Mathematics Education: Reasons to be cheerful?

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UNITED KINGDOM · CHINA · MALAYSIA

Just before Christmas the most comprehensive survey of global educational achievement ever conducted showed just how daunting the challenge is. … But we haven't been progressing relative to our competitors; we've been retreating. In the last ten years we have plummeted in the rankings: from 4th to 16th for science, 7th to 25th for literacy and 8th to 28th for maths.

(Michael Gove, National Curriculum Review launch, 21/1/11)

Current policy

- New GCSE
  - More problem-solving & mathematical reasoning
- Revised National Curriculum
  - Increase emphasis on 'mastery' & multiplicative reasoning
  - Bar raised at primary: "long division"
- Core Maths
  - "Within a decade the vast majority of pupils [will be] studying maths right through to the age of 18"
- Maths Hubs
  - Shanghai teachers, Singapore textbooks, "mastery"

Fast forward to 2015 …

Reform in England since 1970s

- Curriculum:
  - National Curriculum (1989)
  - Revised about every 5 years
  - Framework for teaching mathematics (1999)
- Assessment:
  - Qualifications: GCSE (1988)
  - National testing (1991)
  - Assessment for Learning (c2000)
  - Making Good Progress (2007)
- Accountability & inspection:
  - Ofsted (1992)
  - League tables (1996)
- Pedagogy:
  - Cockcroft (1982)
  - National Strategies (1998)
- Professional Development
  - 20 days courses
  - National Centre (2006)

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Many, many central /national initiatives affecting mathematics classrooms

So has the reform worked?

Earl, L., Watson, N., Levin, B., Leithwood, K., Fullan, M., Torrance, N., . . . Volante, L. (2003). *Watching and learning 3: OISE/UT (Ontario Institute for Studies in Education, University of Toronto) Final report of the external evaluation of England's National Literacy and Numeracy Strategies*. London: Department for Education and Skills.
Fullan, M. (2000). The Return of Large-Scale Reform. *Journal of Educational Change, 1*(1), 5-27. doi: 10.1023/a:1010068703786

LNRP: Progression of Cohorts

C/H	97/98	98/99	99/00	00/01	01/02
1	R	Y1	Y2	Y3	Y4
2	Y4	Y5	Y6	Y7*	

Leverhulme Numeracy Research Programme: The largest study of primary maths in the UK, tracked students across primary in two cohorts. This (fortuitously) allowed a before and after comparison of the effects of the NNS at Y4.

LNRP: Year 4 numeracy test results

	1997-1998	2001-2002	Gain	Equivt age	Effect size
Oct	52%	55%	+3%	+2mth	0.17
Jun	62%	65%	+3%	+2mth	0.18

So the gain was just under 0.2 which is in line with Tymms (and TIMSS):

Tymms, P. (2004). Are standards rising in English primary schools? *British Educational Research Journal, 30*(4), 477-494. doi: 10.1080/0141192042000237194

Tymms, P. (2011). Evidence? The impact of large-scale reform in England. *Zeitschrift für Erziehungswissenschaft, 13*, 105-115.

Brown, M., Askew, M., Millett, A., & Rhodes, V. (2003). The key role of educational research in the development and evaluation of the National Numeracy Strategy. *British Educational Research Journal, 29*(5), 655-672.

TIMSS (Y5)	
1995	484
2003	**531**
2007	**541**
2011	**542**

The primary gain at TIMMS is substantial AND sustained.

TIMSS (Y5)	
1995	484
2003	**531**
2007	**541**
2011	**542**

TIMSS (Y9)	
1995	498
1999	496
2003	498
2007	**513**
2011	507

PISA (age 15)	
2000	500?
2003	508
2006	495
2009	493

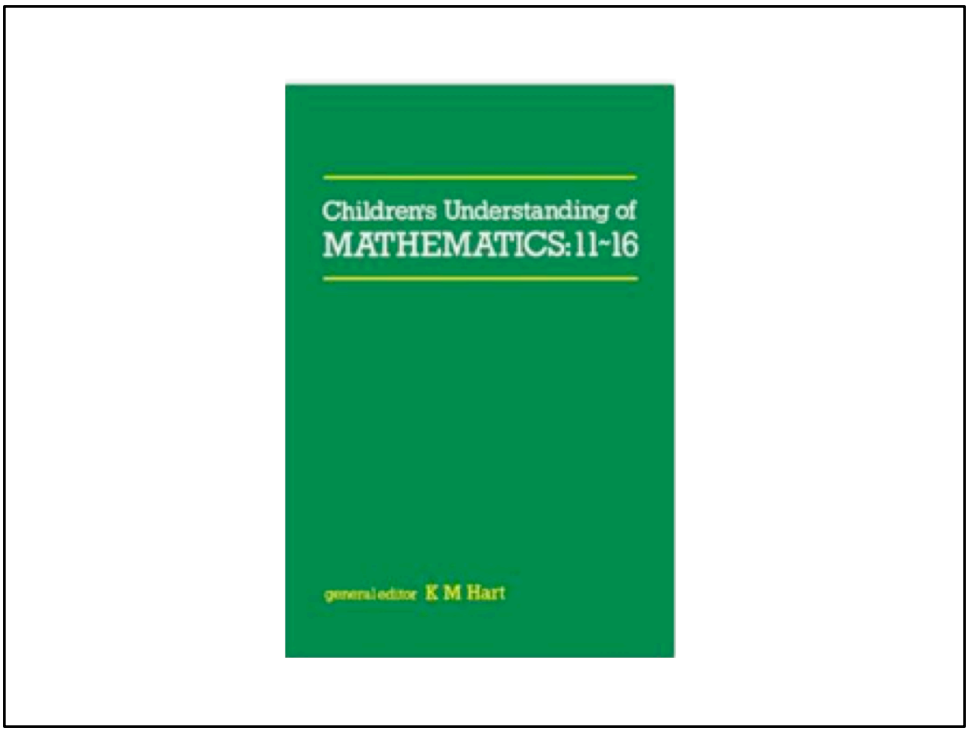
But gains in primary are not sustained into KS3!

LNRP: Mean increase in success rate on common items since the previous test

From	*To*	*Increase*
Year 1 Oct	Year 1 Jun	+20%
Year 1 Jun	Year 2 Oct	+ 5%
Year 2 Oct	Year 2 Jun	+20%
Year 2 Jun	Year 3 Oct	+ 2%
Year 3 Oct	Year 3 Jun	+13%
Year 3 Jun	Year 4 Oct	+ 2%
Year 4 Oct	Year 4 Jun	+10%
Year 4 Oct	*Year 4 Jun*	*+ 10%*
Year 4 Jun	*Year 5 Oct*	*+ 5%*
Year 5 Oct	*Year 5 Jun*	*+11%*
Year 5 Jun	*Year 6 Oct*	*+ 4%*
Year 6 Oct	*Year 6 Jun*	*+11%*
Year 6 Jun	*Year 7 Jun*	*- 2%**

Note the dip from the end of Y6 to the end of Y7. An actual dip in basic primary numeracy

Brown, M., Askew, M., Hodgen, J., Rhodes, V., Millett, A., Denvir, H., & Wiliam, D. (2008). Individual and cohort progression in learning numeracy ages 5-11: Results from the Leverhulme 5-year longitudinal study. In A. Dowker (Ed.), *Mathematical Difficulties: Psychology and Intervention* (pp. 85-108). Oxford: Elsevier.



Revisting the seminal CSMS study:
Hart, K., Brown, M. L., Küchemann, D. E., Kerslake, D., Ruddock, G., & McCartney, M. (Eds.). (1981). *Children's understanding of mathematics: 11-16*. London: John Murray.

ICCAMS

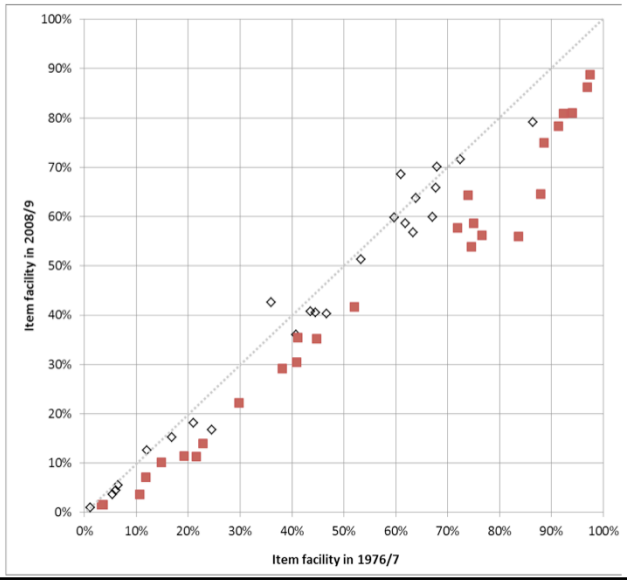
- Phase 1 (2008/11):
  - Nationally representative cross-sectional survey at Y7, Y8 & Y9
  - Longitudinal survey Y7 → Y9
  - Concepts in Secondary Mathematics & Science (CSMS) Tests (1970s): Algebra, Ratio, Number, Fractions

13

The ICCAMS / CSMS tests

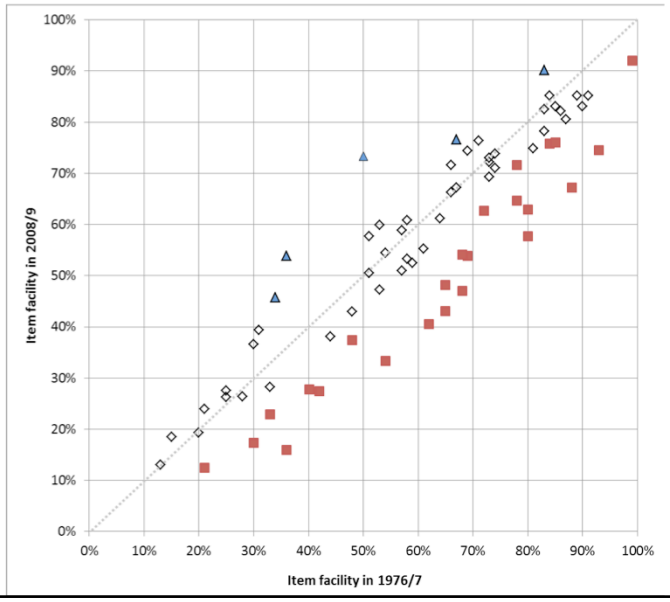
- Conceptual understanding:
  - Problems … recognisably connected to the mathematics curriculum but which would require the child to use methods which were not obviously 'rules'
    - (Hart & Johnson, 1983, p.2)
- Algebra: generalised number & variable
- Ratio: additive → multiplicative reasoning
- Decimals: place value, measurement, rational number

Algebra (Age 14): Change over time

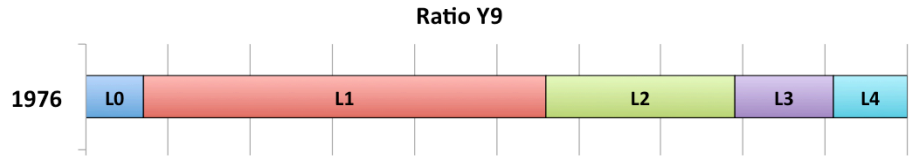


Hodgen, J., Coe, R., Brown, M., & Küchemann, D. E. (Under review). Educational standards over time: changes in mathematical understanding between 1976 and 2009 in England.

Decimals (Age 14): Change over time

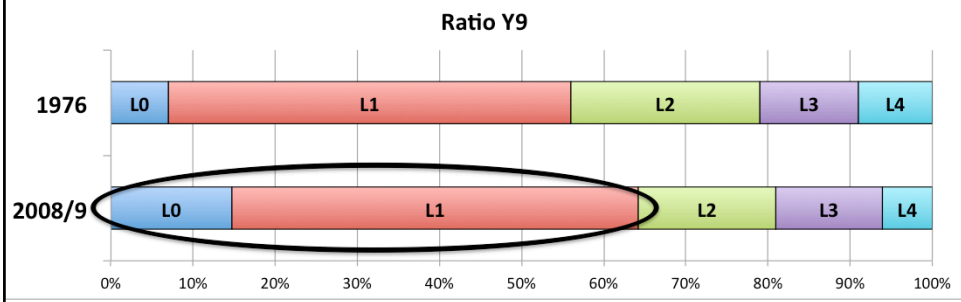


Ratio (Year 9)



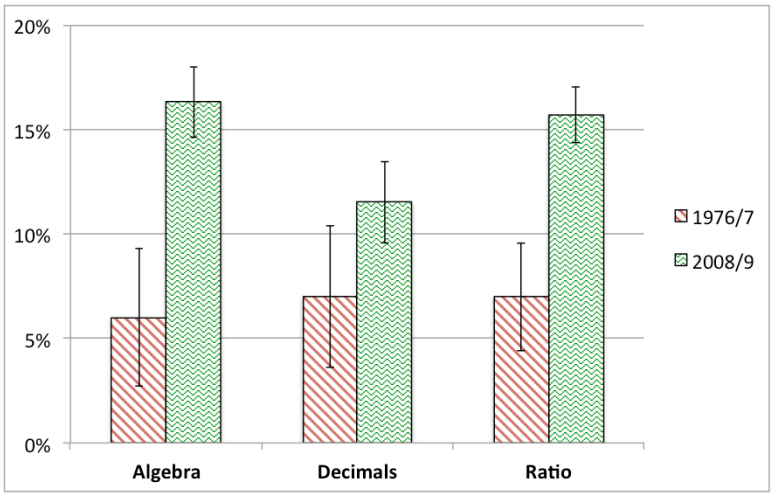
	Level 0	Level 1	Level 2	Level 3	Level 4
1976	7%	49%	23%	12%	9%

Ratio (Year 9)



	Level 0	Level 1	Level 2	Level 3	Level 4
1976	7%	49%	23%	12%	9%
2008/9	15%	49%	17%	13%	6%

Increase on low attaining students



Why is educational change so difficult?

- Are we teaching the 'wrong' things?
  - Times Tables? Long division? Ratio?
- Are we teaching in the 'wrong' way?
  - Textbooks? Direct instruction?
- Are we doing educational policy wrongly?
  - Swings & roundabouts?

Teacher education, teaching time, Examinations, Crowded curriculum

Are we teaching the ‘wrong’ things?

Is it multiplicative?

What is the calculation?

The cost of 6.22 litres of petrol was £4.86.

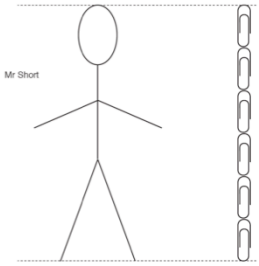
What would the price of one litre be?

$6.22 + 4.86$	$4.86 \div 6.22$
$6.22 \div 4.86$	$4.86 - 6.22$
$6.22 - 4.86$	4.86×6.22

Year 9: 30% (1976) → 17% (2008/9)

Brown, M., Küchemann, D. E., & Hodgen, J. (2010). The struggle to achieve multiplicative reasoning 11-14. In M. Joubert & P. Andrews (Eds.), *Proceedings of the Seventh British Congress of Mathematics Education (BCME7)* (Vol. 30, pp. 49-56). University of Manchester: BSRLM.

Is it multiplicative?



You can see the height of Mr Short measured with paper clips.

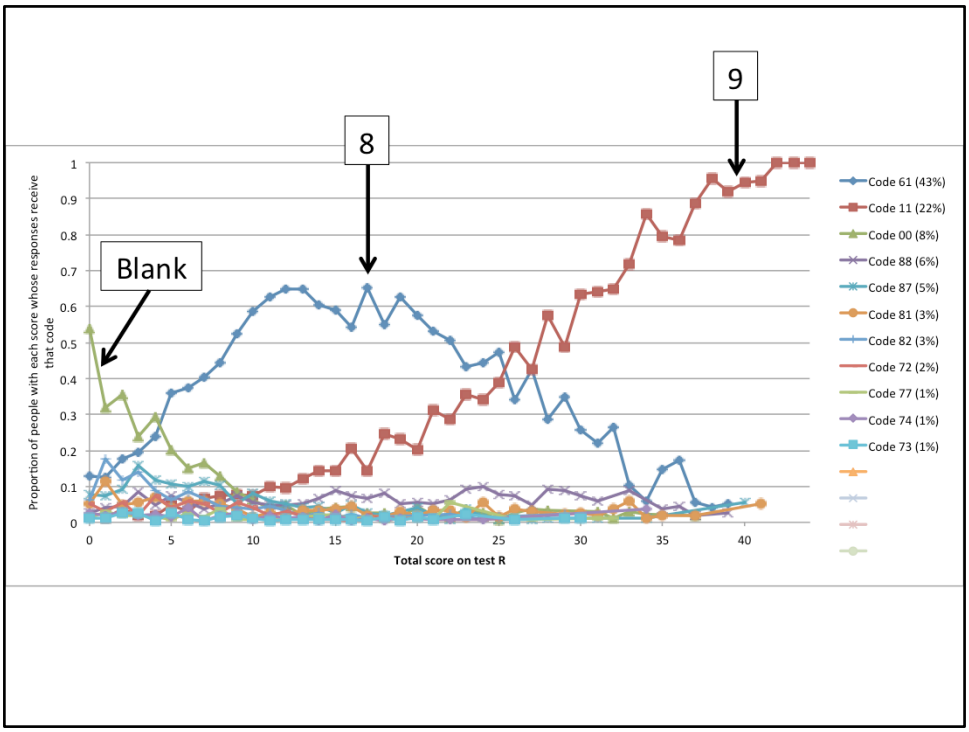
Mr Short has a friend Mr Tall.

When we measure their heights with matchsticks,
Mr Short's height is four matchsticks,
Mr Tall's height is six matchsticks.

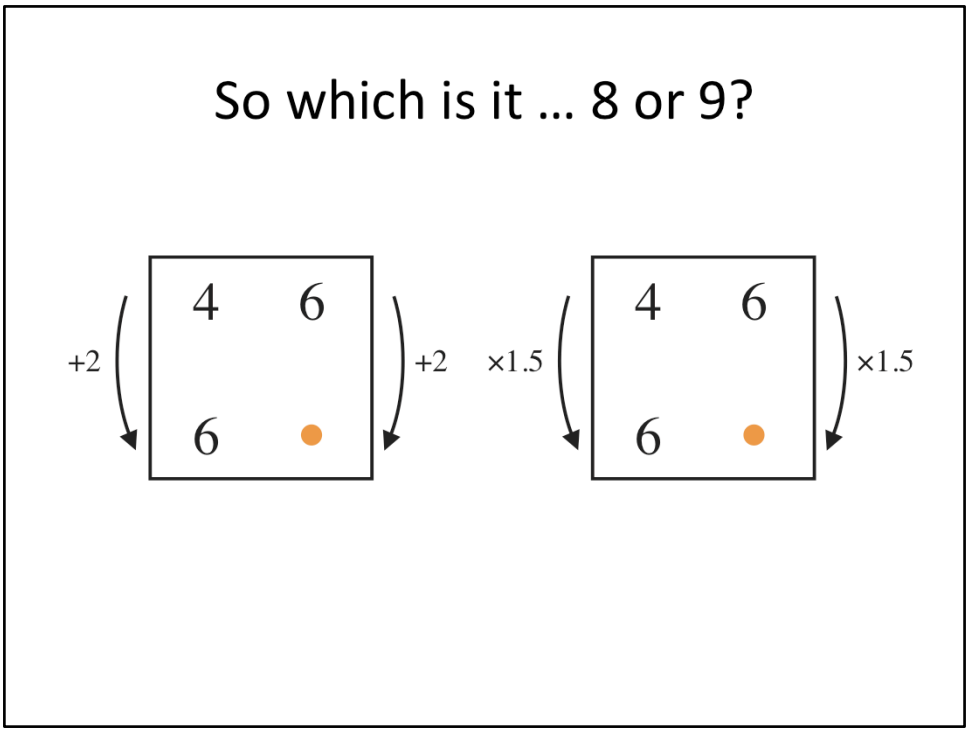
How many paper clips are needed for Mr Tall's height? . .

Year 9: 30% (1976) → 33% (2008/9)

It could be 8 or it could be 9. It depends on what the rule is.



Results for all of KS3



Well, it does depend on what the model is of – and for …

Which is larger, $2n$ or $n+2$?

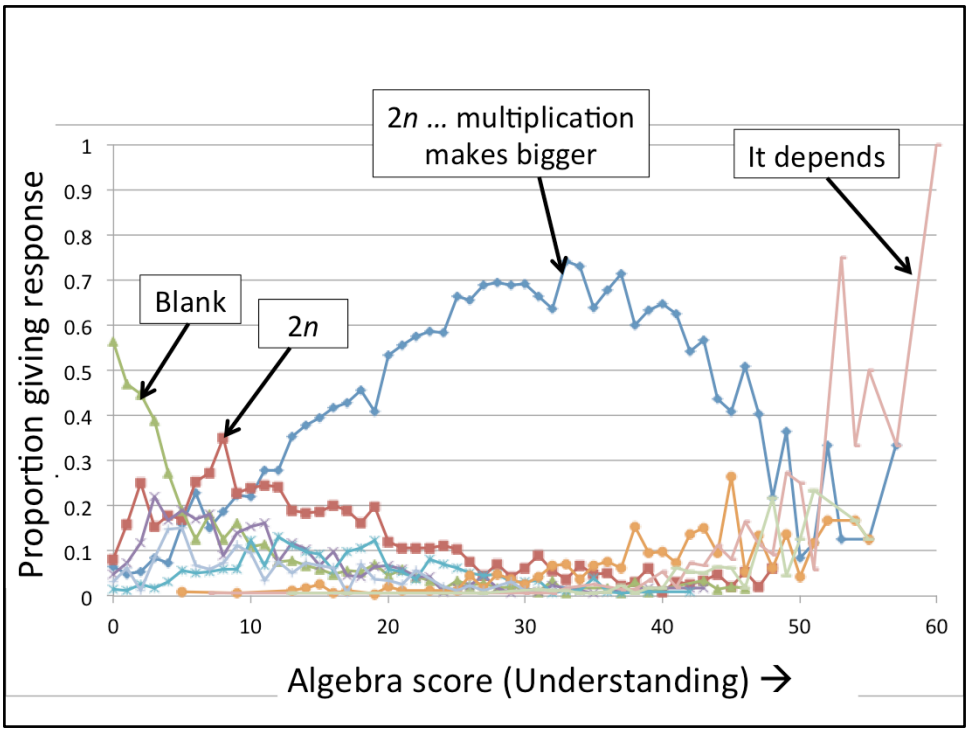
Explain: . . . . . .

Both. "If n=1 then 2n=2 and n+2=3 or if n=5 then 2n=10 and n+2=7"

Which is larger, $2n$ or $n+2$?

Explain: . . . . . **5%** . . . . . .

At Age 14



Results for all of KS3

Hodgen, J., Brown, M., Coe, R., & Küchemann, D. (2012). Surveying lower secondary students' understandings of algebra and multiplicative reasoning: to what extent do particular errors and incorrect strategies indicate more sophisticated understandings? In J. C. Sung (Ed.), *Proceedings of the 12th International Congress on Mathematical Education (ICME-12)* (pp. 6572-6580). Seoul, Korea: International Mathematics Union.

n* multiplied by 4** can be written as **4*n.
Multiply each of these by 4:

$n + 5$ $3n$

. . . .

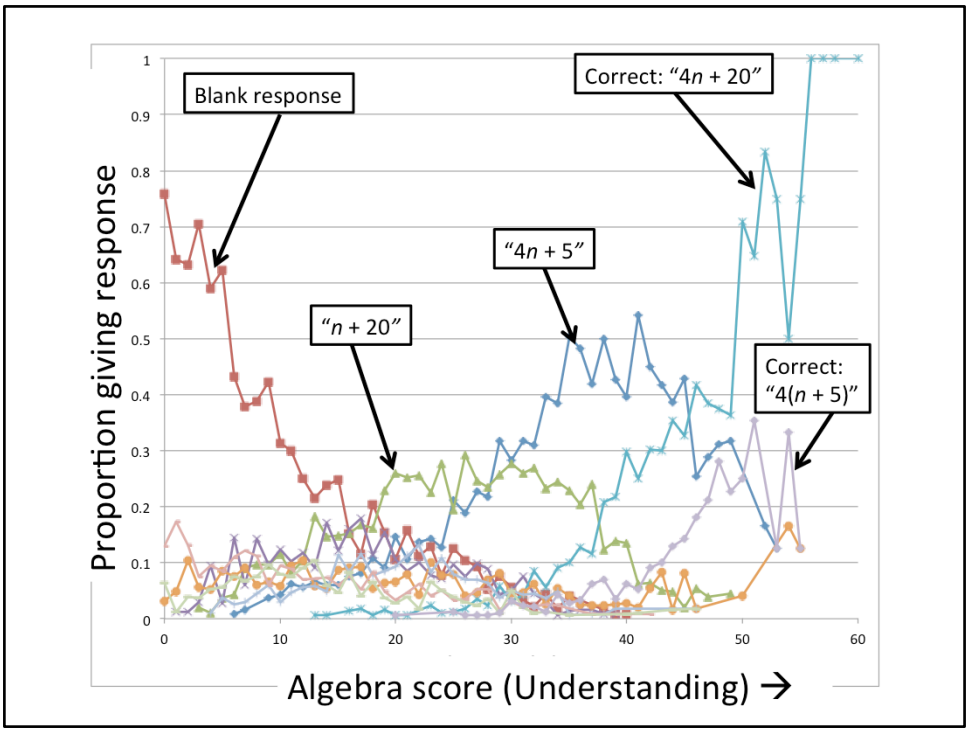
NOT: 4(*n*+5) … 4×(*n*+5) … (*n*+5)×4

n* multiplied by 4** can be written as **4*n.
Multiply each of these by 4:

$n + 5$ **15%**

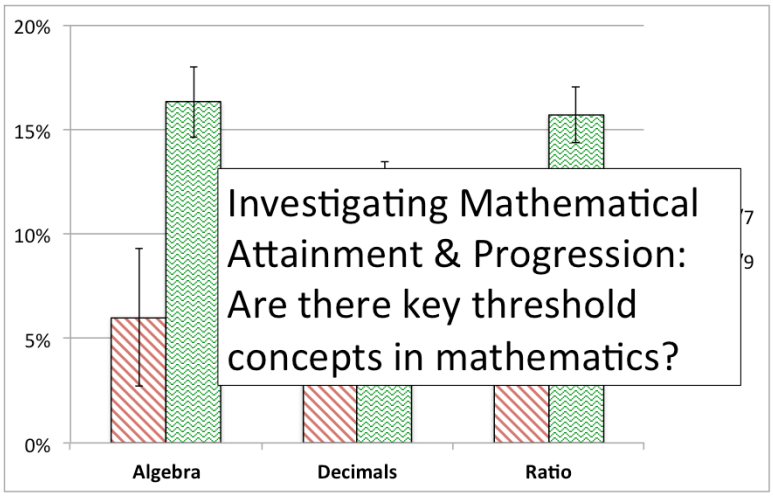
$3n$ **41%**

At age 14



Results for all of KS3

Watch this space …



Are we teaching in the ‘wrong’ way?

A critique of current textbooks

- Textbooks
  - Fragmentary / Disconnected
- Procedural?
  - Sometimes missing the 'mathematical' point
- Poorly chosen problems / examples
  - Often counter to students' "commonsense"
- Problems of emphasis
  - Procedures & algorithms
- Relationships: Multiplication ←→ Algebra
  y=kx … y=kx+c

Current (English) Textbooks

National Curriculum SATs ques

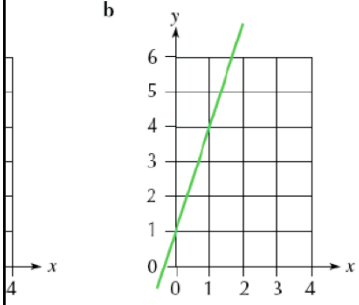
3 For each of the following graphs:

i find the gradient of the coloured line.

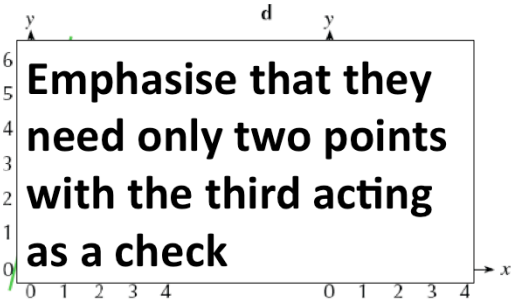
ii write down the coordinates of where the line crosses the y-axis.

iii write down the equation of the line.

b



c **d**

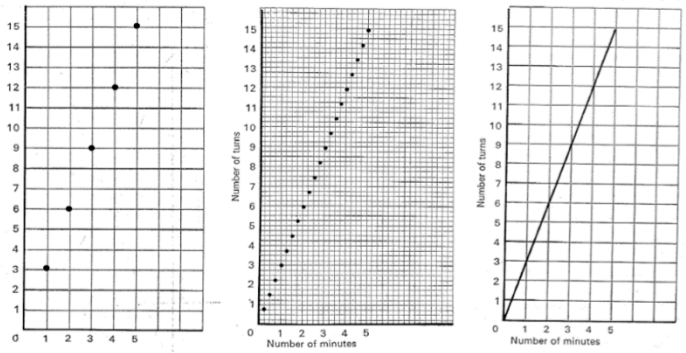


Emphasise that they need only two points with the third acting as a check

Hodgen, Küchemann & Brown (2010)

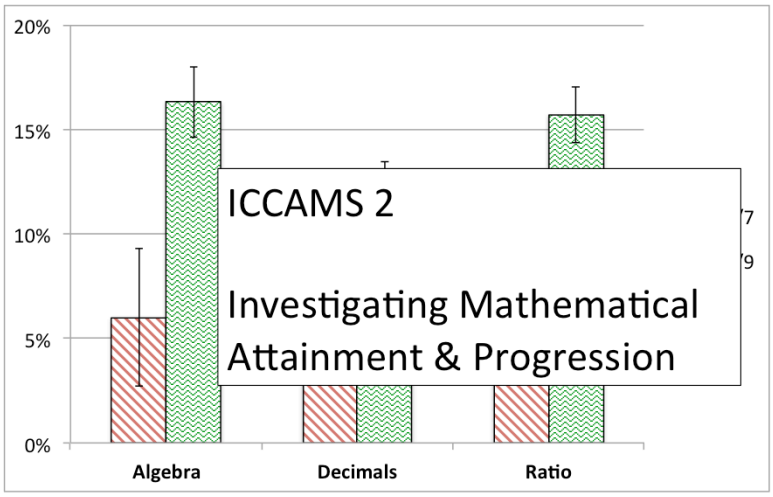
Hodgen, J., Küchemann, D., & Brown, M. (2010). Textbooks for the teaching of algebra in lower secondary school: are they informed by research? *Pedagogies, 5*(3), 187-201. doi: 10.1080/1554480X.2013.739275

School Mathematics Project (1970s)



Immediately a line is drawn, we are attaching meaning to the intermediate points and we must satisfy ourselves that each point on the line satisfies the relation: Is it true that for every point on the line, the second coordinate is always three times the first coordinate?

Watch this space …



Are we doing educational change ‘wrongly’?

Why has reform not worked?

- Wasted effort trying to change the wrong things
- High stakes tests and fragmented-objectives-driven lessons have led to very instrumental teaching *and* learning (Ofsted, 2008)
- What happens in classrooms is not the only factor … decimalisation

Why did the NNS 'work'?

- Pedagogy *and* curriculum?
  - Good tools and models: Counting sticks, Arrays/Area
- Guidance?
- OFSTED & KS2 tests?
- Key government focus?
  - Education, Education, Education
- Involvement of headteachers?
  - Stephen Ball's policy enaction
- But why did we stop???

How could we improve educational standards?

- Teachers tend to re-interpret new initiatives from existing perspectives (Cuban, 1993)
  - Complex & centralising policy environment
  - High stakes accountability regime
  - Obsession with success
- Research fragmented, little scaling up / cost or comparative benefits
- Is it time to forget grand schemes of national change?

Cuban, L. (1993). *How teachers taught: Constancy and change in American classrooms 1880-1990* (2nd ed.). New Yrok: Teachers College Press.

Margaret Brown: BCME 2010

We must focus on releasing the creativity of teachers and others for a 10-year period of innovation. There should be a moratorium on central prescription and teacher unions should organise a 10-year boycott on centrally imposed assessment and inspections.

- Is this impossible? It worked in the 1950s, and it works in other countries.

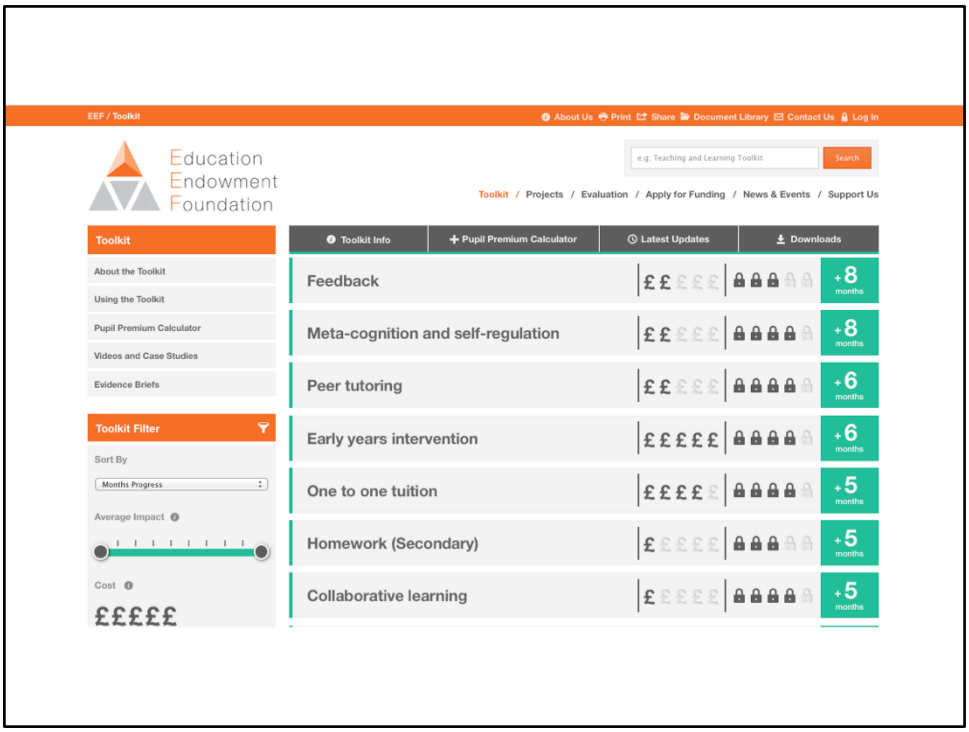
Reasons to be cheerful?

Reasons to be miserable

- Teacher shortage:
  - Y7-Y13: 5500
  - Wolf: 2400
  - Core Maths: 2000
  - Primary???
- Core Maths:
  - "I would rather die" (Brown et al): How are we going to persuade 200,000 to take maths post-16
- Time for CPD?
- Teacher Education?

Reasons to be cheerful

- Stability
- Grassroots teacher movement:
  - @Kris_Boulton @Dannytybrown @BodilUK @Mathsjem …
  - ResearchED
- Maths Hubs?
- Maths in the news
  - Hannah's sweets … Maths on TV
- Focus on research / evidence
  - Education Endowment Foundation
- Core Maths is exciting



Problems in all ‘league table’ metrics: cost, security of evdience and months gain BUT it is much, much better than anything that we’ve had previously.

Reasons to be cheerful

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Paper 1: Mathematical modelling
Refer to *Drug Decay* in the pre-release material.

Model the level of painkiller for a patient who is told to take up to a maximum of 4000 milligrams per day with doses at regular intervals. For example, they could take 1000 milligrams at 8 am, 12 noon, 4pm and 8pm every day.

Consider the effect of the person taking different doses at different regular intervals during the day.

The painkiller has a half-life of between 1 and 4 hours.

Use what you find out from your models to write a brief note to explain to patien how the concentration of a drug in general varies with time.

Show clearly all your working and highlight your final briefing note.

(45 marks)

City & Guilds Core Mathssample Paper 1. Others good to. Look at Pearson!

Questions and comments …