Mastery In Mathematics Education

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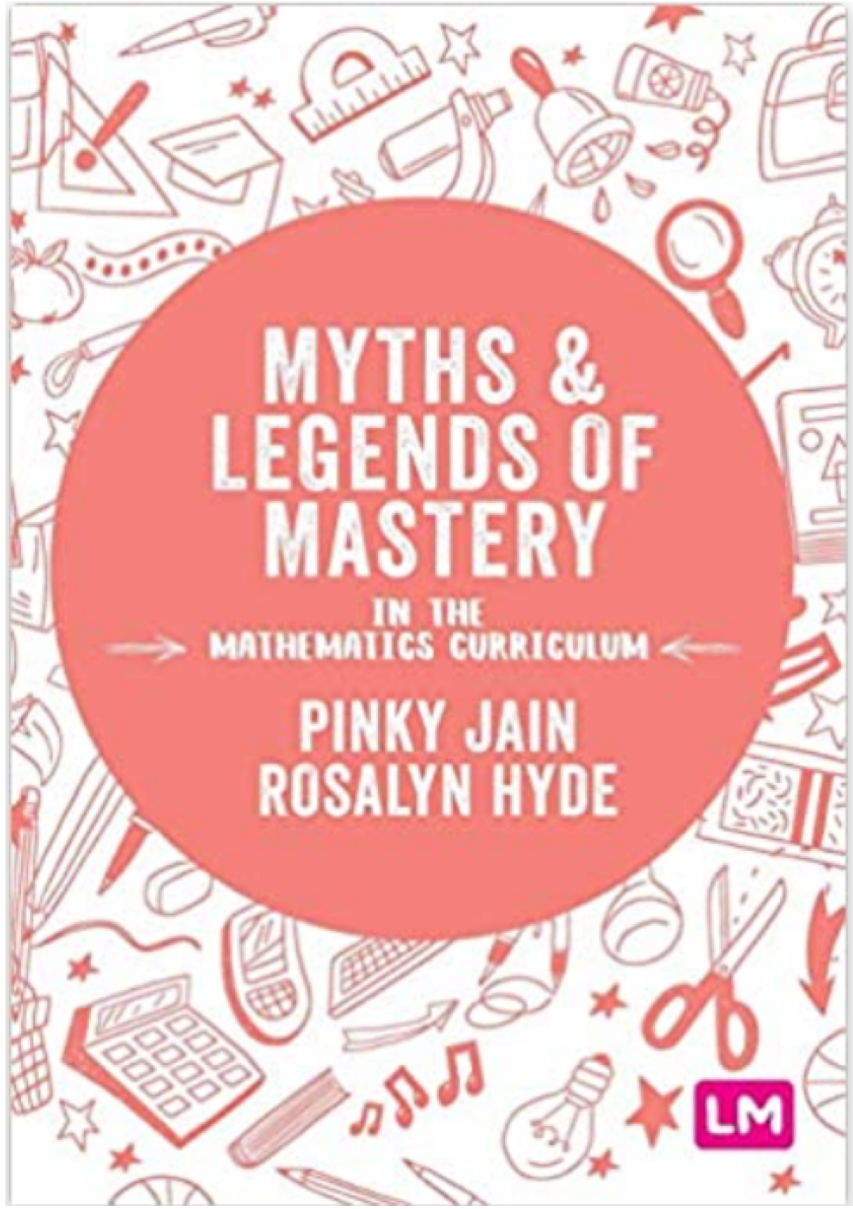
Tuesday 6th April ATM Conference 2021

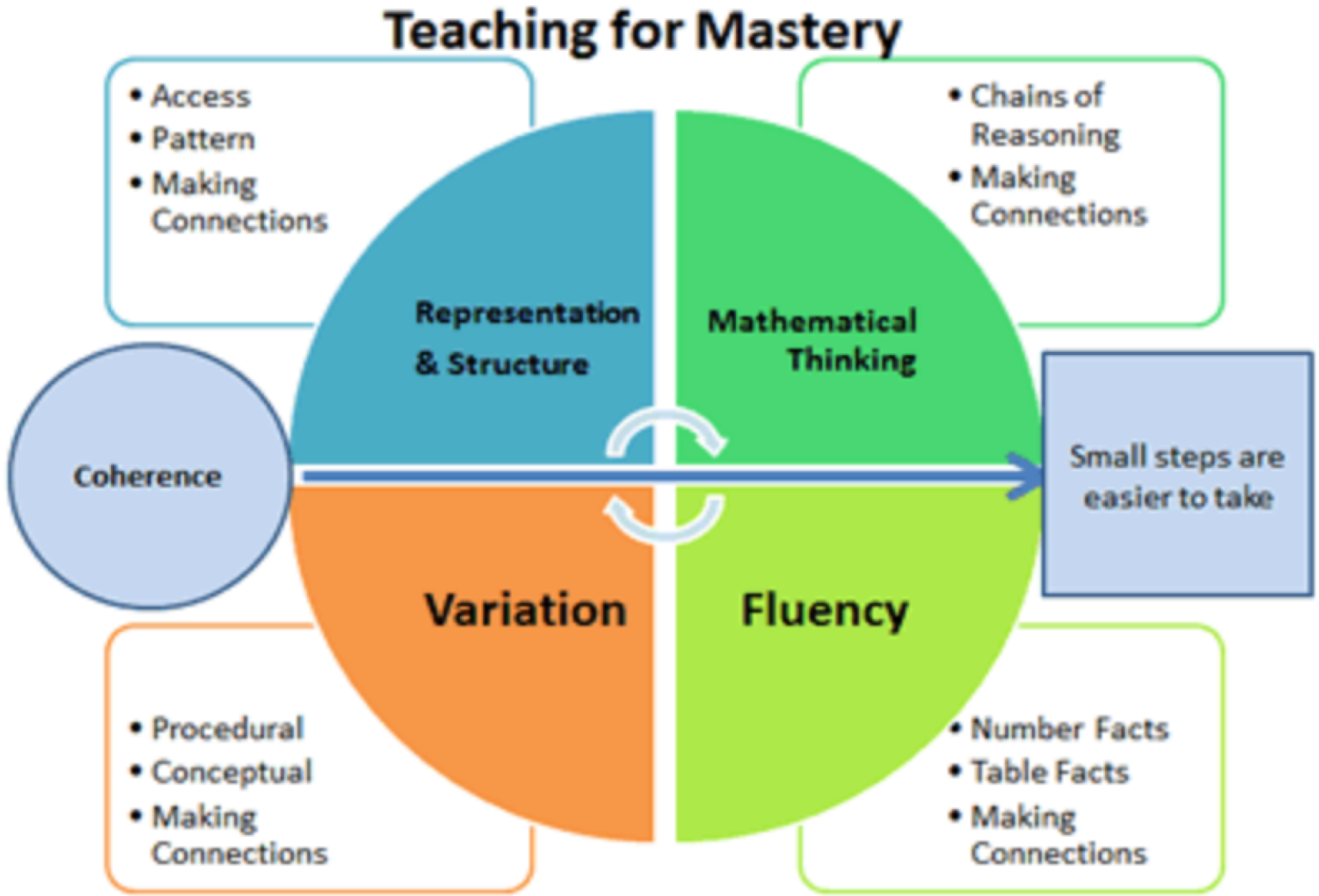
https://www.ametonline.org.uk/ To Join Please email Christine Packer c.packer@chester.ac.uk


MYTHS &
LEGENDS OF
MASTERY
IN THE
MATHEMATICS CURRICULUM
PINKY JAIN
ROSALYN HYDE
LM

Mathematics mastery
What does the concept of mastery really mean?

A mathematical concept or skill has been mastered when, through exploration, clarification, practice and application over time, a person can represent it in multiple ways, has the mathematical language to be able to communicate related ideas, and can think mathematically with the concept so that they can independently apply it to a totally new problem in an unfamiliar situation. (Drury, 2014, p. 9)

Bloom believed that all students could be helped to reach a high criterion of learning if both the instructional methods and time were varied to better match students' individual learning needs. In other words, to reduce variation in the achievement of diverse groups of students and have all students learn well, Bloom argued that educators and teachers must increase variation in instructional approaches and learning time. Bloom labeled the strategy to accomplish this instructional variation and differentiation mastery learning. (Guskey, 2007, p. 9)



https://www.ncetm.org.uk

a) “In education settings, we verify mastery by asking individuals to respond to a series of questions or to perform a sequence of tasks. We then judge the adequacy of their responses or performance as measured against specific criteria. So in essence, we determine mastery through some form of assessment”. Guskey (also above)

b) “To teach for mastery is to teach with the highest expectations for every learner, so that their understanding is deepened, with the aim that they will be able to solve non-standard problems in unfamiliar concepts’. Drury (also above)

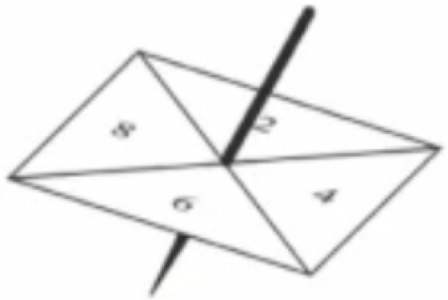
c) “Maths mastery is a teaching and learning approach that aims for pupils to develop deep understanding of maths rather than being able to memorise key procedures or resort to rote learning. The end goal and expectation is for all pupils (with very limited exceptions) to have acquired the fundamental facts and concepts of maths for their year or key stage such that by the end of it they have achieved mastery in the maths they have been taught. At this point they are ready to move confidently on to their next stage of maths”. McCourt

d) TfM (NCETM definition above)

e) Other

Mulligan, A., & Cochrane, D. (2019). Defining mastery. *Myths and Legends of Mastery in the Mathematics Curriculum: Enhancing the Breadth and Depth of Mathematics Learning in Primary Schools*, 4.

Some early attempts to implement mastery learning were based on narrow and inaccurate interpretations of Bloom's ideas. **These programs focused on only low-level skills; attempted to break learning down into small, patchy segments; and insisted that students master each segment before being permitted to move on. Teachers in these programs were regarded as little more than managers of materials and record keepers of student progress. Nowhere in Bloom's writing, however, can this kind of narrowness and rigidity be found.** In fact, Bloom emphasized quite the opposite. He considered thoughtful and reflective teachers vital to the successful implementation of mastery learning and continually stressed flexibility in its application. (Guskey, 2007, p. 12)
https://files.eric.ed.gov/fulltext/EJ786608.pdf



The four-sided spinner is labelled 2, 4, 6, 8
The five-sided spinner is labelled 1, 3, 5, 7, 9

Louise adds the score on the four-sided spinner to the score on the five-sided spinner.
She records the possible total scores in a table.

5-sided spinner \ 4-sided spinner	2	4	6	8
1	3	5	7	9
3	5	7	9	11
5	7	9	11	13
7	9	11		
9	11	13		

Find the probability that Louise's total score is a prime number.

18. (a) Write 1008 as a product of prime factors.
Express your answer in index form.

.....

(3)

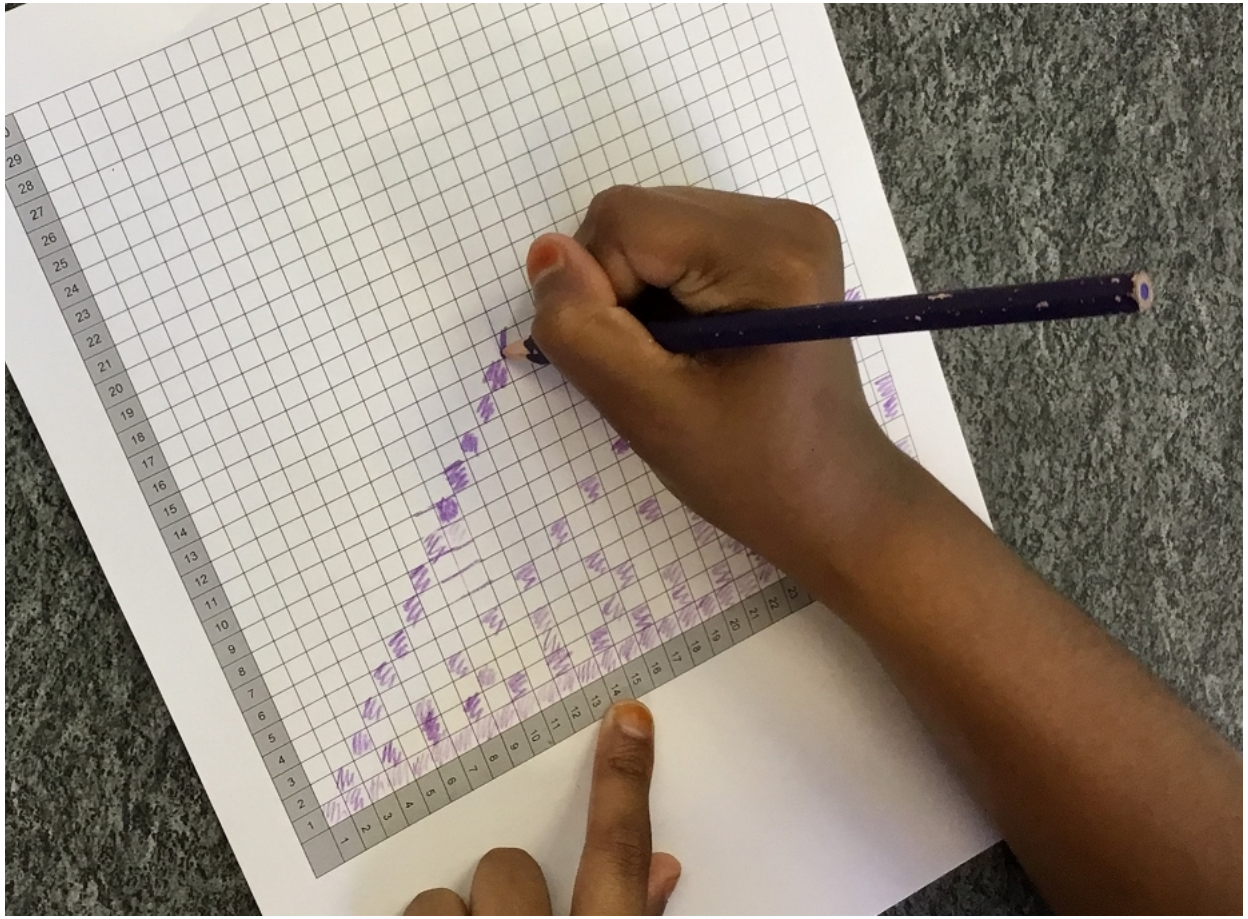
(b) Hence find the **least** number by which 1008 would need to be multiplied by to give a square number.

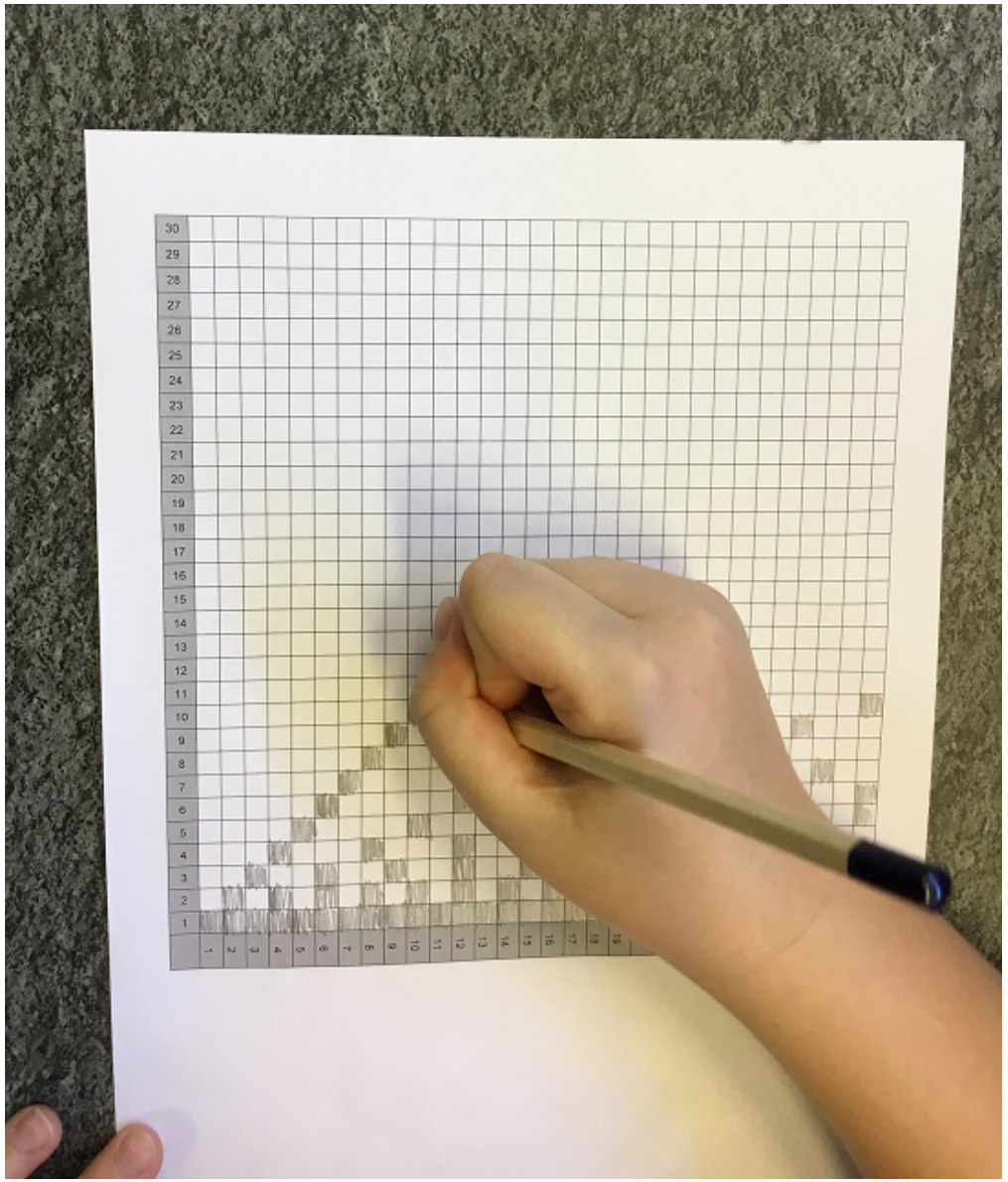
.....

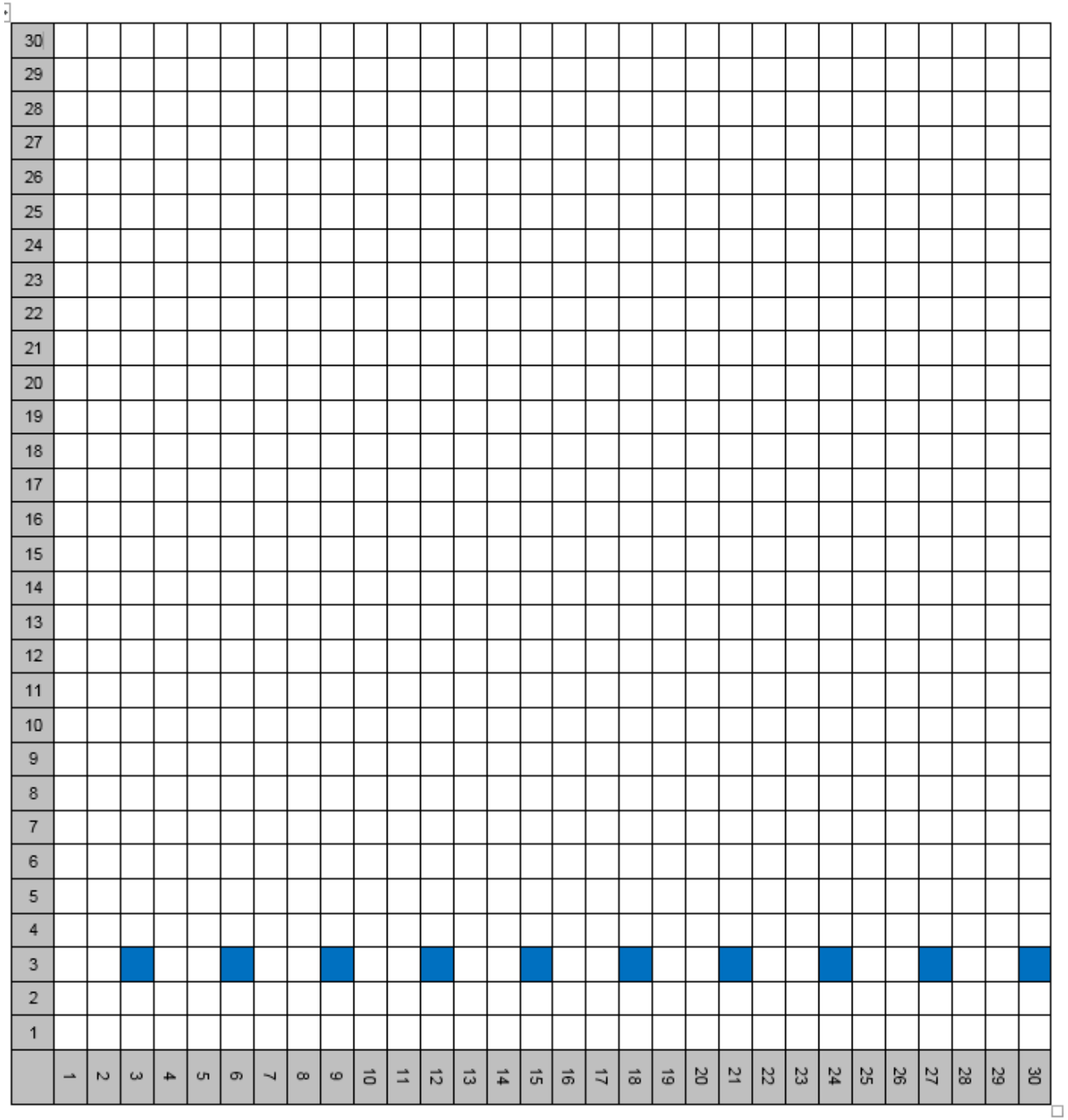
(1)

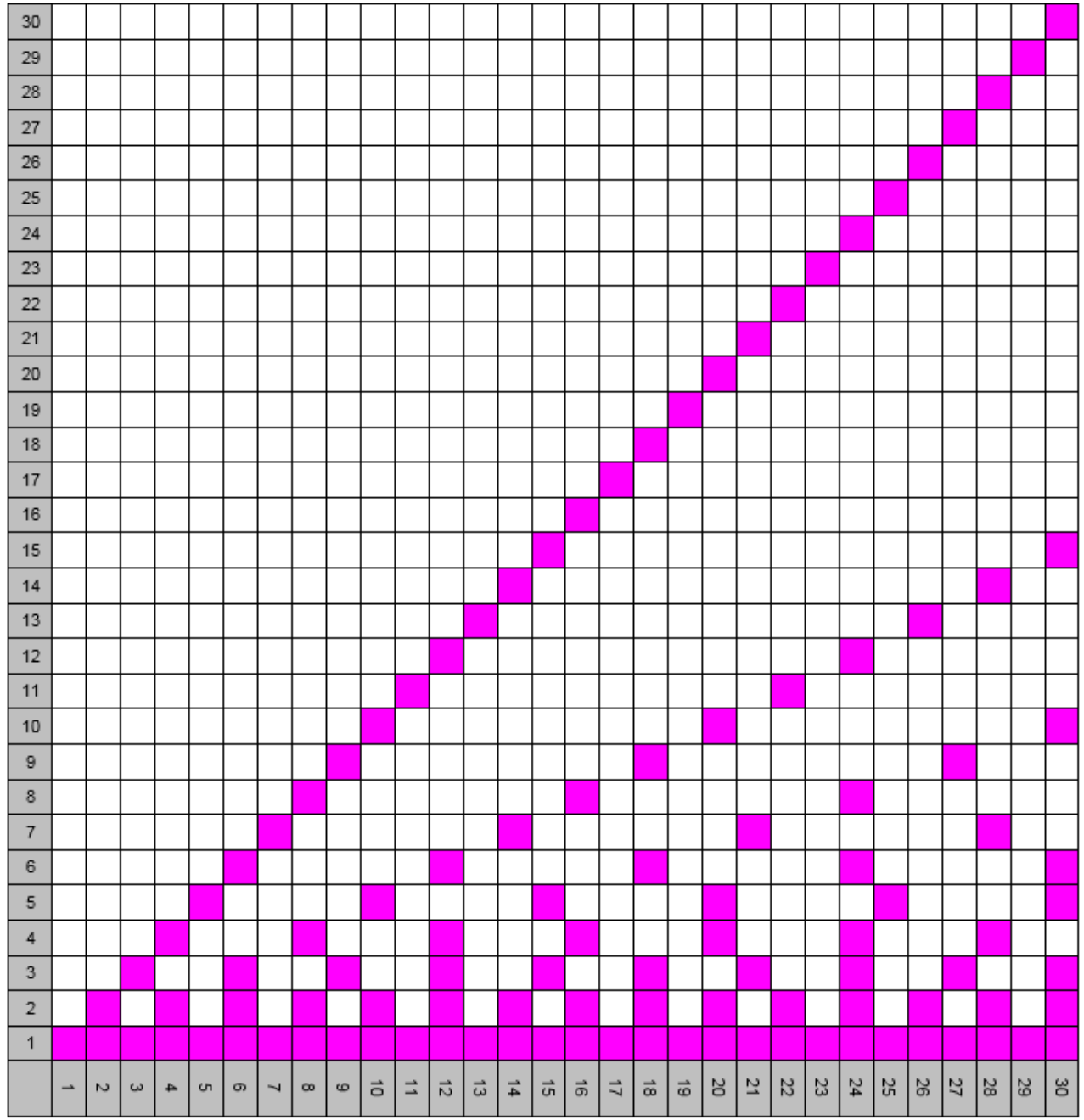
Rule

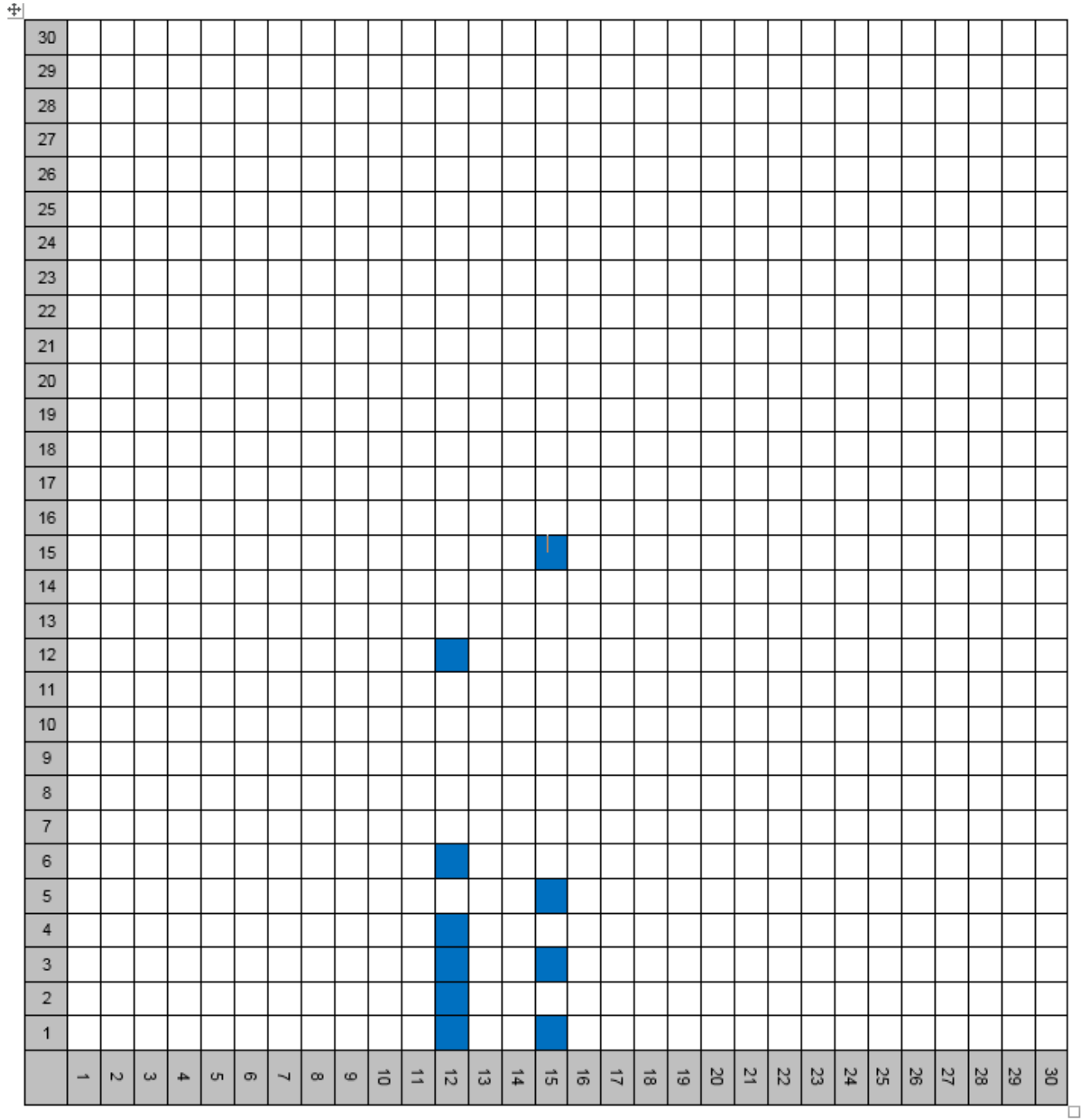
Not Rule

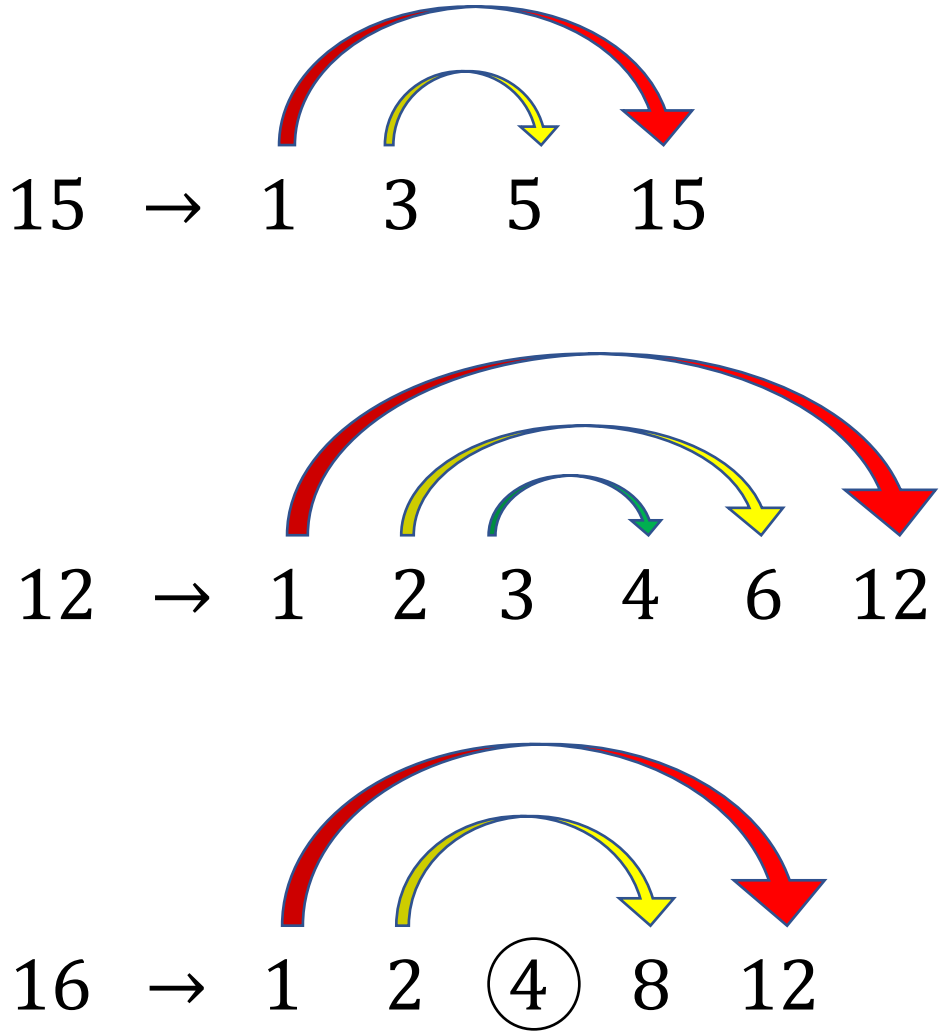











15 → 1 3 5 15
12 → 1 2 3 4 6 12
16 → 1 2 4 8 12

Features of teaching for mastery

Mathematics teaching for mastery rejects the idea that a large proportion of people ‘just can’t do maths’. All students are encouraged by the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.

Facility with procedures and algorithms without a deep and connected understanding does not constitute mastery. Mastery is achieved through developing procedural fluency and conceptual understanding in tandem, since each supports the other.

Where does the idea come from?

Central to the current mastery promotion is the idea that mathematics should be taught in a way similar to the one used in East Asia (Shanghai and Singapore in particular) since these countries do well in international comparisons (PISA).

Increasingly educational policy is shaped by international forces of global comparisons, market or semi-market orientations, and political ideologies that shape what happens in school ….. (Boylan 2020)

Features of teaching for mastery

- the use of variation (see Watson and Mason, 2006) and using variation to help learners generalise and capture key mathematical ideas;
- a high level of interaction in lesson, where children are encouraged to work together, discuss mathematical content, express their ideas and learn from each other;
- the use of manipulatives, imagery and models; the making of connections between different areas of mathematics;
- differentiation through using tasks that can be accessed at different levels of depth, rather than acceleration into new content, with scaffolding intervention for these students who might struggle to make sure they don’t fall behind.

We are clearer about the required standard

We know the best-performing places - like Singapore or Shanghai - have high expectations for every pupil.

Classes are ‘taught to the top’ - and then struggling students are given extra support to keep up.

Of course individual needs are attended to. But there is **no false differentiation: they don’t set out, right from the beginning, with the assumption that some children just won’t make the grade**.

https://www.gov.uk/government/speeches/elizabeth-truss-speaks-about-improving-teaching

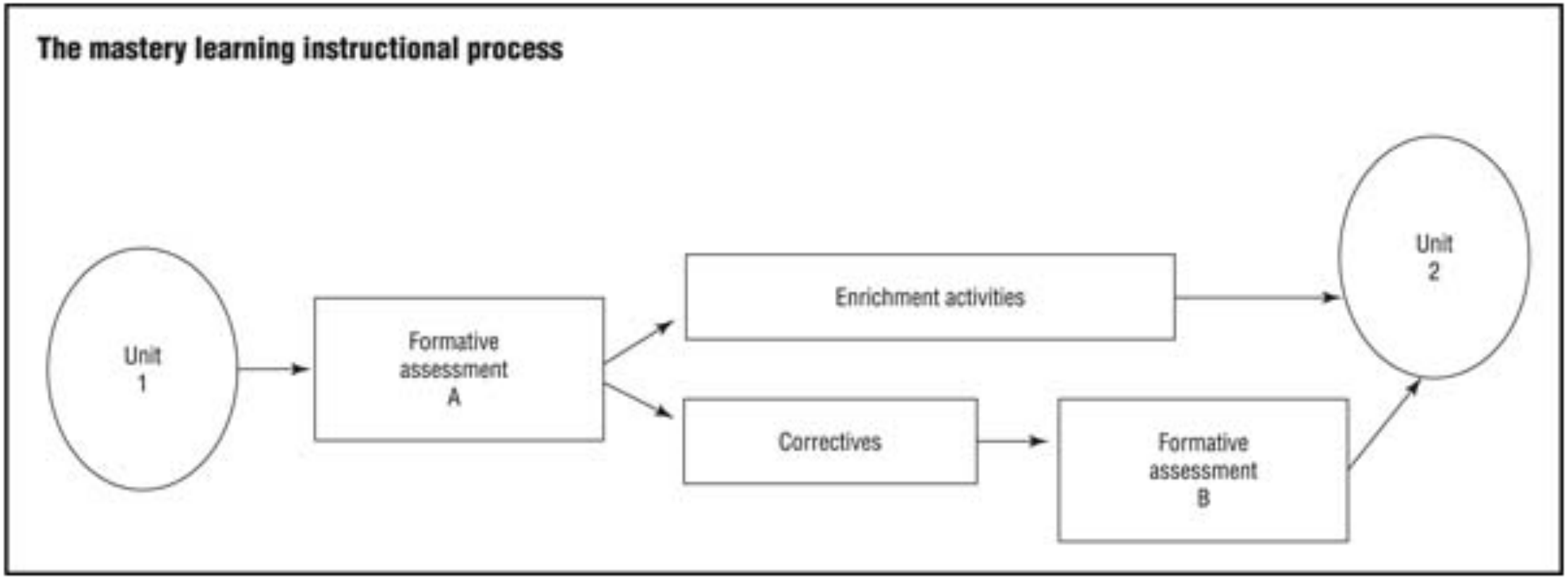


Figure 2ILLUSTRATION BY GGS INFORMATION SERVICES. CENGAGE LEARNING, GALE.

Guskey, 2015https://tguskey.com/wp-content/uploads/Mastery-Learning-1-Mastery-Learning.pdf

Bloom, B. S. (1971a). Mastery learning. In J. H. Block (Ed.), *Mastery learning: Theory and practice* (pp. 47–63). New York: Holt, Rinehart & Winston.

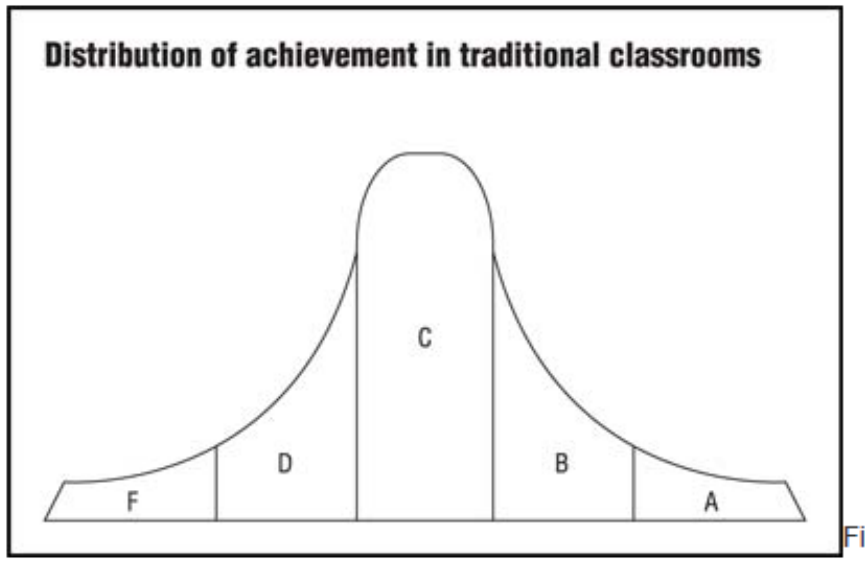


Figure 1ILLUSTRATION BY GGS INFORMATION SERVICES. CENGAGE LEARNING, GALE.

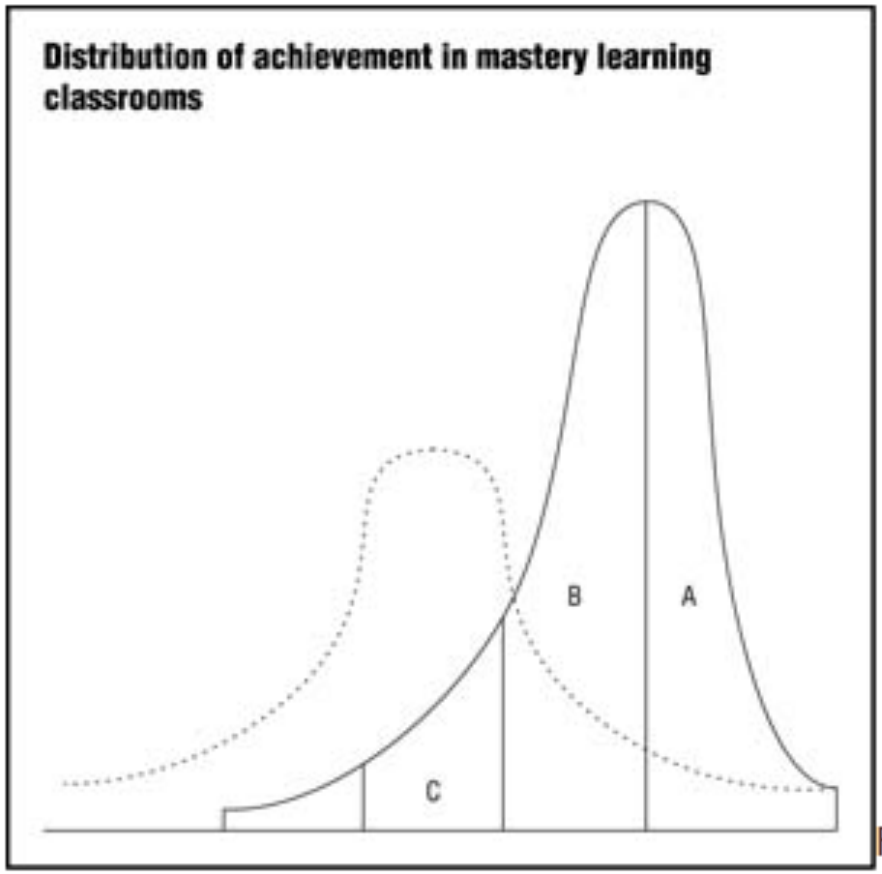


Figure 3ILLUSTRATION BY GGS INFORMATION SERVICES. CENGAGE LEARNING, GALE.

Guskey, 2015
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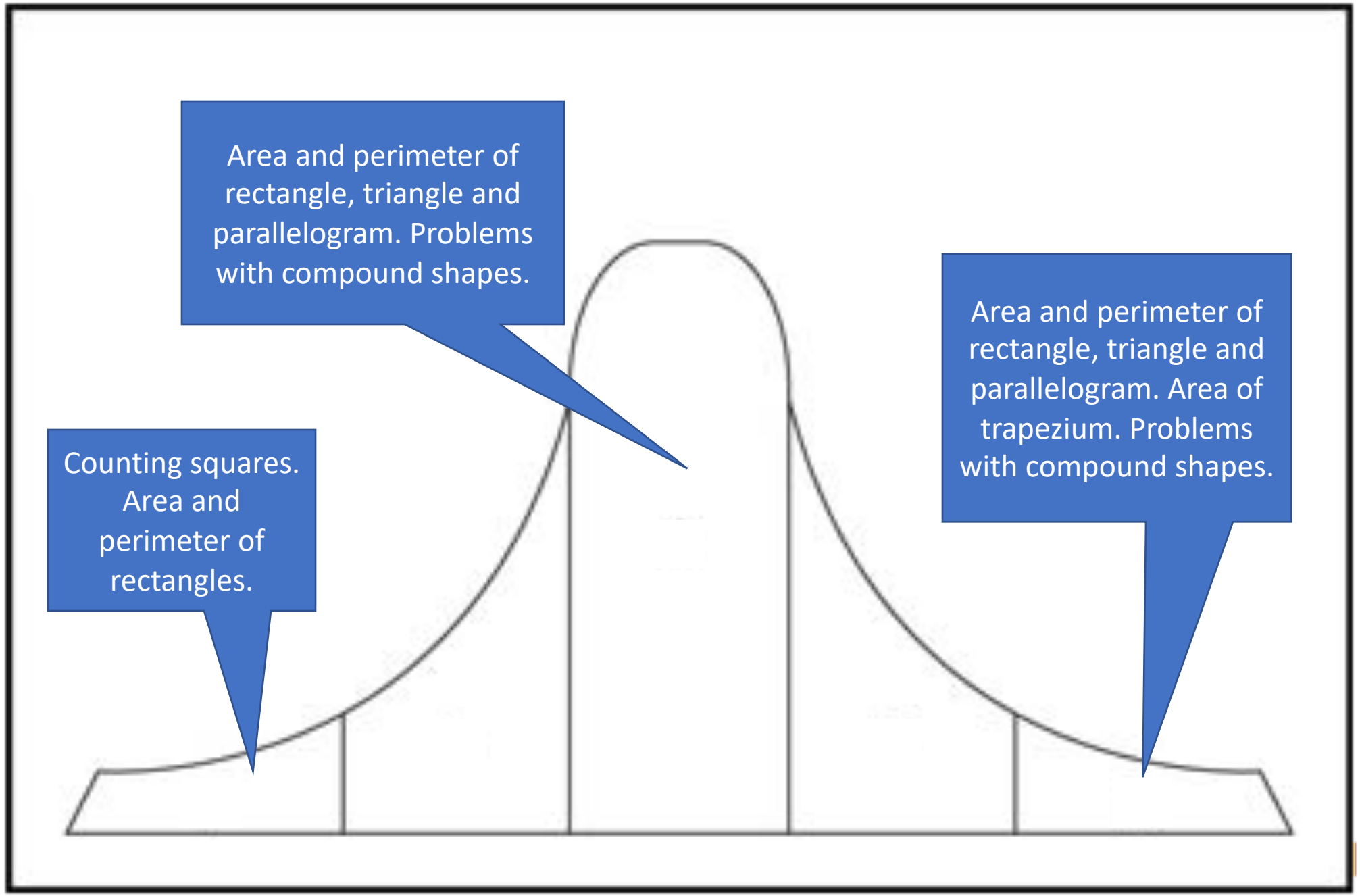
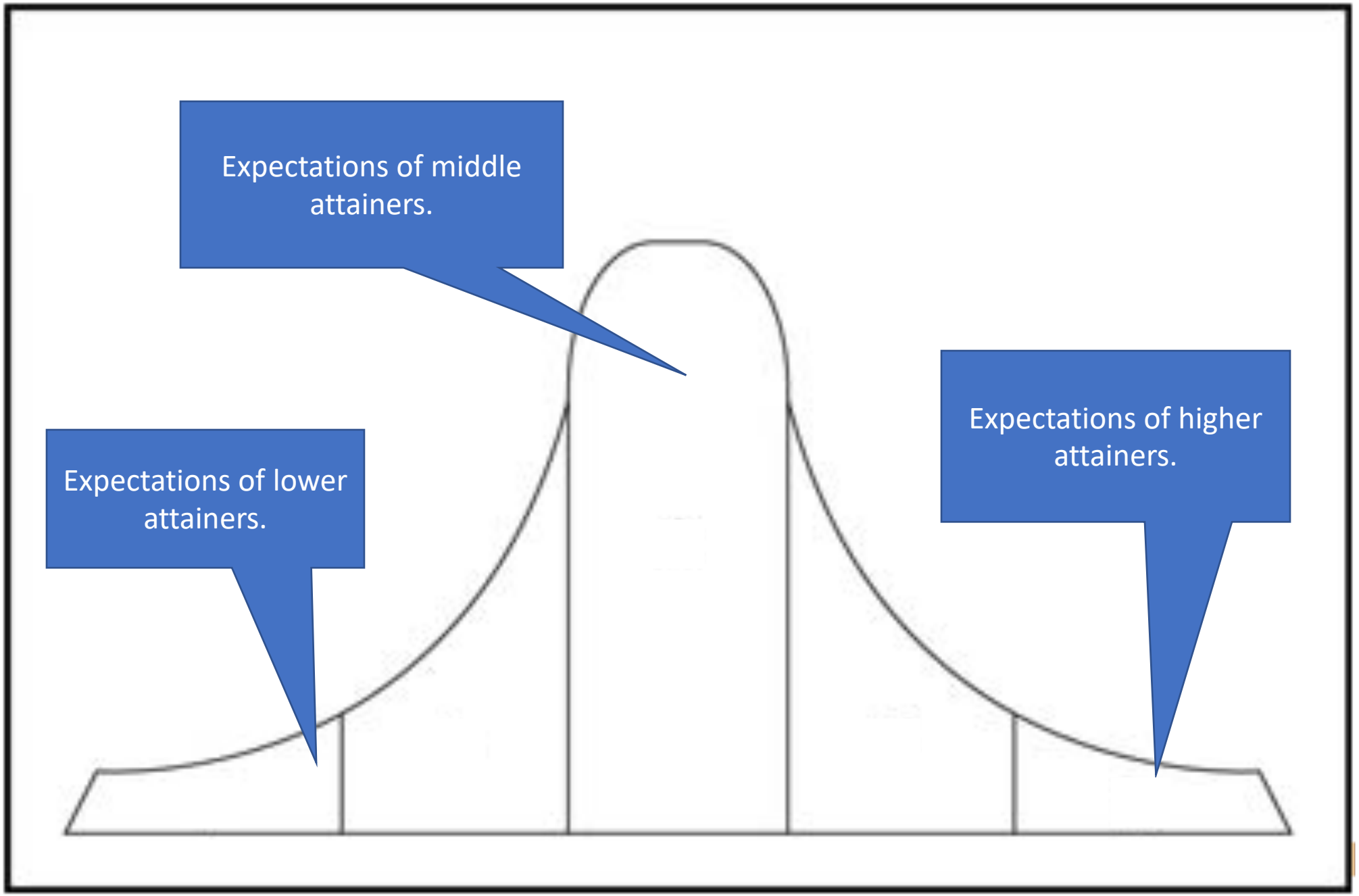

Area and perimeter of rectangle, triangle and parallelogram. Problems with compound shapes.
Counting squares. Area and perimeter of rectangles.
Area and perimeter of rectangle, triangle and parallelogram. Area of trapezium. Problems with compound shapes.


Expectations of middle attainers.
Expectations of lower attainers.
Expectations of higher attainers.

Intended Objective(s) for pupils' learning; **What do I want my pupils to learn this lesson, given what I know about their prior knowledge, skills and understanding?**	
Inclusion; how will you **support** pupils so that everyone can access the lesson? **What support do I need to put into place so that everyone has access to my objective?**	Inclusion; how will you **challenge** pupils who meet/have met the learning objective? **How do I enrich the learning for those pupils who have already mastered the objective or master it readily in my lesson?**

In the mastery learning model teachers' knowledge of enactive and iconic representations of concepts and language models. Knowledge of how children learn.

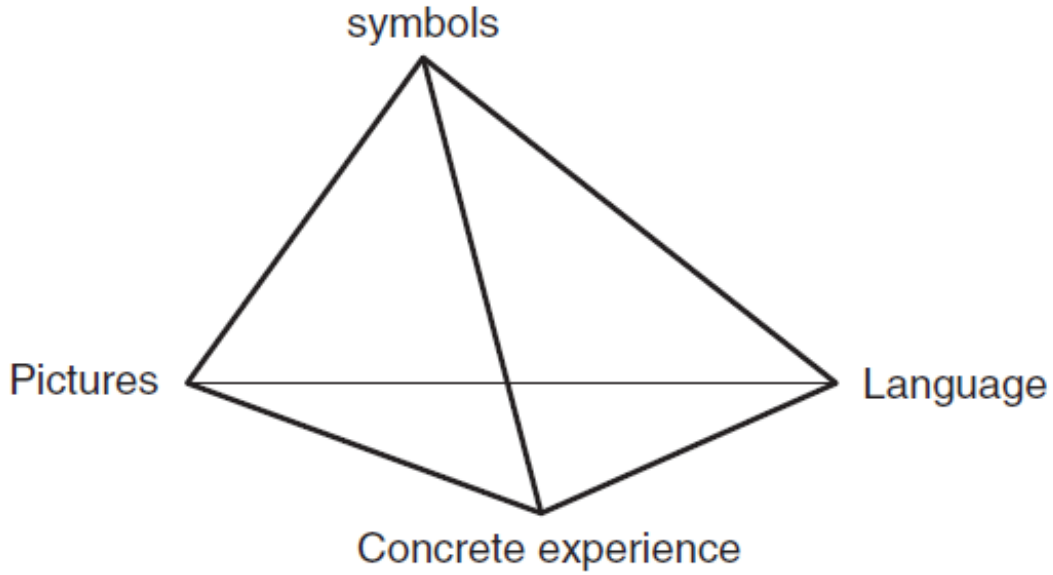
In the mastery learning model pupils do not move on to the next level or next topic in order to be challenged. Their learning is enriched within the current unit in order to gain a deeper understanding.

Bruner's Representations of Knowledge

- Enactive
- Iconic
- Symbolic

Singapore's Ministry of Education

- Concrete
- Pictorial
- Abstract



(Haylock & Thangata, 2007)

Leong, Y. H., Ho, W. K., & Cheng, L. P. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator*, 16(1), 1-18. Retrieved from http://math.nie.edu.sg/ame/matheduc/tme/tmeV16_1/TME16_1.pdf

A unit of work detailing what everyone should learn.
Alignment between teaching, content and learners.

Formative assessment of attainment in that unit of work. Identifying what has been mastered and targets for improvement.

Intervention

Eg. Solving problems using other representations.

Enrichment

Deepening understanding of the concepts within the unit (not extension to the next concept)

Separate goal or separate tasks?

Variation Theory

You cannot know what something is, without knowing what it is not.

Ference Marton, June 2012

Variation Theory

You cannot know what something is, without knowing what it is not. If you have only heard English all your life, you cannot know what “English” means. It is simply “language” for you (and not *a* language). Similarly, … you cannot understand what linear equations are without having come across other kinds of equation. In the same way, you cannot understand what “a lively style of writing” is by considering only examples of a lively style; you would need to have encountered more and less lively styles.

It is the patterns of variation and invariance among examples, instances, cases, illustrations and so on, which is the aspect of teaching that Variation Theory singles out as ***a key to better learning.***

Ference Marton, June 2012

https://gupea.ub.gu.se/bitstream/2077/29645/5/gupea_2077_29645_5.pdf

Variation Theory

Watson, and others, talk about variation in task design:

https://www.atm.org.uk/write/MediaUploads/Journals/MT252/MT252-16-04.pdf

How many ‘x’ do you need in this expression

$$x^2 + ?\, x + 12$$

To make a rectangle with no gaps and no tiles leftover?

What are the dimensions of your rectangle?

Amcan: I will learn ...

1) $10(\frac{1}{2}x + 5) = 10x + 50$

2) $3(x + 7) = 3x + 21$

3) $5(2x + 1) = 10x + 5$

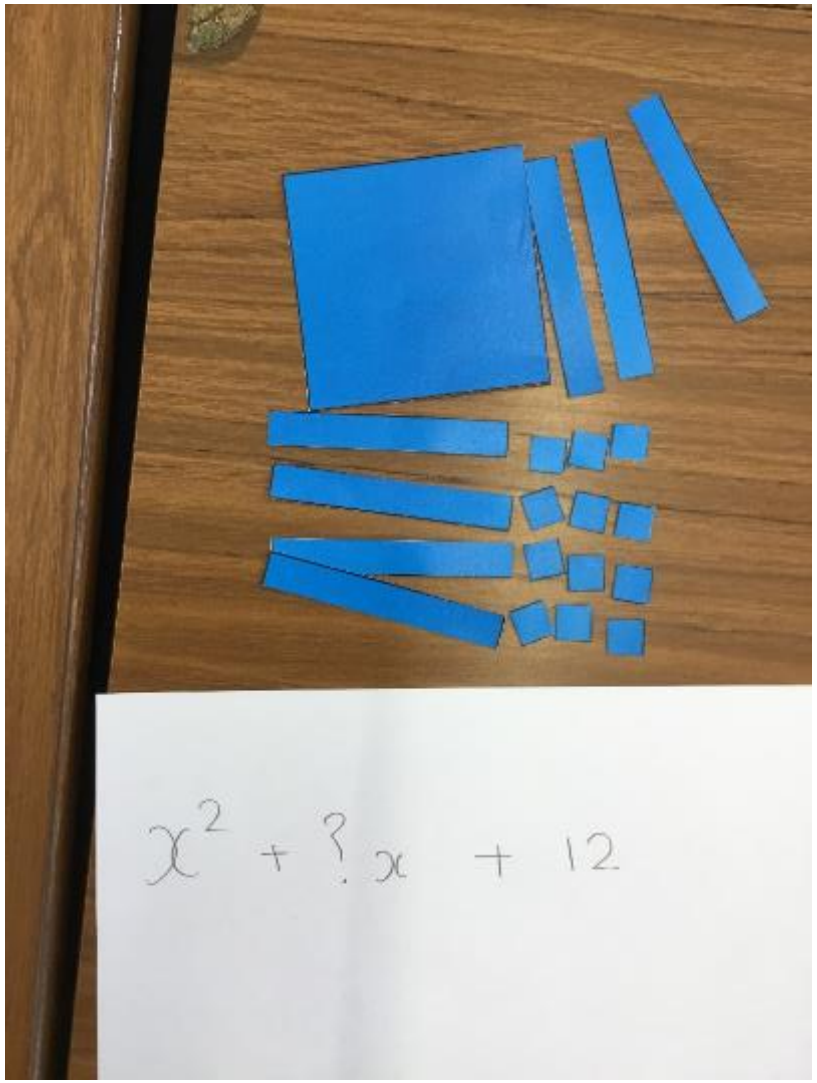
$(x+9)(x+1) = x^2 + 10x + 9$

$(x+8)(x+2) = x^2 + 10x + 16$

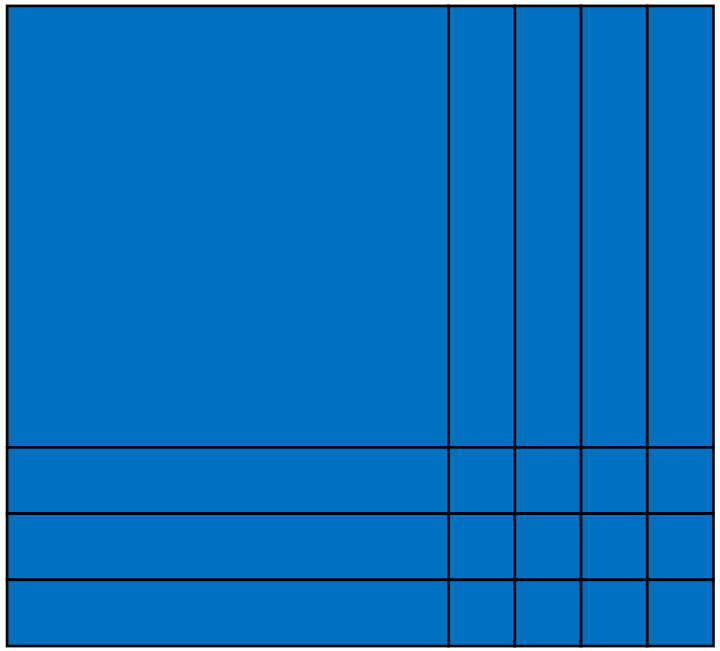
$(x+7)(x+3) = x^2 + 10x + 21$

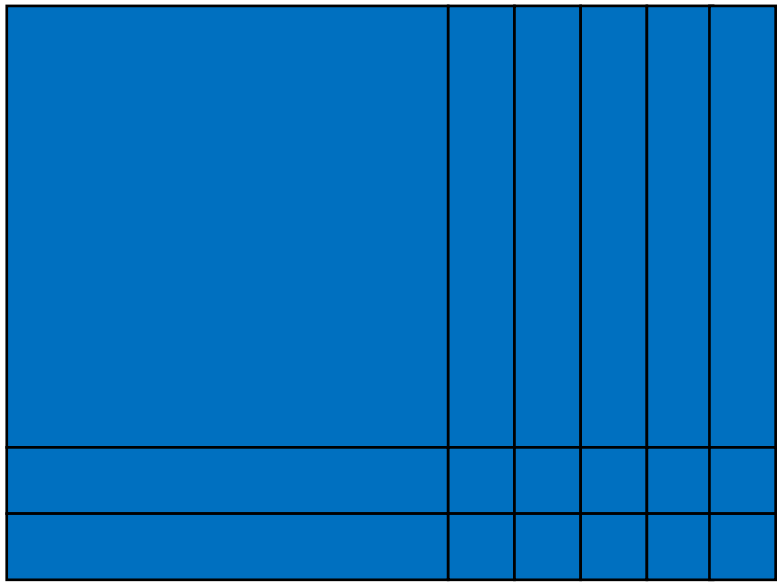
$(x+6)(x+4) = x^2 + 10x + 24$

$(x+5)(x+5) = x^2 + 10x + 25$



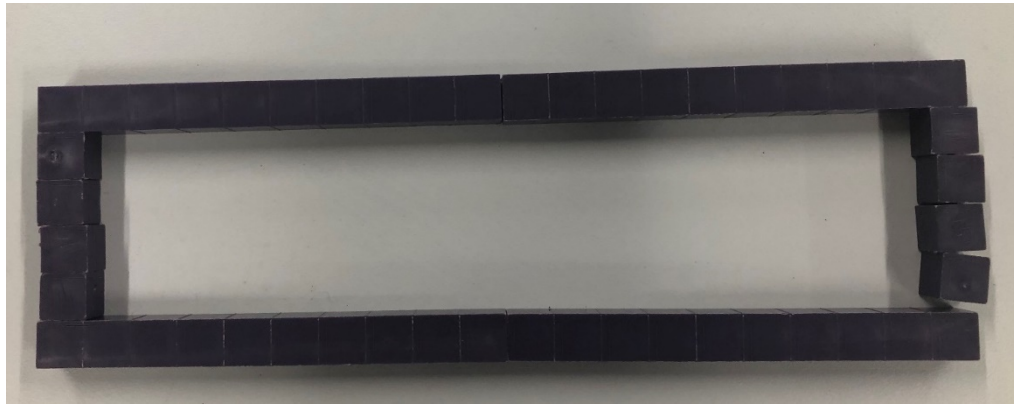
$$x^2 + 7x + 12$$

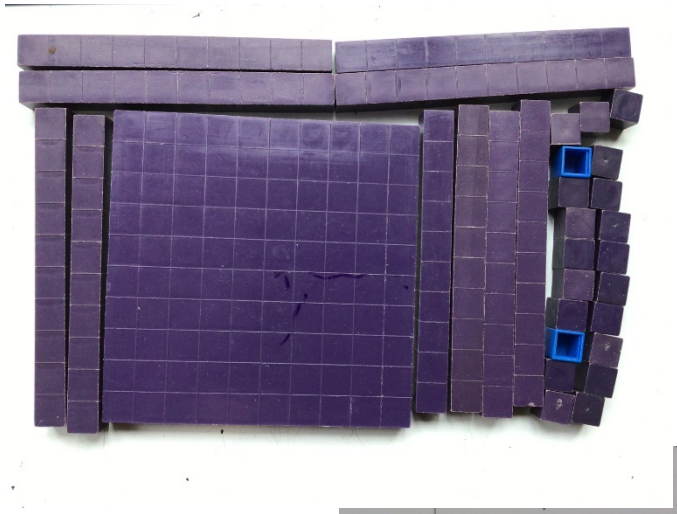


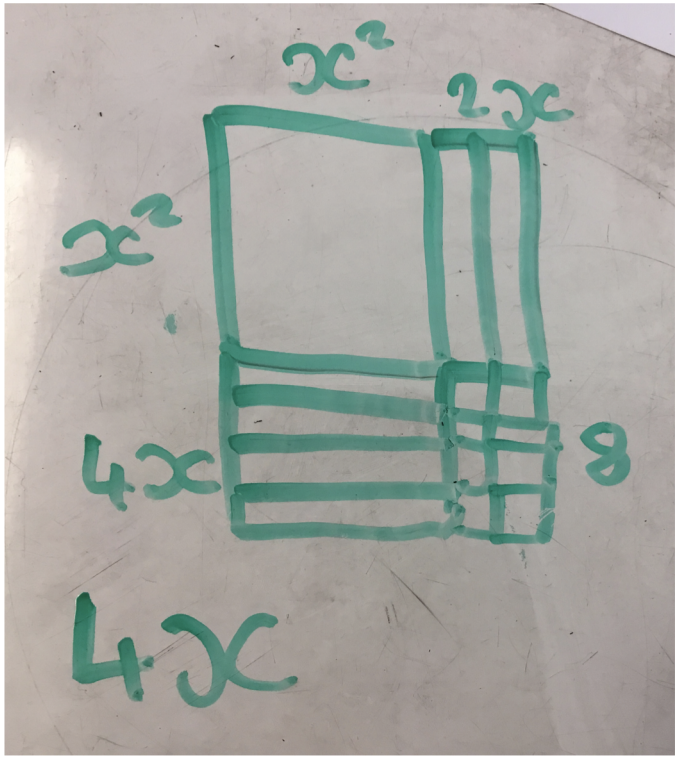


Both images involve $x^2 + 7x$.

Which image represents the expression?



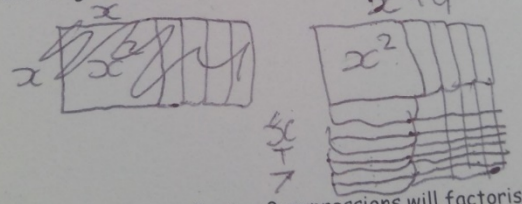




$x^2+11x+28$

$(x+7)(x+4)$

as a diagram. What is the equivalent expression?



5. Which 2 of the following 3 expressions will factorise?

The NCETM https://www.ncetm.org.uk is funded by the government to promote and implement government policy in mathematics education (which includes the mastery approach)

We must distinguish between the aim – mastery – and how it is achieved – teaching. Mastery is not a way of teaching!

How did it start?

Interest in East Asian approaches lead to two study visits to Shanghai (funded by the DfE) in 2012/13. By 2014 the NCETM had adopted the word ‘mastery’ and started writing about it. The government funded the first teacher exchange with Shanghai in 2014/15. From this the TfM programme developed. This programme (led by the NCETM) consists of professional development programme to train mastery specialists supported by textbooks and exchange visits with Shanghai. This was followed by Teresa May’s visit to Shanghai in 2018 where she talked about exporting to England mathematics teaching and English teaching to China.

There are significant differences between the education systems

	Shanghai	England
Teaching approach	Whole class teaching. Focus on questioning. Carefully chosen examples with variation. Emphasis on correct mathematical language	Teacher transmission followed by individual practice.
	Lesson starts with a mathematical problem	Teacher tries to maximise content covered in a lesson
Organisation	35 minutes lesson, practice as homework	One hour lesson with practice in lesson
Curricular progress	Nationally adopted textbooks, writing is supported by research of how children learn. Content covered in fine detail before progression	Low attaining students progress slowly, high attaining students are accelerated
Teacher preparation	Both primary and secondary teacher have extensive mathematical knowledge. Teach 2 35 minutes lessons a day. 340 to 560 hours of professional development in the first five years of teaching	Limited opportunities for mathematics specific professional development

Ineson, G., & Povey, H. (Eds.). (2020). *Debates in Mathematics Education*. Routledge.

Where do the ideas come from? Is it all new?

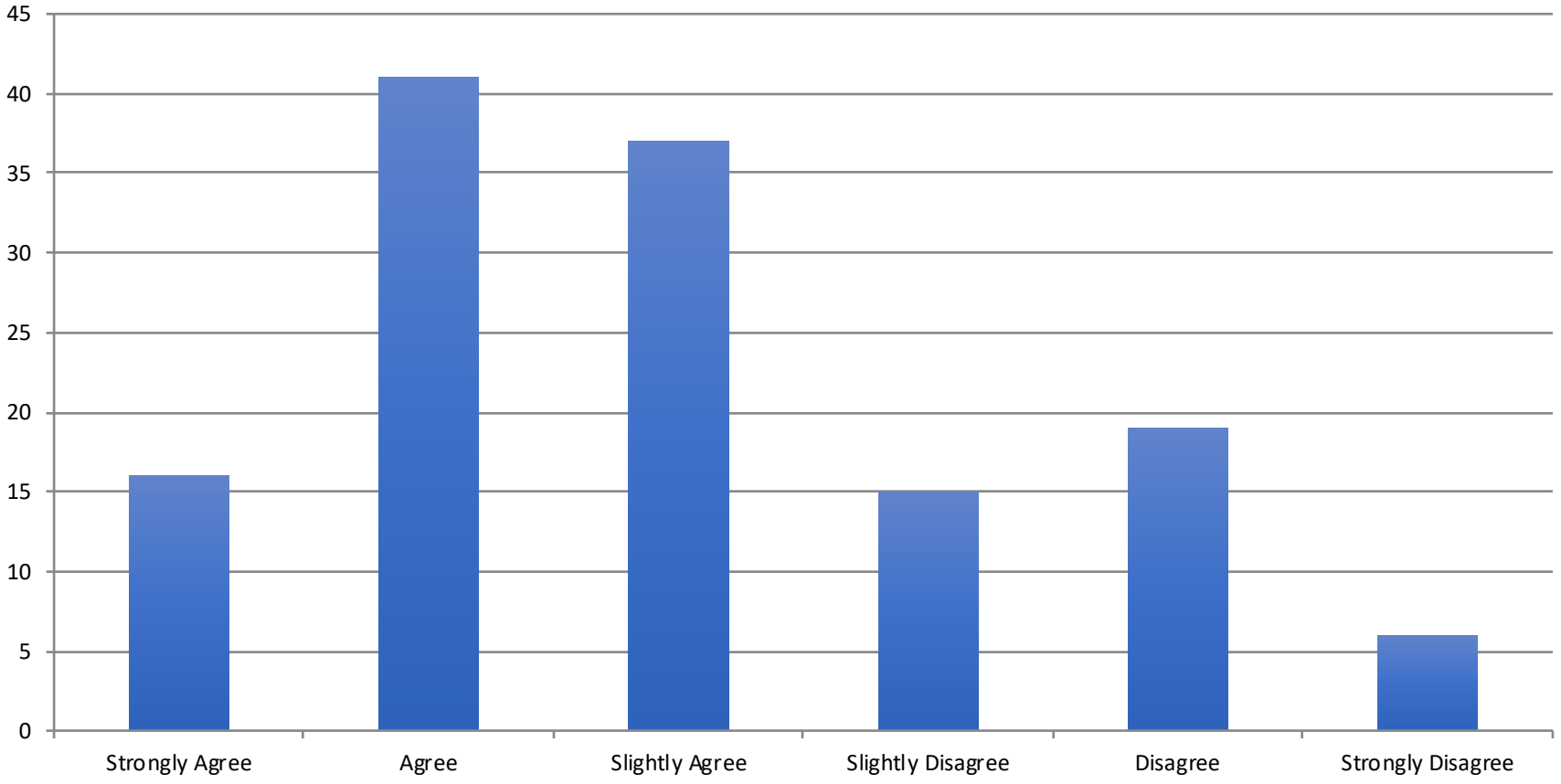
The idea of sequenced learning as concrete-pictorial-abstract is a Singaporean interpretation of Bruner's (1966) theory of enactive iconic and symbolic and the importance of using multiple representations in learning mathematics. The variation theory that seems to be a distinctive aspect of Shanghai teaching has been promoted in England for a long time by educationalists like Anne Watson, Mike Askew and John Mason. While the emphasis on teacher pupil interaction described in Shanghai practices is reminiscent of dialogic teaching promoted in England (Williams and Ryan)

What does it look like in a school/ classroom setting and how can it be assessed?
How can we teach across a range of abilities and ensure all children are mastering?

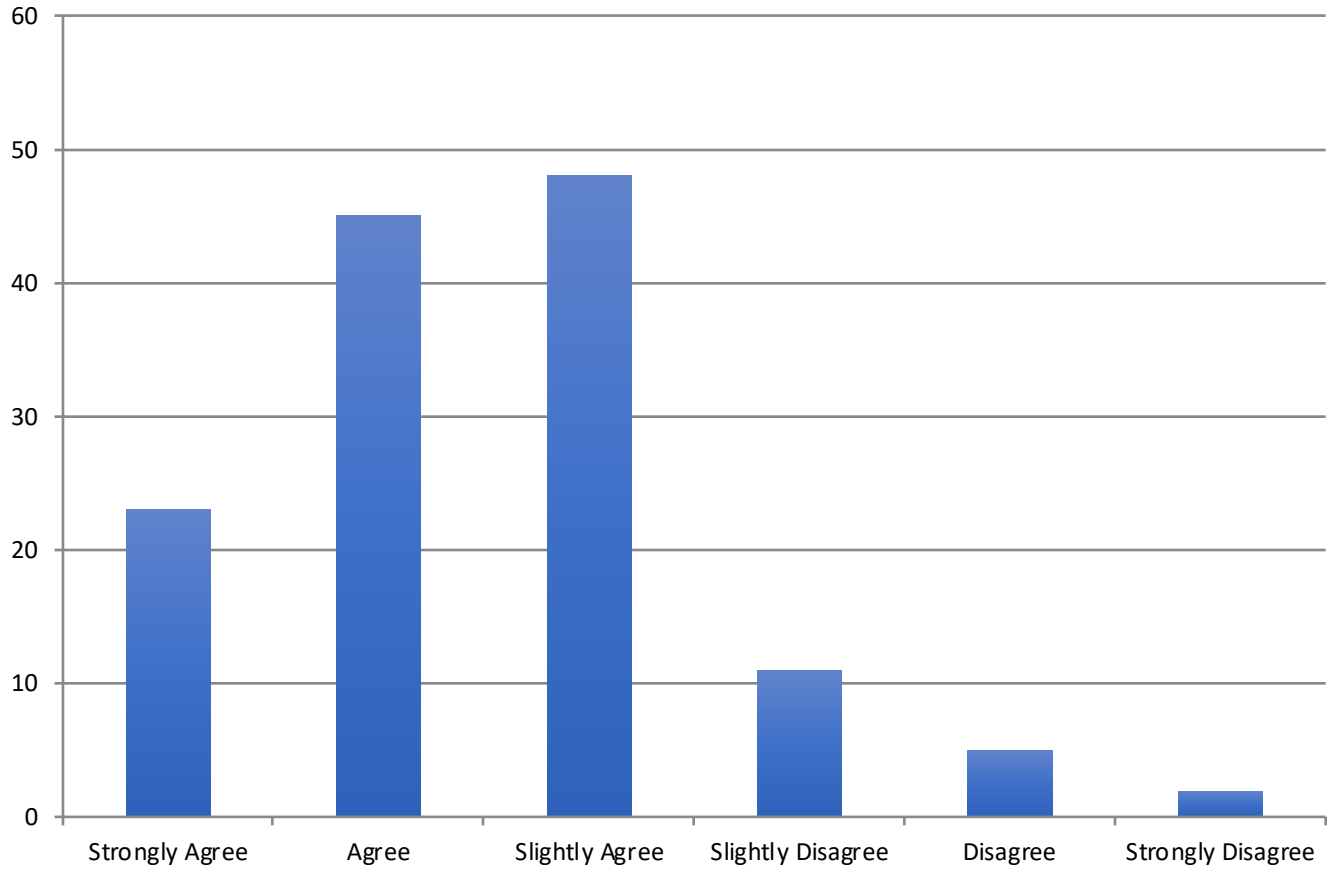
Results of survey

135 trainee teachers from different HEIs (primary and secondary responded to the survey.

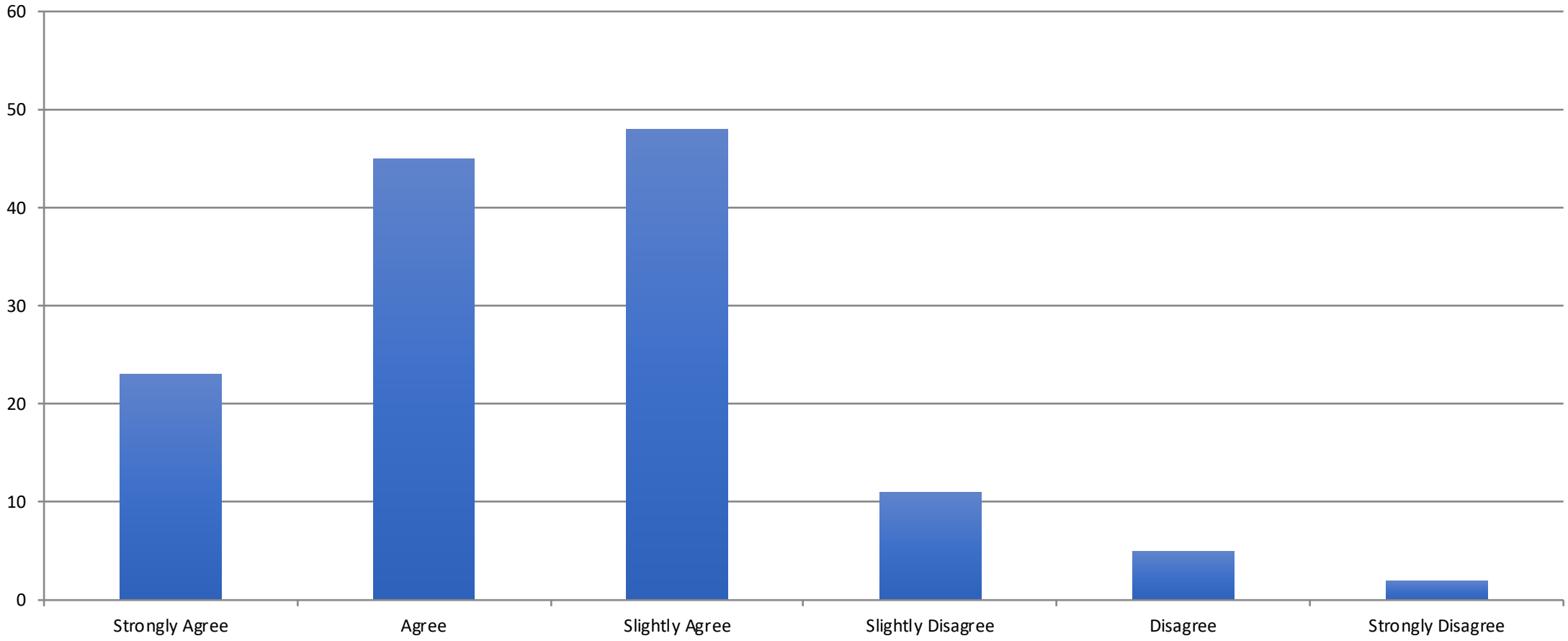
Mathematics is a collection of rules and procedure that prescribe how to solve problems



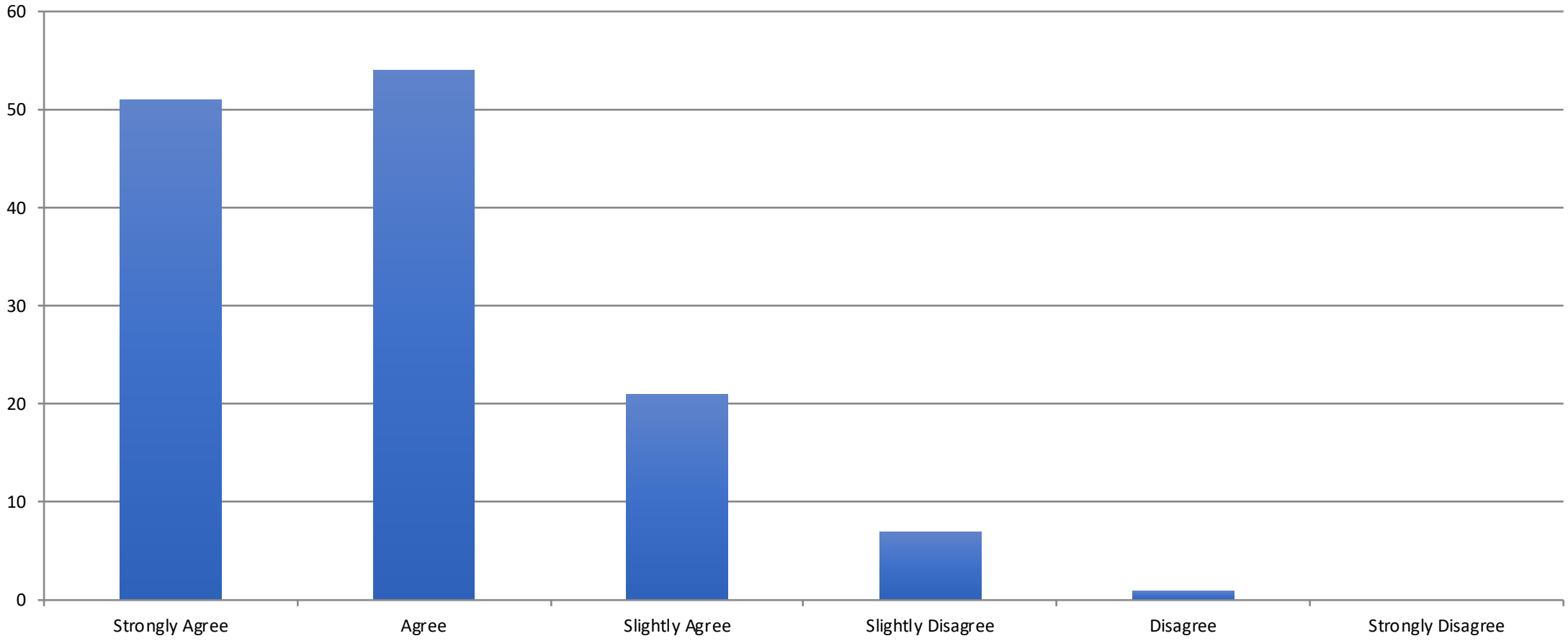
Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures



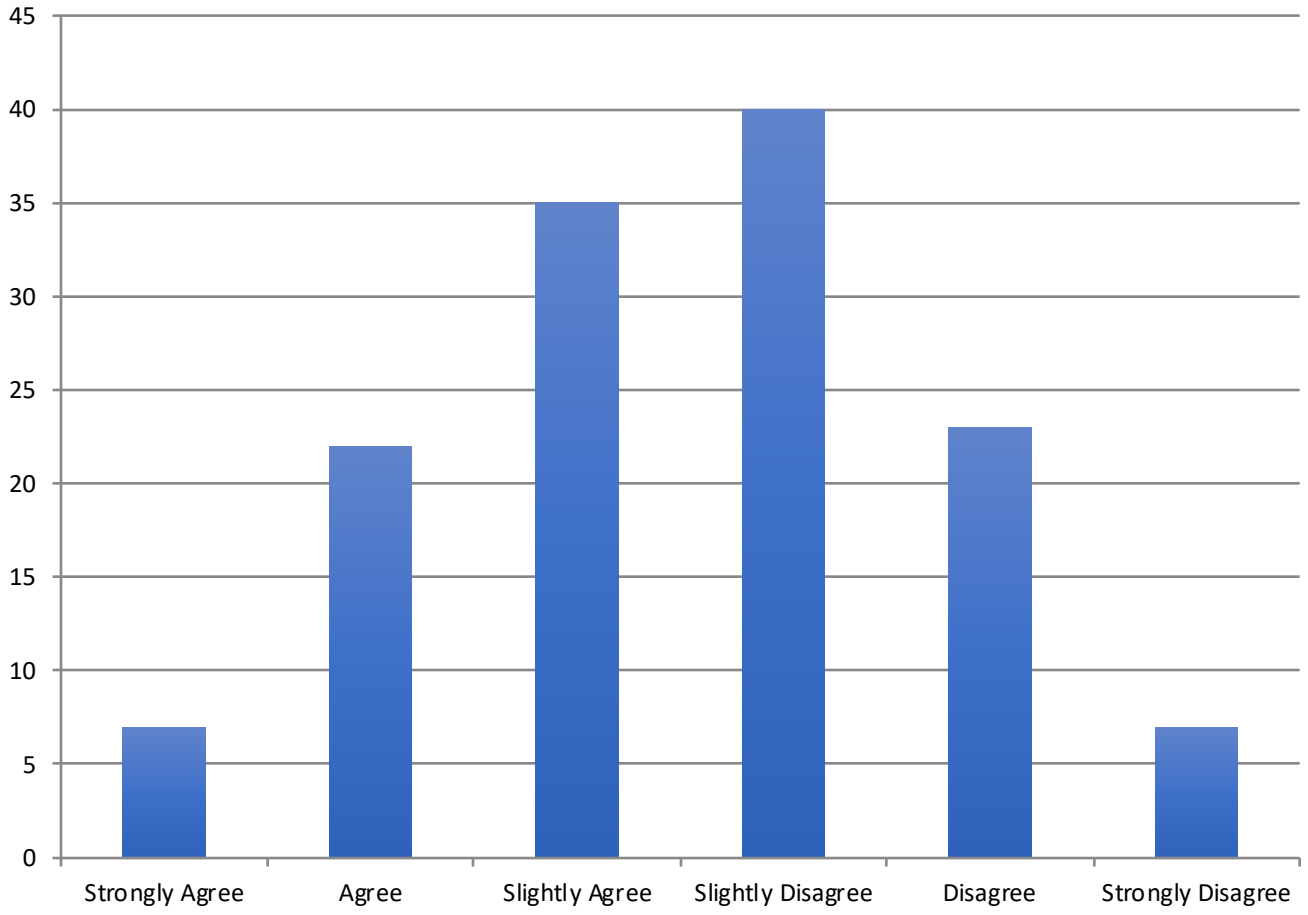
Mathematics involves creativity and new ideas.



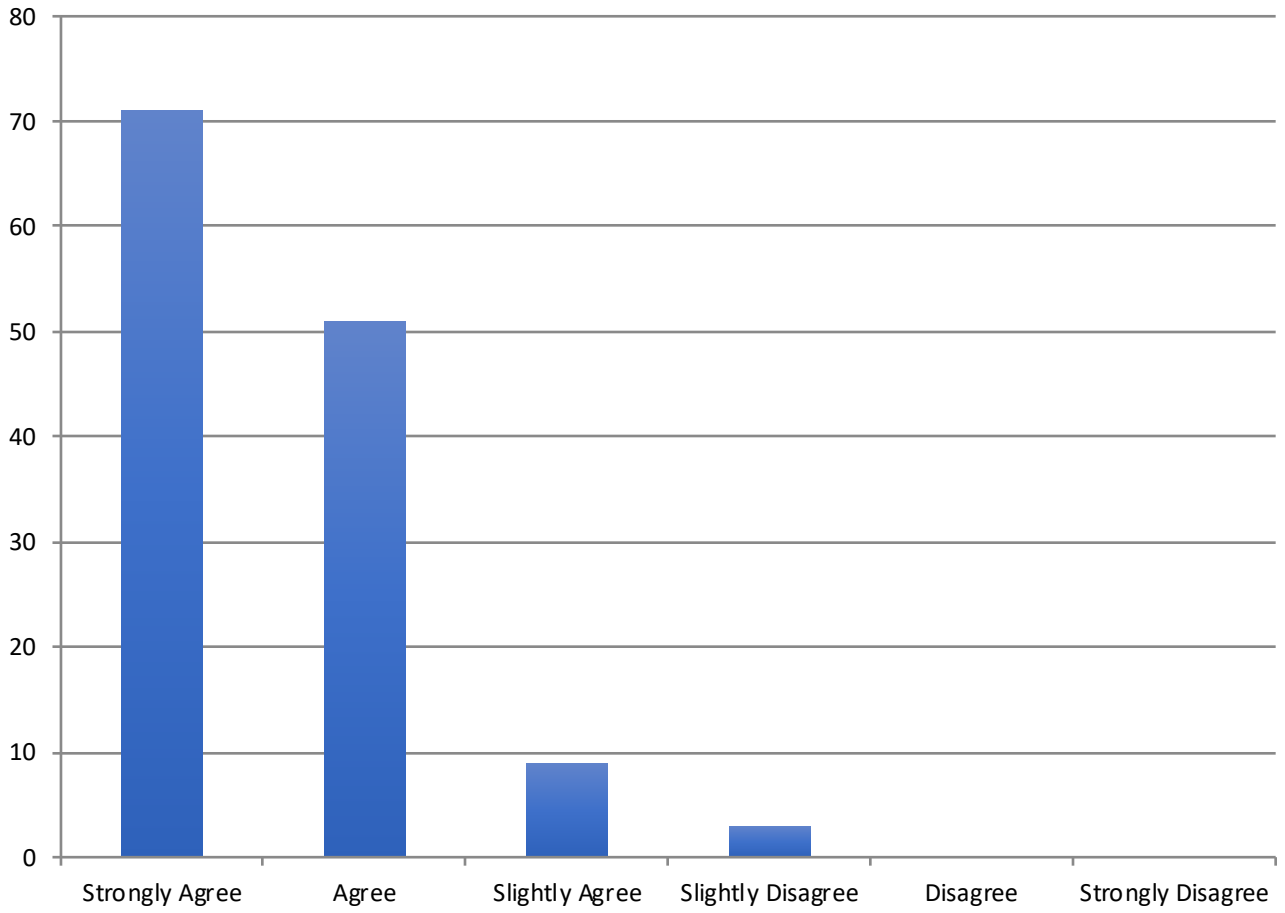
In mathematics many things can be discovered and tried by oneself



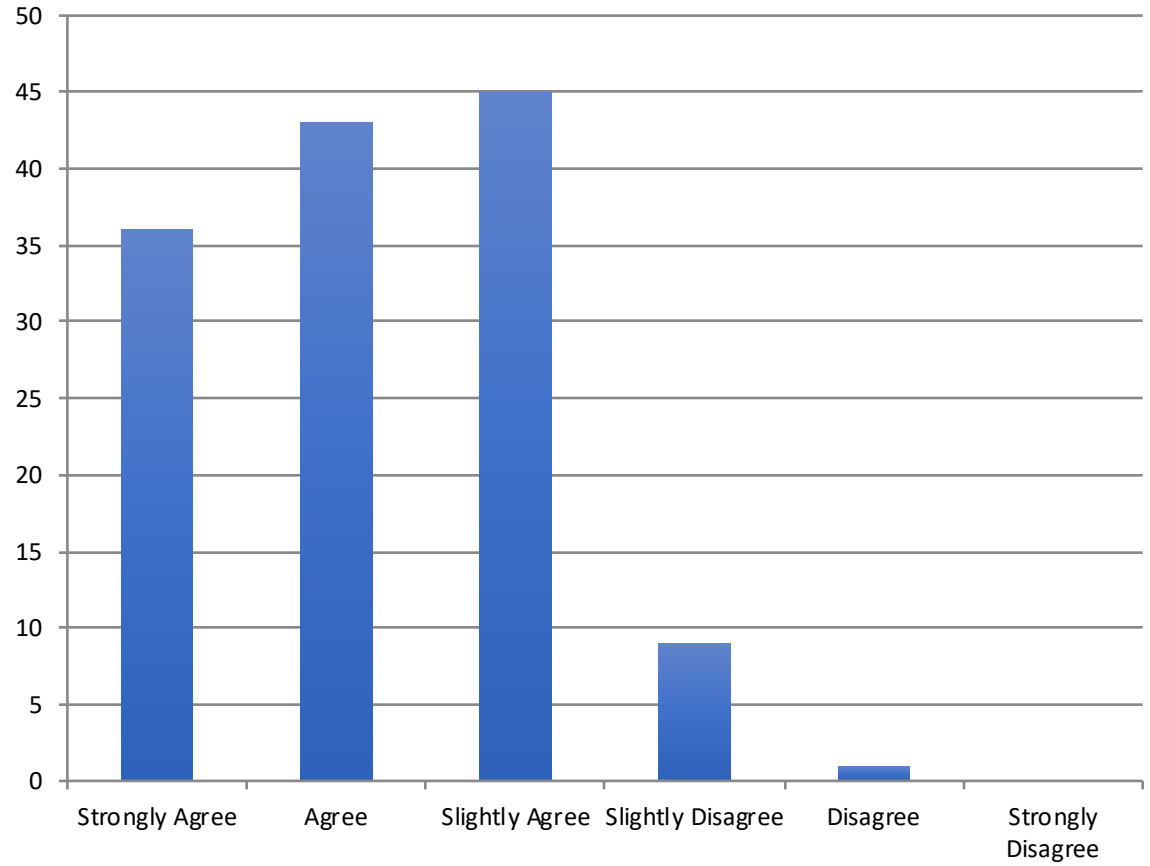
When solving mathematical tasks you need to know the correct procedure else you would be lost



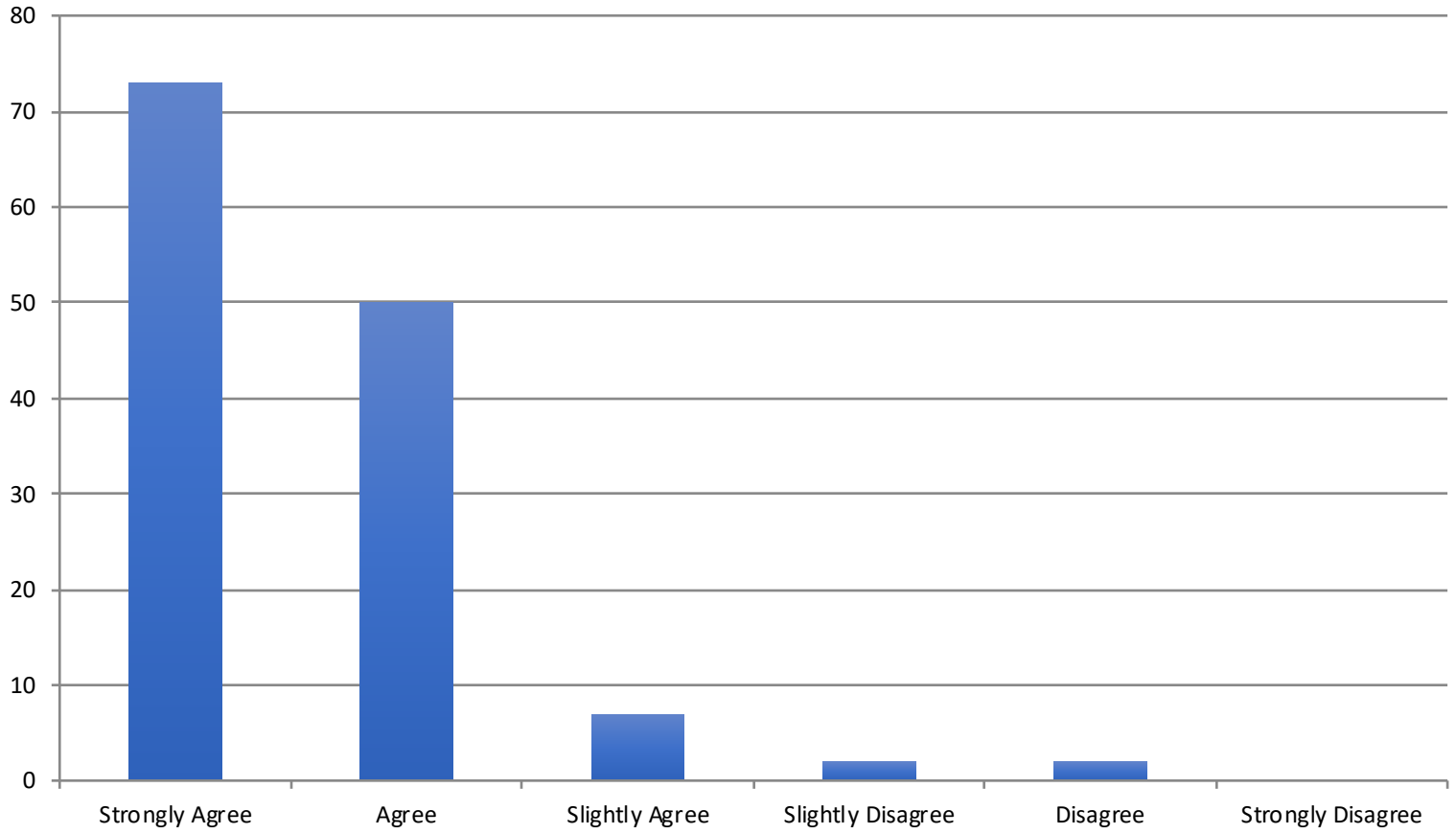
If you engage in mathematical tasks, you can discover new things (e.g. connections, rules, concepts)



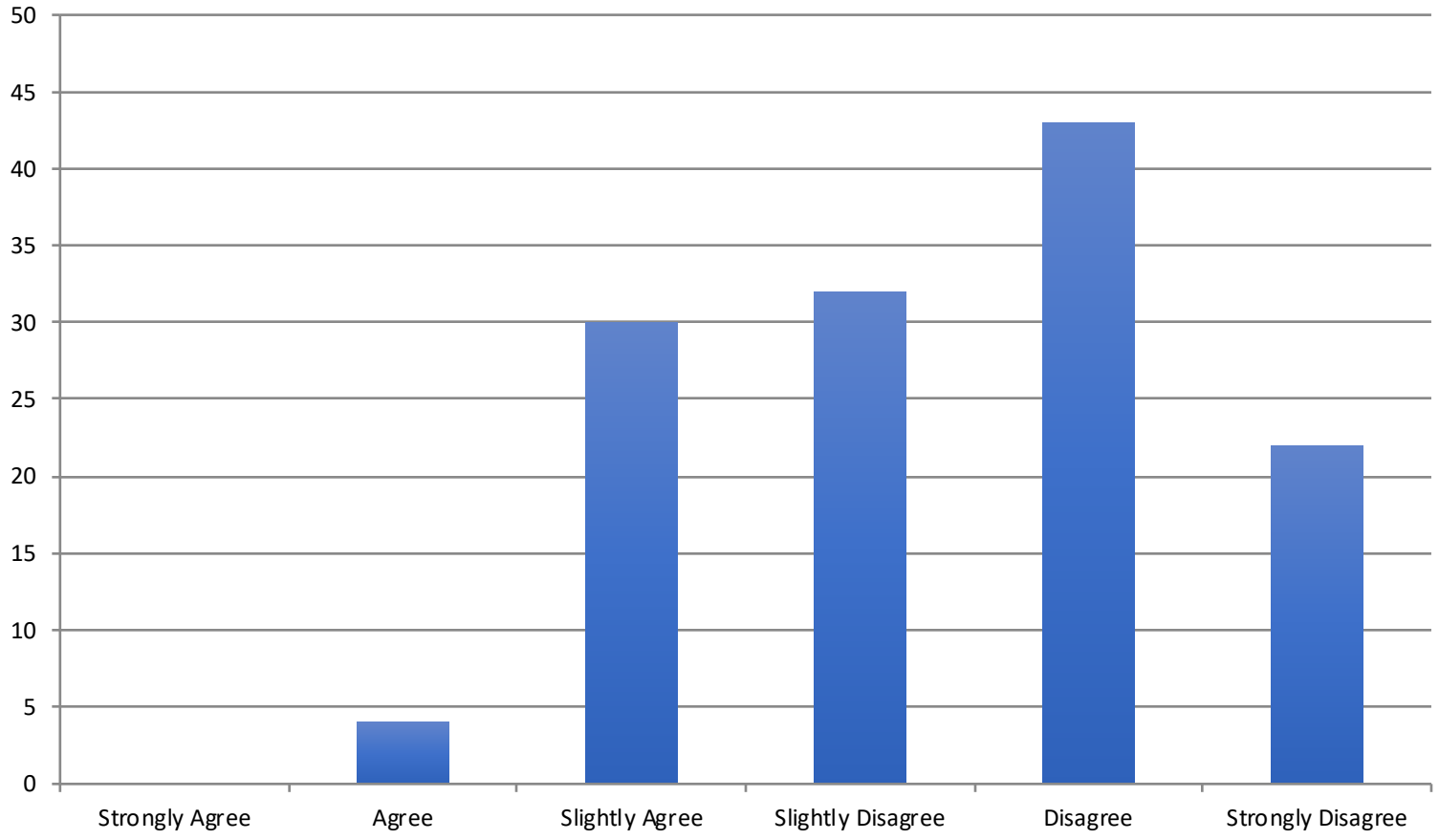
Fundamental to mathematics is its logical rigor and preciseness



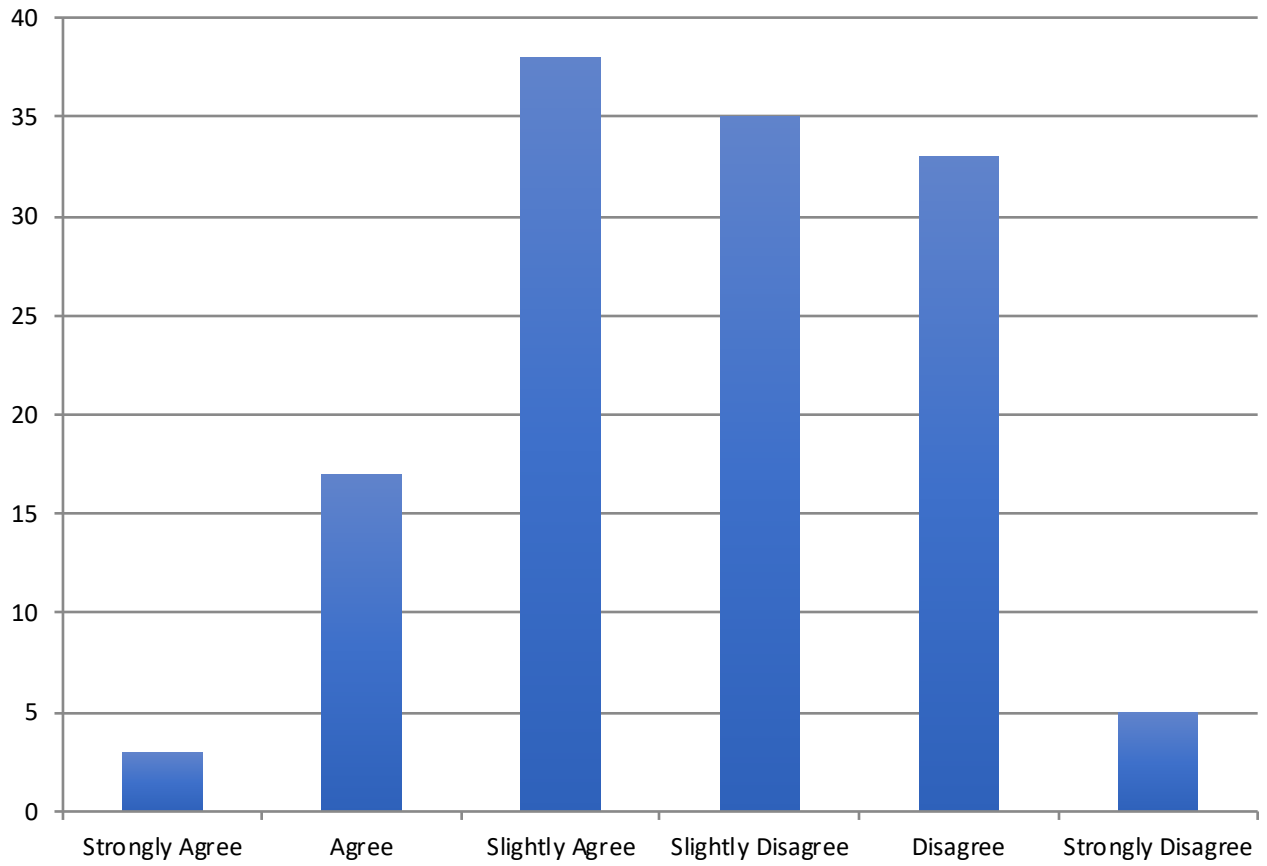
Many aspects of mathematics have practical relevance



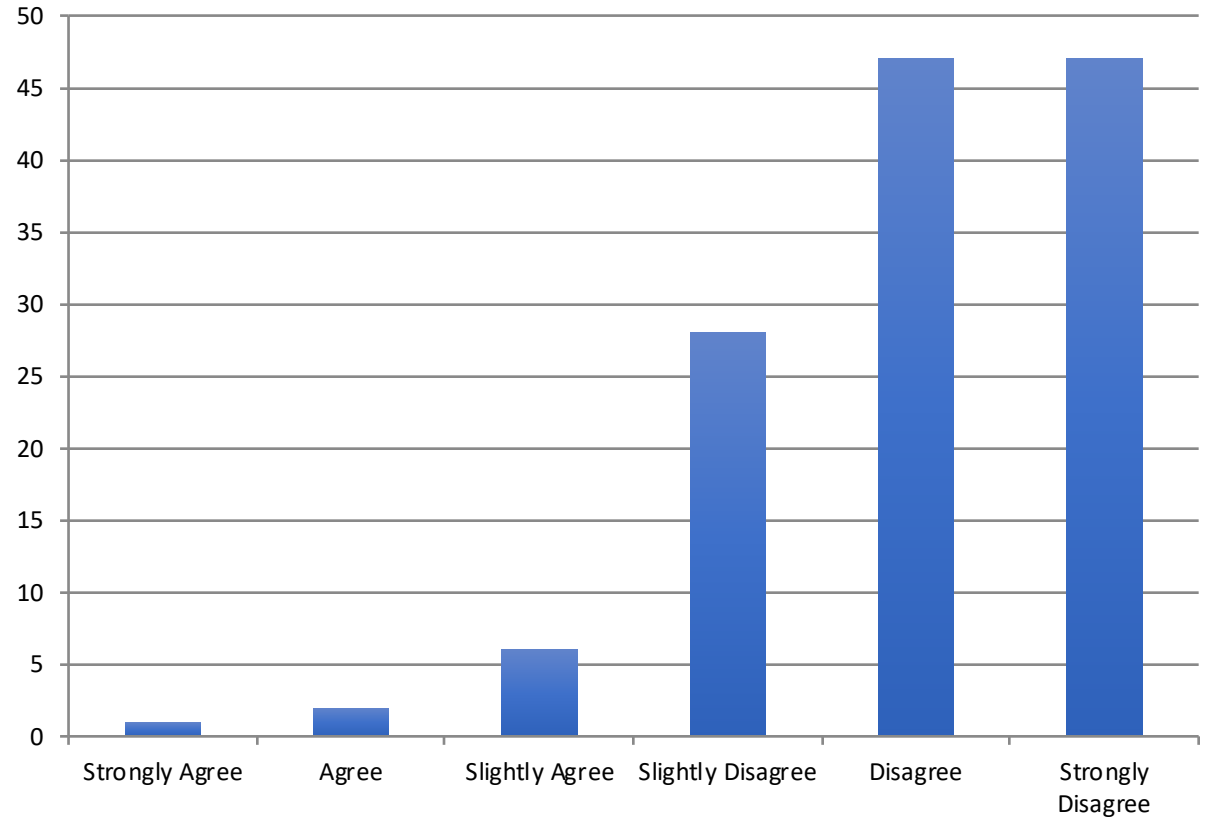
The best way to do well in mathematics is to memorise all the formulas



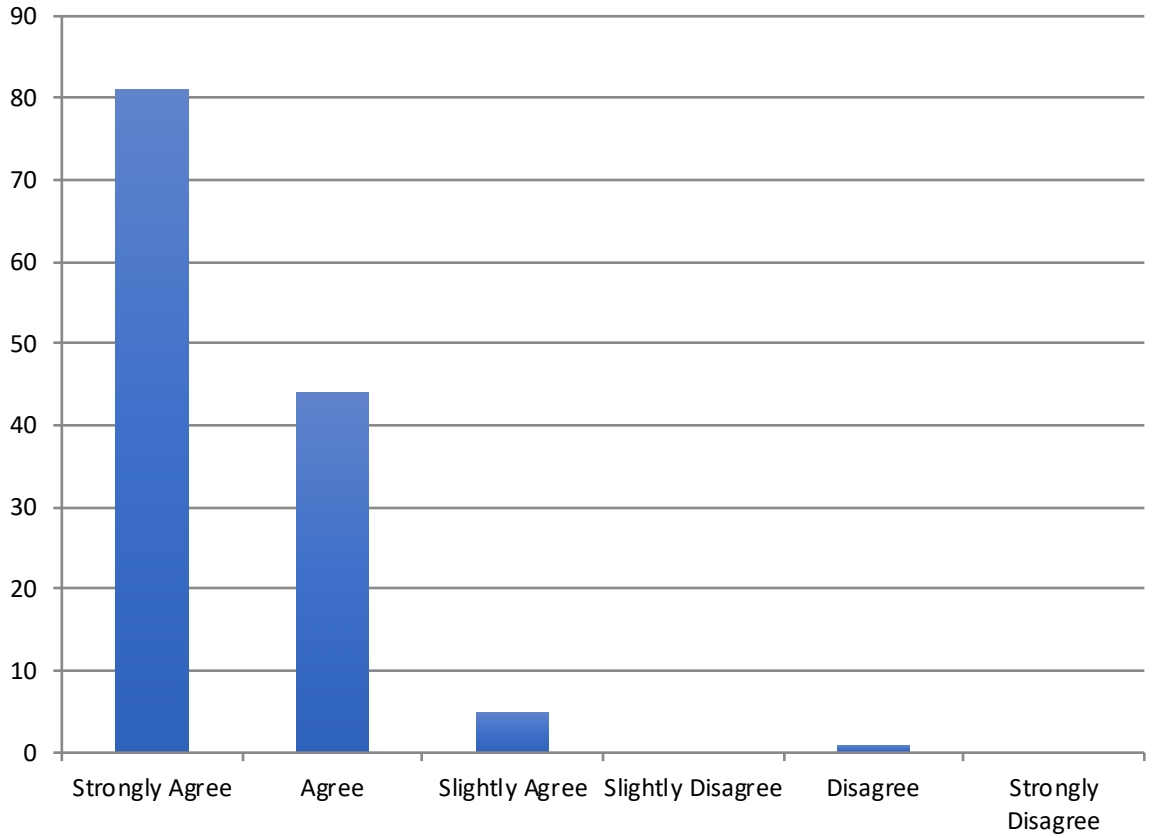
Pupils need to be taught exact procedures for solving mathematical problems



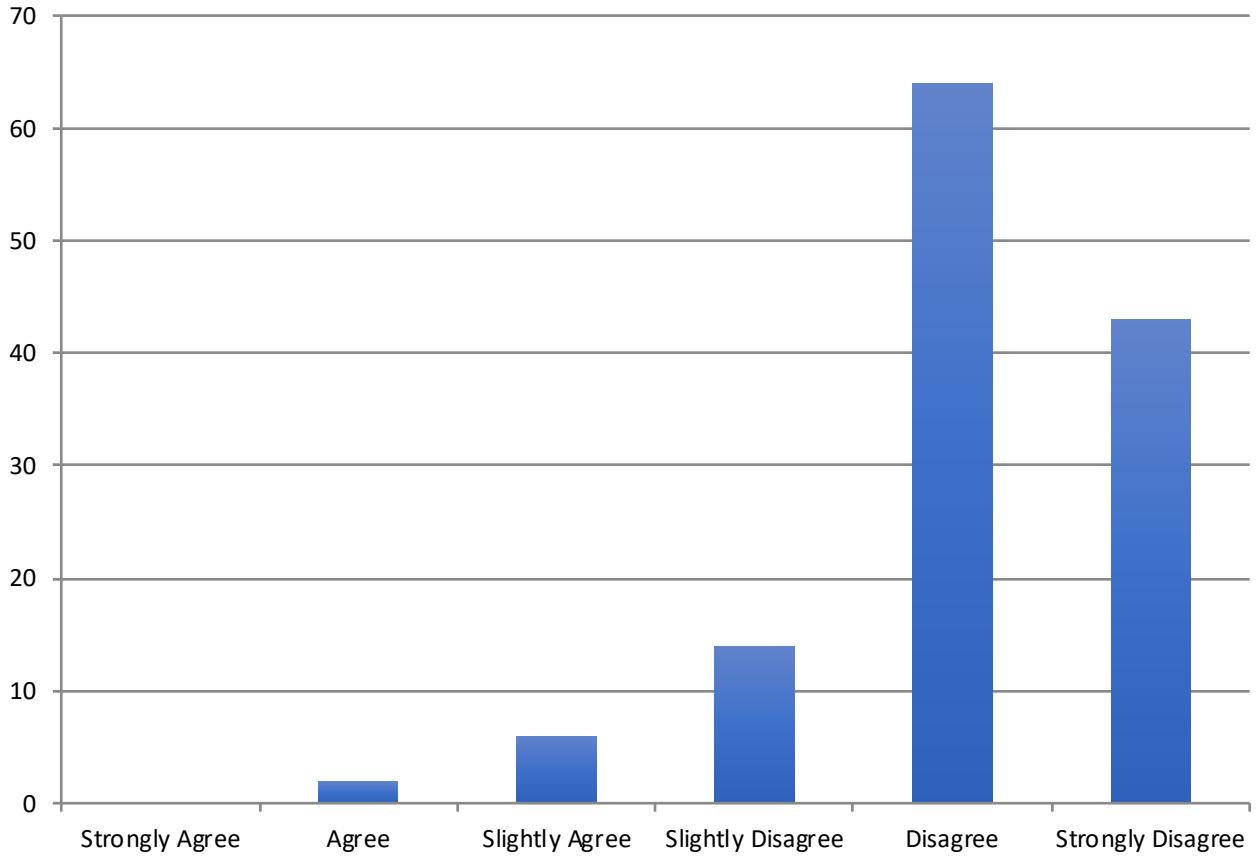
It doesn't really matter if you understand a mathematical problem, if you can get the right answer



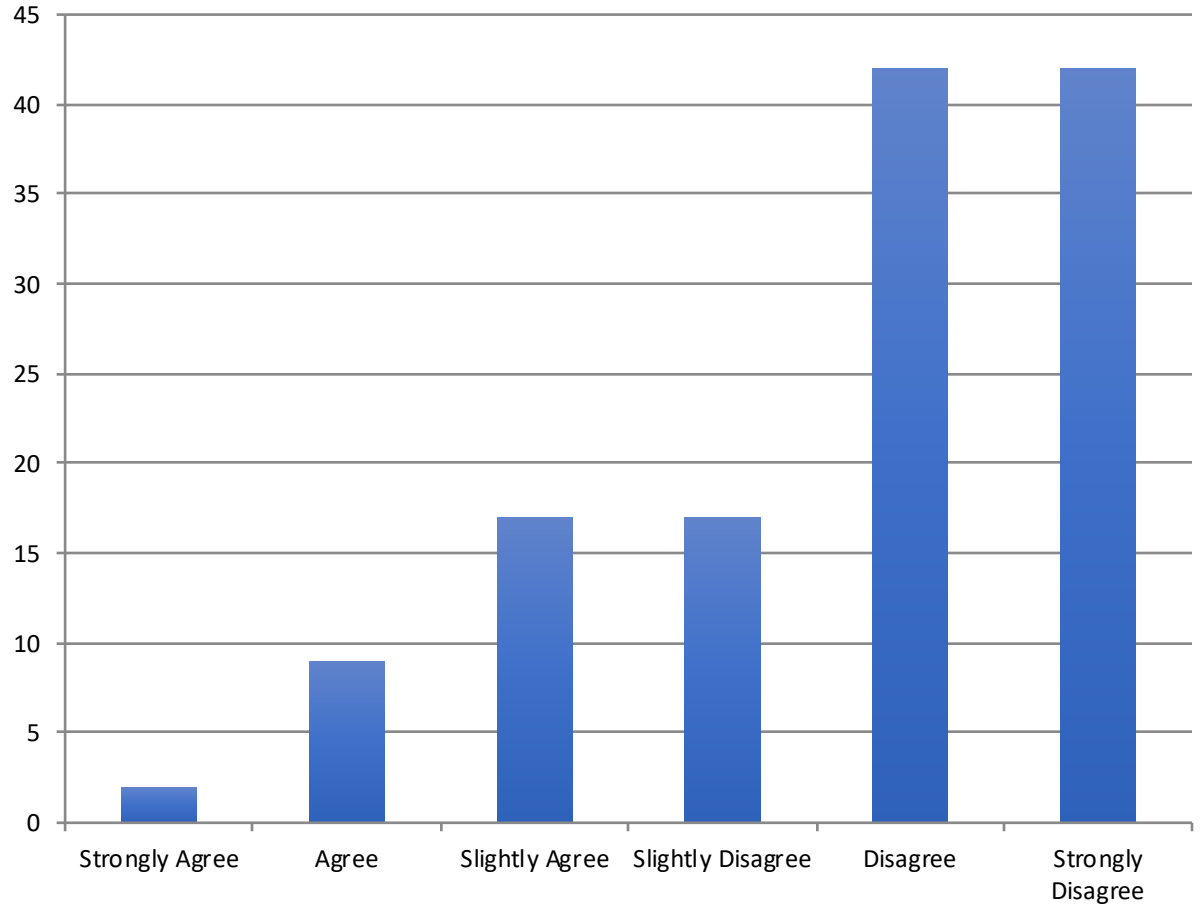
It is helpful for pupils to discuss different ways to solve particular problems

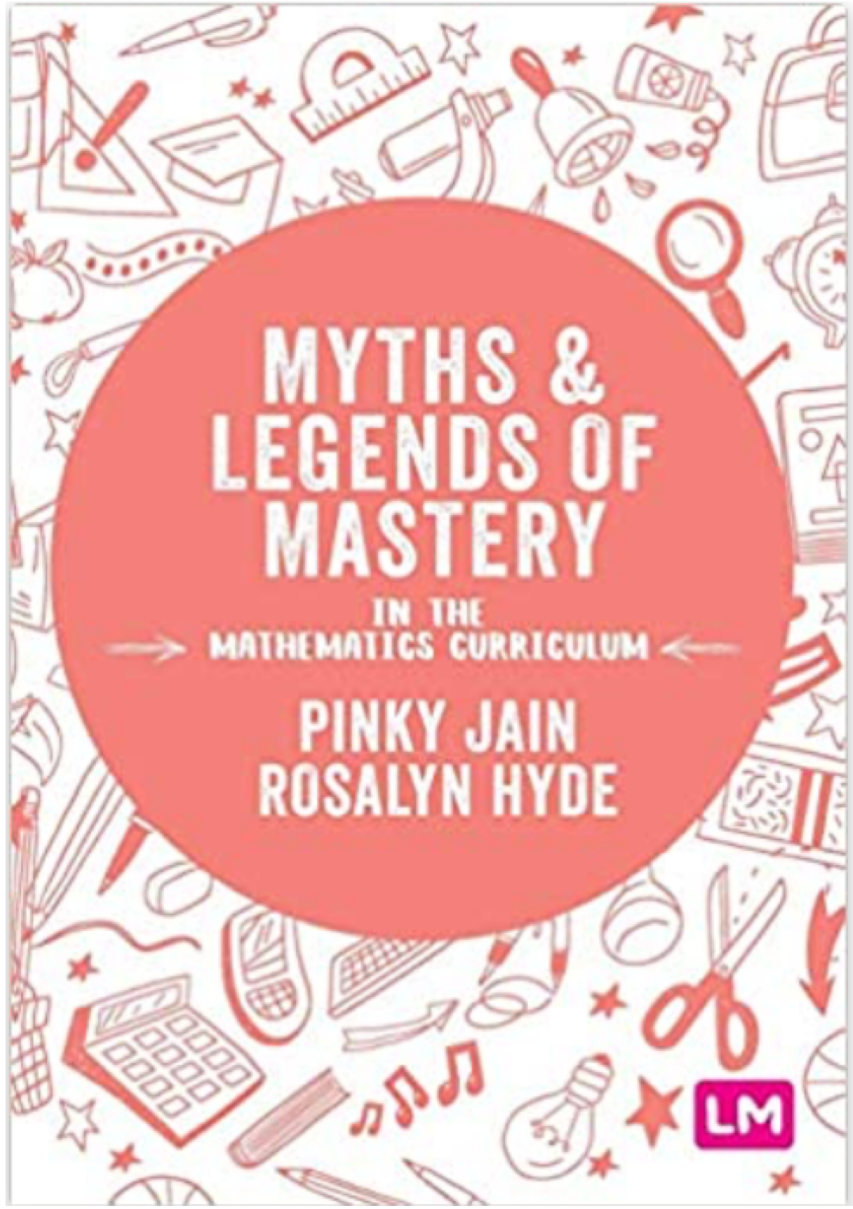


In primary/lower secondary mathematics? (Only the more able pupils can participate in multi-step problem solving activities)



In primary/lower secondary mathematics? (Mathematical ability is something that remains relatively fixed throughout a person's life)




MYTHS &
LEGENDS OF
MASTERY
IN THE
MATHEMATICS CURRICULUM
PINKY JAIN
ROSALYN HYDE
LM