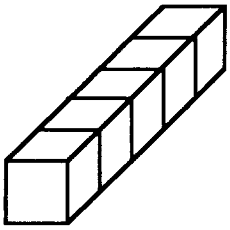
Considerable discussion ensued with a number of the helpers firmly disagreeing with her comment that the concept was natural. A little deeper probing showed that those who did feel the concept was simple to grasp could remember no difficulty with it when they first encountered it, while those who thought it was a difficult concept could recall the problems they had had with it initially, coming up with a variety of different reasons why they found it hard to acc~pt. For ~xample, one postgraduate student indicated that as the concept image he had of powers when he first met 20 was one of 'multiplying a number by itself the power's number of times', he had been quite convinced that 2^0 was 0 initially, coming up with a variety of different reasons why they found it hard to accept. For example, one postgraduate student indicated that as the concept image he had of powers when he first met 20 was one of 'multiplying a number by itselfthe power's number of times', he had been quite convinced that 20 was ° 'since it is zero two's multiplied together'. From his experience 20 = ° was the expected answer and, as in our next anecdote on 'natural' misconceptions, it required a conflict (possibly with an example like 22 + 22 = 20) to change his perception.

Again we concluded that each person's experience colours their expectations; what we found hard we expect others to have difficulty with and what we grasped easily we expect others to find trivial. The two anecdotes together highlighted for us just how much our experience affects the way we work, the strategies we use and the expectations we have of others.

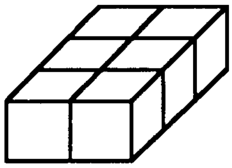
'They're made of the same pieces"

One of the tasks assigned on a *worksheet is to make two different polyhedra, each consisting of two pentagons and ten triangles, which most groups manage after* a *little time. They are then asked to explain why the polyhedra have the same volume. Ignoring the advice to use two extra polyhedra to aid their explanation, they almost invariably answer, "They are made of the same pieces."*

This anecdote highlights a serious misconception which seems to be widespread amongst the pupils we see at the workshops but also hints at deeper issues about the conflict between what individuals consider 'natural' and 'true'. When we first met this reaction, we were quite unprepared for it - and a large number of helpers met it at roughly the same time in the first workshop that contained the question. Whilst all the helpers were fully aware that polyhedra made from the same polygons do not necessarily have the same volume, not one of this group of professional and student mathematicians could quickly come up with a counter-example! Luckily the workshop leader thought for a moment and came up with two cuboids each 'made' from 22 squares, with different volumes:



Volume is 5 cubes



Volume is 6 cubes.

There is something very natural to the pupils about saying that the same pieces give the same volume and they seem to need a counterexample like this before they see that it isn't true. As we develop mathematically, however, our concept of what is 'natural' changes; we seem to filter out thoughts like these before we are conscious of them. Since, for the helpers, there was no longer a sense of conflict between the 'natural' explanation and a valid one, a counter-example was hard to come by quickly.

Conclusion

Incidents like these helped us to become aware of the filter through which we are seeing mathematics and helped us notice that other people do not see things in the same way we do. The notions of the differing filters through which one sees mathematics and the conflict between an individual's 'natural' explanation and the body of accepted mathematics goes far too deep for a discussion here to be useful, but just having an awareness that there may be differing ways of seeing a problem is valuable in helping us communicate mathematically. Our experience indicates that - whatever the eventual effect of this venture on the take-up rates of mathematics at more advanced levels among Humberside pupils (and this would be very difficult to evaluate accurately) - the workshops have a profound effect on the presenters and helpers themselves: in forcing mathematicians and students to think about the processes involved in learning mathematics, rather than simply focusing on content, the workshops have produced valuable outcomes which few expected at the outset. Our workshops continue to evolve in response to reflections such as those presented in this article, and to comments from pupils and teachers.

Acknowledgements

We wish to thank all the teachers who have provided pupils for the workshops, all the pupils who have attended, and our many student and staff helpers who made it possible. *Further information on the Workshops may be obtained from the Workshops Secretary: Alex Adams, School of Mathematics, University of Hull, Hull HU6* 7RX, *UK*

This article is converted from the print version published by the Association of Mathematics Education Teachers (AMET)

Articles available online at www.amet.ac.uk

Original pagination of this article – pp44-55